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Some Extensions of Coefficient Problems for Bi-Univalent Ma-Minda Starlike and Convex Functions

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Abstract. Motivated by the works of H.M.Srivastava et al. [7], we introduce and investigate two new general subclasses $\mathcal{H}_{\mathcal{T}}(\varphi,\psi,\alpha)$,$\mathcal{H}_{\mathcal{T}}^{h,p}(\alpha)$ of bi-starlike and bi-convex of Ma-Minda type functions. Bounds on the first two coefficients $|a_2|$ and $|a_3|$ for functions in $\mathcal{H}_{\mathcal{T}}(\varphi,\psi,\alpha)$ and $\mathcal{H}_{\mathcal{T}}^{h,p}(\alpha)$ are given. The results here generalize and improve the corresponding earlier works done by Ali et al.[1] and Brannan et al.[2].

1. Introduction

Let $\mathcal{A}$ denote the class of functions which are analytic in $\Delta=\{z\in\mathbb{C}:|z|<1\}$ and normalized by the conditions: $f(0)=f'(0)-1=0$. Let Ω be the class consisting of functions $w(z):w(0)=0,|w(z)|<1,w(z)\in\mathcal{A}$. An analytic function f is subordinate to an analytic function g, written $f(z)\prec g(z)$, if there is an analytic function $w(z)\in\Omega$ such that $f(z)=g(w(z))$.

Ma and Minda [6] unified various subclasses of usual starlike and convex functions for which either of the quantity $\frac{zf'(z)}{f(z)}$ or $1+\frac{zf''(z)}{f'(z)}$ is subordinate to a superordinate function.

A function $f\in\mathcal{A}$ is said to be bi-univalent in Δ if both f and f^{-1} are univalent in Δ. Let $\mathcal{T}$ denote the class of bi-univalent functions. It is well known that every univalent function $f\in\mathcal{A}$ has an inverse f^{-1}, defined by $f^{-1}(f(z))=z(z\in\Delta)$ and $f(f^{-1}(\omega))=\omega(|\omega|<r_0(f);r_0(f)\geqslant\frac{1}{4})$, moreover, the function f^{-1} is given by

$$f^{-1}(\omega)=\omega-a_2\omega^2+(2a_2^2-a_3)\omega^3-(5a_2^3-5a_2a_3+a_4)\omega^4+... \tag{1.1}$$

Recently, H.M.Srivastava et al. [7] introduced two new subclasses of the function class $\mathcal{T}$ and obtained the bounds on coefficients $|a_2|$ and $|a_3|$ with them. Subsequently, several authors studied the coefficient problems for kinds of subclasses of functions related $\mathcal{T}$(see[1,3,5,8,9,10]).

Definition 1.1. *A function $f\in\mathcal{T}$ is said to be in the $\mathcal{H}_{\mathcal{T}}(\varphi,\psi,\alpha)$ if the following conditions are satisfied:*

$$\left(\frac{zf'(z)}{f(z)}\right)^{\alpha}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\alpha}\prec\varphi(z)$$

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and

$$\left(\frac{\omega g'(\omega)}{g(\omega)}\right)^{\alpha}\left(1+\frac{\omega g''(\omega)}{g'(\omega)}\right)^{1-\alpha} \prec \psi(\omega),$$

where $g(\omega) = f^{-1}(\omega), \alpha \geqslant 0$, φ *and* ψ *are analytic functions with positive real part in* Δ, *satisfying* $\varphi(0) = \psi(0) = 1, \varphi'(0) > 0, \psi'(0) > 0$.

Without loss of generality, let $\varphi(z)$ and $\psi(z)$ has a series expansion of the form

$$\varphi(z) = 1 + A_1 z + A_2 z^2 + A_3 z^3 + ..., (A_1 > 0) \tag{1.2}$$

and

$$\psi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + ..., (B_1 > 0). \tag{1.3}$$

In fact, we can note that $\mathcal{H}_{\mathcal{T}}(\varphi,\varphi,\alpha) \equiv \mathcal{L}_{\mathcal{T}}(\alpha,\varphi)$, $\mathcal{H}_{\mathcal{T}}(\varphi,\varphi,1) \equiv ST_{\mathcal{T}}(\varphi)$,$\mathcal{H}_{\mathcal{T}}(\varphi,\varphi,0) \equiv CV_{\mathcal{T}}(\varphi)$, where the classes $\mathcal{L}_{\mathcal{T}}(\alpha,\varphi), ST_{\mathcal{T}}(\varphi), CV_{\mathcal{T}}(\varphi)$ were defined by Ali et al. [1].

Definition 1.2. *A function* $f \in \mathcal{T}$ *is said to be in the* $\mathcal{H}_{\mathcal{T}}^{h,p}(\alpha)$ *if the following conditions are satisfied:*

$$\left(\frac{zf'(z)}{f(z)}\right)^{\alpha}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\alpha} \in h(\Delta)(z \in \Delta)$$

and

$$\left(\frac{\omega g'(\omega)}{g(\omega)}\right)^{\alpha}\left(1+\frac{\omega g''(\omega)}{g'(\omega)}\right)^{1-\alpha} \in p(\Delta)(\omega \in \Delta),$$

where $g(\omega) = f^{-1}(\omega), \alpha \geqslant 0$, *the functions* $h, p : \Delta \to \mathbb{C}$ *are constrained that*

$$\min\{\mathcal{R}(h(z)), \mathcal{R}((p(z))\} > 0 (z \in \Delta)$$

and $h(0) = p(0) = 1$.

In particular, taking $\alpha = 1$ and $\alpha = 0$ in Definition 1.2, we can obtain the subclasses $\mathcal{S}_{\mathcal{T}}(h,p)$ and $\mathcal{C}_{\mathcal{T}}(h,p)$ of bi-starlike of Ma-Minda type and bi-convex of Ma-Minda type functions, respectively.

In this paper, some coefficient problems on functions classes $\mathcal{H}_{\mathcal{T}}(\varphi,\psi,\alpha)$ and $\mathcal{H}_{\mathcal{T}}^{h,p}(\alpha)$ are discussed, which would generalize and improve some recent works and earlier works(see, for details, Remark 2.2 and Remark 2.6 below.).

2. Coefficient Bounds for the Functions Classes $\mathcal{H}_{\mathcal{T}}(\varphi,\psi,\alpha)$ and $\mathcal{H}_{\mathcal{T}}^{h,p}(\alpha)$

Theorem 2.1. *If the function* $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{H}_{\mathcal{T}}(\varphi,\psi,\alpha)$, *then*

$$|a_2| \leqslant \frac{\sqrt{2(A_1+B_1)}A_1B_1}{\sqrt{|2A_1^2B_1^2(\alpha^2-3\alpha+4)+2(2-\alpha)^2[A_1^2(B_1-B_2)+B_1^2(A_1-A_2)]|}}, \tag{2.1}$$

$$|a_3| \leqslant \frac{|\alpha^2-11\alpha+16|[B_1^2|A_2-A_1|+A_1^3]+|\alpha^2+5\alpha-8|A_1^2[|B_1-B_2|+B_1]}{|4A_1^2(3-2\alpha)(\alpha^2-3\alpha+4)|}. \tag{2.2}$$

Proof. Since $f(z) \in \mathcal{H}_\tau(\varphi, \psi, \alpha)$, then following the Define 1.1 , there are analytic functions $w_1(z), w_2(z) \in \Omega$, such that

$$\Big(\frac{zf'(z)}{f(z)}\Big)^\alpha\Big(1+\frac{zf''(z)}{f'(z)}\Big)^{1-\alpha} = \varphi(w_1(z)), z \in \Delta, \tag{2.3}$$

$$\Big(\frac{\omega g'(\omega)}{g(\omega)}\Big)^\alpha\Big(1+\frac{\omega g''(\omega)}{g'(\omega)}\Big)^{1-\alpha} = \psi(w_2(z)), \omega \in \Delta, \tag{2.4}$$

where $g(\omega) = f^{-1}(\omega)$. In order to prove this Theorem, we define the functions $\mathcal{Y}_1(z)$ and $\mathcal{Y}_2(z)$ as

$$\mathcal{Y}_1(z) = \frac{1+w_1(z)}{1-w_1(z)} = 1 + r_1 z + r_2 z^2 + \ldots \prec \frac{1+z}{1-z}, z \in \Delta, \tag{2.5}$$

$$\mathcal{Y}_2(z) = \frac{1+w_2(z)}{1-w_2(z)} = 1 + s_1 z + s_2 z^2 + \ldots \prec \frac{1+z}{1-z}, z \in \Delta. \tag{2.6}$$

It notes that $\mathcal{Y}_1(0) = \mathcal{Y}_2(0) = 1$, moreover, $\mathcal{Y}_1(z)$ and $\mathcal{Y}_2(z)$ have a positive real part in Δ. In fact,

$$w_1(z) = \frac{\mathcal{Y}_1(z)-1}{\mathcal{Y}_1(z)+1} = \frac{1}{2}\Big(r_1 z + (r_2 - \frac{r_1^2}{2})z^2 + \ldots\Big), \tag{2.7}$$

$$w_2(z) = \frac{\mathcal{Y}_2(z)-1}{\mathcal{Y}_2(z)+1} = \frac{1}{2}\Big(s_1 z + (s_2 - \frac{s_1^2}{2})z^2 + \ldots\Big). \tag{2.8}$$

So, following (1.2),(1.3),(2.7) and (2.8), we have

$$(\frac{zf'(z)}{f(z)})^\alpha(1+\frac{zf''(z)}{f'(z)})^{1-\alpha} = \varphi\Big(\frac{\mathcal{Y}_1(z)-1}{\mathcal{Y}_1(z)+1}\Big) = 1 + \frac{1}{2}A_1 r_1 z + \Big(\frac{1}{2}A_1(r_2 - \frac{r_1^2}{2}) + \frac{1}{4}A_2 r_1^2\Big)z^2 + \ldots, \tag{2.9}$$

$$(\frac{\omega g'(\omega)}{g(\omega)})^\alpha(1+\frac{\omega g''(\omega)}{g'(\omega)})^{1-\alpha} = \psi\Big(\frac{\mathcal{Y}_2(\omega)-1}{\mathcal{Y}_2(\omega)+1}\Big) = 1 + \frac{1}{2}B_1 s_1 \omega + \Big(\frac{1}{2}B_1(s_2 - \frac{s_1^2}{2}) + \frac{1}{4}B_2 s_1^2\Big)\omega^2 + \ldots \tag{2.10}$$

Moreover, following (1.1), we have

$$\Big(\frac{zf'(z)}{f(z)}\Big)^\alpha\Big(1+\frac{zf''(z)}{f'(z)}\Big)^{1-\alpha} = 1 + (2-\alpha)a_2 z + (2(3-2\alpha)a_3 + \frac{(\alpha-2)^2 - 3(4-3\alpha)}{2}a_2^2)z^2 + \ldots, \tag{2.11}$$

$$\Big(\frac{\omega g'(\omega)}{g(\omega)}\Big)^\alpha\Big(1+\frac{\omega g''(\omega)}{g'(\omega)}\Big)^{1-\alpha} = 1 - (2-\alpha))a_2\omega + ((8(1-\alpha) + \frac{1}{2}\alpha(\alpha+5))a_2^2 - 2(3-2\alpha)a_3)\omega^2 + \ldots \tag{2.12}$$

Now, equating the coefficients of (2.9)-(2.12), we have

$$(2-\alpha)a_2 = \frac{1}{2}A_1 r_1, \tag{2.13}$$

$$2(3-2\alpha)a_3 + \frac{(\alpha-2)^2 - 3(4-3\alpha)}{2}a_2^2 = \frac{1}{2}A_1(r_2 - \frac{r_1^2}{2}) + \frac{1}{4}A_2 r_1^2, \tag{2.14}$$

$$-(2-\alpha)a_2 = \frac{1}{2}B_1 s_1, \tag{2.15}$$

$$(8(1-\alpha) + \frac{1}{2}\alpha(\alpha+5))a_2^2 - 2(3-2\alpha)a_3 = \frac{1}{2}B_1(s_2 - \frac{s_1^2}{2}) + \frac{1}{4}B_2 s_1^2. \tag{2.16}$$

Following (2.13) and (2.15), we have

$$r_1 = -\frac{B_1}{A_1}s_1. \tag{2.17}$$

Also, from (2.14),(2.15) and (2.16), we get

$$a_2^2 = \frac{\frac{1}{2}A_1 r_2 + \frac{1}{2}B_1 s_2}{\frac{(\alpha-2)^2-3(4-3\alpha)}{2} + 8(1-\alpha) + \frac{1}{2}\alpha(\alpha+5) - (2-\alpha)^2\frac{B_2-B_1}{B_1^2} - (2-\alpha)^2\frac{A_2-A_1}{A_1^2}}$$

$$= \frac{A_1^2 B_1^2(A_1 r_2 + B_1 s_2)}{2A_1^2 B_1^2(\alpha^2-3\alpha+4) - 2(2-\alpha)^2[A_1^2(B_2-B_1)+B_1^2(A_2-A_1)]}.$$

Since $\mathcal{Y}_1(z) \in \mathcal{P}, \mathcal{Y}_2(z) \in \mathcal{P}$, where the $\mathcal{P}$ denotes the class of functions with positive real part. So $|r_1| \leqslant 2, |s_1| \leqslant 2$(see [4]), which gives us the desired estimate on $|a_2|$ as asserted in (2.1). Next, in order to find the bound on $|a_3|$, by following (2.14),(2.16) and (2.17), we get

$$2(3-2\alpha)a_3 = \frac{1}{2}A_1(r_2 - \frac{r_1^2}{2}) + \frac{1}{4}A_2 r_1^2 - \frac{\alpha^2+5\alpha-8}{2}.\frac{\frac{1}{2}B_1(s_2-\frac{s_1^2}{2}) + \frac{1}{4}B_2 s_1^2 + 2(3-2\alpha)a_3}{8(1-\alpha)+\frac{1}{2}\alpha(\alpha+5)}$$

So

$$a_3 = \frac{(\alpha^2-11\alpha+16)[B_1^2(A_2-A_1)s_1^2 + 2A_1^3 r_2] + (\alpha^2+5\alpha-8)A_1^2[(B_1-B_2)s_1^2 - 2B_1 s_2]}{16A_1^2(3-2\alpha)(\alpha^2-3\alpha+4)},$$

which gives us the desired estimate on $|a_3|$ as asserted in (2.2). This completes the proof of Theorem 2.1. □

Remark 2.2. *(1)Taking $\varphi(z) = \psi(z)$ in Theorem 2.1, we can obtain the results proved by Ali et al.[1, Theorem 2.4].*
(2)Taking $\varphi = \psi, \alpha = 1$ in Theorem 2.1, we can obtain the results proved by Ali et al.[1, Corollary 2.1].
(3)Taking $\varphi = \psi, \alpha = 0$ in Theorem 2.1, we can obtain the results proved by Ali et al.[1, Corollary 2.2].
(4)Taking $\varphi(z) = \psi(z) = (\frac{1+z}{1-z})^\gamma = 1 + 2\gamma z + 2\gamma^2 z^2 + ...(0 < \gamma \leqslant 1)$ and $\alpha = 1$ in Theorem 2.1, we can reduce to the estimates proved by Brannan et al.[2, Theorem 2.1].
(5)Taking $\varphi(z) = \psi(z) = \frac{1+(1-2\gamma)z}{1-z} = 1 + 2(1-\gamma)z + 2(1-\gamma)z^2 + ...(0 < \gamma \leqslant 1)$ and $\alpha = 1$ in Theorem 2.1, we can reduce to the estimates proved by Brannan et al.[2, Theorem 3.1].
(6)Taking $\varphi(z) = \psi(z) = \frac{1+(1-2\gamma)z}{1-z} = 1 + 2(1-\gamma)z + 2(1-\gamma)z^2 + ...(0 < \gamma \leqslant 1)$ and $\alpha = 0$ in Theorem 2.1, we can reduce to the estimates proved by Brannan et al.[2, Theorem 4.1].

Theorem 2.3. *If the function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{H}_{\mathcal{T}}^{h,p}(\alpha)$, then*

$$|a_2| \leqslant \sqrt{\frac{|h''(0)| + |p''(0)|}{2|\alpha^2-3\alpha+4|}}, \tag{2.18}$$

$$|a_3| \leqslant |\frac{\alpha^2-11\alpha+16}{2(\alpha^2-3\alpha+4)}||h''(0)| + |\frac{\alpha^2+5\alpha-8}{2(\alpha^2-3\alpha+4)}||p''(0)|. \tag{2.19}$$

Proof. Since $f \in \mathcal{H}_{\mathcal{T}}^{h,p}(\alpha)$, it follows from the Definition 1.2 that

$$(\frac{zf'(z)}{f(z)})^\alpha (1 + \frac{zf''(z)}{f'(z)})^{1-\alpha} = h(z)(z \in \Delta)$$

and

$$(\frac{\omega g'(\omega)}{g(\omega)})^\alpha (1 + \frac{\omega g''(\omega)}{g'(\omega)})^{1-\alpha} = p(\omega)(\omega \in \Delta),$$

where h and p satisfy the hypotheses of Definition 1.2. Here, let the functions $h(z)$ and $p(\omega)$ have the following series expansions:

$$h(z) = 1 + h_1 z + h_2 z^2 + ... \tag{2.20}$$

and

$$p(\omega) = 1 + p_1\omega + p_2\omega^2 + ..., \tag{2.21}$$

respectively. Moreover, following (1.1), we have

$$(\frac{zf'(z)}{f(z)})^{\alpha}(1 + \frac{zf''(z)}{f'(z)})^{1-\alpha} = 1 + (2-\alpha)a_2 z + (2(3-2\alpha)a_3 + \frac{(\alpha-2)^2 - 3(4-3\alpha)}{2}a_2^2)z^2 + ..., \tag{2.22}$$

$$(\frac{\omega g'(\omega)}{g(\omega)})^{\alpha}(1 + \frac{\omega g''(\omega)}{g'(\omega)})^{1-\alpha} = 1 - (2-\alpha))a_2\omega + ((8(1-\alpha) + \frac{1}{2}\alpha(\alpha+5))a_2^2 - 2(3-2\alpha)a_3)\omega^2 + ... \tag{2.23}$$

Now, equating the coefficients of (2.20)-(2.23), we have

$$(2-\alpha)a_2 = h_1, \tag{2.24}$$

$$2(3-2\alpha)a_3 + \frac{(\alpha-2)^2 - 3(4-3\alpha)}{2}a_2^2 = h_2, \tag{2.25}$$

$$-(2-\alpha)a_2 = p_1, \tag{2.26}$$

$$(8(1-\alpha) + \frac{1}{2}\alpha(\alpha+5))a_2^2 - 2(3-2\alpha)a_3 = p_2. \tag{2.27}$$

From (2.25) and (2.27), We find that

$$(\alpha^2 - 3\alpha + 4)a_2^2 = h_2 + p_2,$$

So

$$a_2^2 = \frac{h_2 + p_2}{\alpha^2 - 3\alpha + 4},$$

which gives us the desired estimate on $|a_2|$ as asserted in (2.18). Next, in order to find the bound on $|a_3|$, by subtracting (2.27) from (2.25), we get

$$4(3-2\alpha)a_3 + (8\alpha - 12)a_2^2 = h_2 - p_2.$$

So

$$a_3 = \frac{\alpha^2 - 11\alpha + 16}{\alpha^2 - 3\alpha + 4}h_2 - \frac{\alpha^2 + 5\alpha - 8}{\alpha^2 - 3\alpha + 4}p_2.$$

This completes the proof of Theorem 2.3. □

Corollary 2.4. *If the function* $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{S}_{\mathcal{T}}(h, p)$*, then*

$$|a_2| \leqslant \frac{1}{2}\sqrt{|h''(0)| + |p''(0)|}, \quad |a_3| \leqslant \frac{3}{2}|h''(0)| + \frac{1}{2}|p''(0)|.$$

Proof. Taking $\alpha = 1$ in Theorem 2.3, it is easily seen to yield Corollary 2.4. □

Corollary 2.5. *If the function* $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in C_{\mathcal{T}}(h, p)$*, then*

$$|a_2| \leqslant \sqrt{\frac{|h''(0)| + |p''(0)|}{8}}, \quad |a_3| \leqslant 2|h''(0)| + |p''(0)|.$$

Proof. Taking $\alpha = 0$ in Theorem 2.3, it is easily seen to yield Corollary 2.5. □

Remark 2.6. *(1) Taking* $h(z) = p(z) = (\frac{1+z}{1-z})^{\gamma} = 1 + 2\gamma z + 2\gamma^2 z^2 + ...(0 < \gamma \leqslant 1)$ *in Theorem 2.3, then we have*

$$|a_2| \leqslant \frac{2\gamma}{\sqrt{|\alpha^2 - 3\alpha + 4|}}, \quad |a_3| \leqslant 2\gamma^2 \left|\frac{\alpha^2 - 11\alpha + 16}{\alpha^2 - 3\alpha + 4}\right| + 2\gamma^2 \left|\frac{\alpha^2 + 5\alpha - 8}{\alpha^2 - 3\alpha + 4}\right|.$$

Furthermore, if $\alpha = 1$, *then we have* $|a_2| \leqslant \sqrt{2}\gamma$, $|a_3| \leqslant 8\gamma^2$, *in fact, this result is an improvement of Brannan et al.[2, Theorem 2.1].*
(2) Taking $h(z) = p(z) = \frac{1+(1-2\gamma)z}{1-z} = 1 + 2(1-\gamma)z + 2(1-\gamma)z^2 + ...(0 < \gamma \leqslant 1)$ *in Theorem 2.3, then we have*

$$|a_2| \leqslant 2\sqrt{\frac{1-\gamma}{|\alpha^2 - 3\alpha + 4|}}, \quad |a_3| \leqslant 2(1-\gamma)\left|\frac{\alpha^2 - 11\alpha + 16}{\alpha^2 - 3\alpha + 4}\right| + 2(1-\gamma)\left|\frac{\alpha^2 + 5\alpha - 8}{\alpha^2 - 3\alpha + 4}\right|.$$

Furthermore, if $\alpha = 1$, *then we have* $|a_2| \leqslant \sqrt{2(1-\gamma)}$, $|a_3| \leqslant 8(1-\gamma)$, *in fact, this result is an improvement of Brannan et al.[2, Theorem 3.1].*

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