



Stable Attribute Reduction for Neighborhood Rough Set

Shaochen Liang^a, Xibei Yang^{a,b}, Xiangjian Chen^a, Jingzheng Li^a

^aSchool of Computer, Jiangsu University of Science and Technology, Zhenjiang, Jiangsu, 212003, P.R. China

^bIntelligent Information Processing Key Laboratory of Shanxi Province, Shanxi University, Taiyuan, Shanxi, 030006, P.R. China

Abstract. In neighborhood rough set theory, traditional heuristic algorithm for computing reducts does not take the stability of the selected attributes into account, it follows that the performances of the reducts may not be good enough if the perturbations of data occur. To fill the gap, the mechanism of acquiring the most significant attribute is realized by two steps in the reduction process: firstly, several important attributes are derived in each iteration based on several radii which are close to the given radius for computing reduct; secondly, the most significant attribute is selected from them by a voting strategy. The experiments verify that such method can effectively improve the stabilities of the reducts, and it does not require too much attributes for constructing the reducts.

1. Introduction

Neighborhood rough set firstly specifies a radius for each sample. Then for a given sample, those samples within the scope determined by the radius are considered as its neighbors. By such distance strategy, neighborhood rough set can deal with continuous data or even mixed data [8].

Similar to other generalized rough sets, attribute reduction [2, 10, 19, 23] plays a fundamental role in neighborhood rough set. Recently, a great number of evaluation criteria such that neighborhood decision error rate and conditional entropy, have been deeply explored in neighborhood rough set [7, 9, 24]; how to accelerate the speeds of computations of the reducts was investigated in Refs. [3, 14]; some scholars even managed to apply attribute reduction to other research domains or practical problems [12, 13, 22]. Nevertheless, to our best knowledge, the stabilities of the reducts in neighborhood rough set were rarely studied. Based on the results shown in Refs. [1, 5, 11], such topic is important and should be carefully addressed.

Up to now, the voting strategies in ensemble learning have been demonstrated useful for stable feature selection [4, 15, 16, 18]. Since attribute reduction is a kind of feature selection, it is possible to introduce ensemble voting into attribute reduction.

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Email addresses: lshc940302@163.com (Shaochen Liang), yangxibei@hotmail.com (Xibei Yang), cxj831209@163.com (Xiangjian Chen), maxlijingzheng@163.com (Jingzheng Li)

2. Preliminary knowledge

2.1. Neighborhood rough set

Without loss of generality, a neighborhood information system can be represented as $S = \langle U, A \rangle$, in which U is samples' set, and A is condition attributes' set.

Given a neighborhood information system S , $\forall B \subseteq A$, a neighborhood binary relation is then defined as $\delta_B = \{(x, y) \in U \times U : \Delta_B(x, y) \leq \delta\}$, where $\Delta_B : U \times U \rightarrow [0, 1]$ is the distance function which returns the normalized Euclidean distance [20] of two samples with respect to B and $\delta \in [0, 1]$ is the given radius. Furthermore, if $(x, y) \in \delta_B$, then $\delta_B(x, y) = 1$; otherwise $\delta_B(x, y) = 0$. It follows that δ_B satisfies reflexive and symmetry and then δ_B can be used to characterize the similarity among samples. Specially, $\delta_B(x) = \{y \in U : \delta_B(x, y) = 1\}$ contains all the samples which are similar to x .

From the viewpoint of granular computing, $\delta_B(x)$ is not only regarded as an information granule, but also can construct approximations. Therefore, $\forall X \subseteq U$, Hu et al. [8] defined the neighborhood rough set of X as $[\underline{\delta}_B(X), \overline{\delta}_B(X)]$ in which $\underline{\delta}_B(X) = \{x \in U : \delta_B(x) \subseteq X\}$ is lower approximation of X and $\overline{\delta}_B(X) = \{x \in U : \delta_B(x) \cap X \neq \emptyset\}$ is upper approximation of X .

2.2. Neighborhood decision system

To process data with classification information, the neighborhood information system needs to be expanded such that $NDS = \langle U, A, D \rangle$ known as the neighborhood decision system. In NDS , D is one decision attribute. It can partition U into some collections with respect to the classification information. In rough set theory, each of these collections is called a decision class. Without loss of generality, the k -th decision class is denoted by X_k .

Assume that there are p decision classes in NDS , then $\forall B \subseteq A$, the lower and upper approximations of D with respect to B are $\underline{\delta}_B(D) = \bigcup_{k=1}^p \underline{\delta}_B(X_k)$ and $\overline{\delta}_B(D) = \bigcup_{k=1}^p \overline{\delta}_B(X_k)$, where $\underline{\delta}_B(X_k) = \{x \in U : \delta_B(x) \subseteq X_k\}$ and $\overline{\delta}_B(X_k) = \{x \in U : \delta_B(x) \cap X_k \neq \emptyset\}$.

3. Attribute reduction

In this section, we will present the details of the traditional heuristic algorithm and our proposed ensemble heuristic algorithm for computing reducts.

3.1. Heuristic algorithm

Definition 1. Given a neighborhood decision system $NDS = \langle U, A, D \rangle$, $\forall B \subseteq A$, the approximate quality of NDS with respect to B is

$$\gamma_B^\delta(D) = \frac{|\underline{\delta}_B(D)|}{|U|}, \tag{1}$$

where δ is the radius employed by the neighborhood rough set and $|X|$ is the cardinality of the set X .

Definition 2. Given a neighborhood decision system $NDS = \langle U, A, D \rangle$, $\forall B \subseteq A$, B is an approximate quality reduct of A iff

- (1) $\gamma_A^\delta(D) = \gamma_B^\delta(D)$;
- (2) $\forall C \subset B, \gamma_C^\delta(D) \neq \gamma_B^\delta(D)$.

Definition 3. Given a neighborhood decision system $NDS = \langle U, A, D \rangle$, if $B \subset A$, then $\forall a \in A \setminus B$, its significance is

$$\text{Sig}^\delta(a, B, D) = \gamma_{B \cup \{a\}}^\delta(D) - \gamma_B^\delta(D), \tag{2}$$

where δ is the radius employed by the neighborhood rough set.

The Eq. (2) reflects the variation of the approximate quality when attribute a is added into B . Obviously, a higher value of $\text{Sig}^\delta(a, B, D)$ indicates that the attribute a is more significant for B , because the approximate quality witnesses a remarkable increase. By $\text{Sig}^\delta(a, B, D)$, a forward heuristic attribute reduction algorithm [6, 8] is designed:

Algorithm 1. Heuristic Algorithm (HA)

Inputs: $NDS = \langle U, A, D \rangle$, radius δ , and approximate quality threshold ϵ ;

Outputs: An approximate quality reduct B .

1. $B \leftarrow \emptyset$;
 2. Compute $\gamma_A^\delta(D)$;
 3. **Do**
 - 1) $\forall a \in A \setminus B$, compute $\text{Sig}^\delta(a, B, D)$; // $\gamma_\emptyset^\delta(D) = 0$
 - 2) Select b such that $\text{Sig}^\delta(b, B, D) = \max \{\text{Sig}^\delta(a, B, D) : \forall a \in A \setminus B\}$;
 - 3) $B \leftarrow B \cup \{b\}$;
 - 4) Compute $\gamma_B^\delta(D)$;
 - Until** $\gamma_A^\delta(D) - \gamma_B^\delta(D) \leq \epsilon \cdot \gamma_A^\delta(D)$;
 4. **Return** B .
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In Algorithm 1, step 3 selects the most significant attribute b in each iteration. Then it checks whether the set of the selected attributes satisfies the definition of the approximate quality reduct. Since the definition is too strict, an approximate quality threshold $\epsilon \in [0, 1]$ is employed in the algorithm.

3.2. Ensemble heuristic algorithm

As a whole, the key step of Algorithm 1 is to select the most significant attribute. Nevertheless, the changing of data is inevitable in real world applications [21], e.g., the condition attribute values of some samples are updated. Then, the most significant attribute may no longer be the one obtained before the variation of data, and it follows that a completely different reduct occurs. For such reason, an algorithm that can generate a more stable reduct is desired.

The reason why Algorithm 1 fails to obtain a stable reduct is that the factor of the variation of data is not considered in its steps of searching the most significant attribute, so we firstly need to simulate the variation of data and then attempt to take such factor into account.

Although it is impossible to predict how data will change, it is known that neighborhood rough set can adjust its radius δ to deal with different data. Therefore, a slight variation of the radius δ employed by neighborhood rough set can simulate the variation of data.

The next step is to concentrate on the computation of the most significant attribute. By Eq. (2), we can obtain several important attributes when different radii are respectively adopted. Then inspired by the voting strategy in ensemble learning, an ensemble heuristic algorithm is proposed. The detailed steps of it are presented as follows.

Algorithm 2. Ensemble Heuristic Algorithm (EHA)

Inputs: $NDS = \langle U, A, D \rangle$, radius δ , approximate quality threshold ϵ , stepsize ω and a positive integer α ;

Outputs: An approximate quality reduct B .

1. $B \leftarrow \emptyset$;
2. Compute $\gamma_A^\delta(D)$;
3. **Do**
 - 1) Temporary pool $T \leftarrow \emptyset$;
 - 2) **For** $j = -\alpha; j \leq \alpha; j = j + \omega$
 - $\delta' \leftarrow \delta + \omega \cdot j$;
 - $\forall a \in A \setminus B$, compute $\text{Sig}^{\delta'}(a, B, D)$; // $\gamma_\emptyset^{\delta'}(D) = 0$

Select b such that $\text{Sig}^{\delta'}(b, B, D) = \max \{\text{Sig}^{\delta'}(a, B, D) : \forall a \in A \setminus B\}$;
 Add b into T ;
End For
 3) Select attribute c by ensemble voting in T ;
 4) $B \leftarrow B \cup \{c\}$;
 5) Compute $\gamma_B^{\delta}(D)$;
Until $\gamma_A^{\delta}(D) - \gamma_B^{\delta}(D) \leq \epsilon \cdot \gamma_A^{\delta}(D)$;
4. Return B .

Algorithm 2 firstly adopts a set of radii which are close to the given radius δ . Then the important attributes separately derived from different radii are stored in the temporary pool T . Subsequently, in each looping, voting strategy regards the attribute c that has maximal frequency of occurrences in T as the most significant attribute. In this way, a reduct is finally generated.

Note that the radius utilized by the terminal condition of the iteration in step 3 is δ . It implies that the reduct generated by our proposed algorithm also maintains the change of approximate quality of the neighborhood decision system in a small range determined by ϵ , while the difference compared to Algorithm 1 lies in how to select the most significant attribute.

4. Experiment

In this section, we will analyze whether Algorithm 2 is effective by the stabilities and lengths of reducts.

4.1. Data sets

To test the performance of our proposed algorithm, several UCI data sets are employed to conduct the experiment. Their information is displayed in Table 1. Note that the ‘‘Attributes’’ column in Table 1 refers to the condition attributes.

Table 1: Data sets description

ID	Data set name	Samples	Attributes	Decision classes
1	Climate Model Simulation Crashes	540	20	2
2	Connectionist Bench (Sonar, Mines vs. Rocks)	208	60	2
3	Forest Type Mapping	523	27	4
4	Ionosphere	351	34	2
5	Parkinsons	195	23	7
6	Wine	178	13	3

4.2. Experimental configurations

For both Algorithm 1 and Algorithm 2, the approximate quality threshold is set by $\epsilon = 0.05$ and the radius of neighborhood rough set is set by $\delta = 0.18, 0.21, \dots, 0.30$. In Algorithm 2, the stepsize is set by $\omega = 0.03$ and the positive integer is set by $\alpha = 2$.

4.3. Experimental results and experimental analyses

To describe the stability of the reduct, all samples in U are firstly divided into t groups of equal sizes. Then, in each time, $t-1$ groups of them are combined and used to compute a reduct. Therefore, t reducts are finally obtained.

If these groups are respectively denoted by U_1, U_2, \dots, U_t , the reduct derived from U_2, U_3, \dots, U_t is denoted by red_1 , the reduct derived from U_1, U_3, \dots, U_t is denoted by red_2, \dots , and the reduct derived from U_1, U_2, \dots, U_{t-1} is denoted by red_t , then the stability of the reduct [17] can be defined as:

$$ReductStability = \frac{2}{t \cdot (t - 1)} \sum_{i=1}^{t-1} \sum_{j=i+1}^t \frac{|red_i \cap red_j|}{|red_i \cup red_j|}. \tag{3}$$

Obviously, a higher value of *ReductStability* indicates a better stability. Moreover, in our experiment, t is set to 5.

The stabilities of reducts separately generated from Algorithm 1 and Algorithm 2 on all the data sets are displayed in Tables 2-7. In these tables, the better performances between the two algorithms are highlighted in boldface. In addition, the average lengths of reducts separately generated from Algorithm 1 and Algorithm 2 on all the data sets can be observed in Tables 8-13. Note that each value in these tables refers to the average length of the t reducts.

Table 2: Stability of reduct on Climate Model Simulation Crashes

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	0.5487	0.4057	0.3996	0.4057	0.3637
Algorithm 2	0.4456	0.4435	0.4322	0.4206	0.5744

Table 3: Stability of reduct on Connectionist Bench (Sonar, Mines vs. Rocks)

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	0.1874	0.2111	0.2468	0.3001	0.3383
Algorithm 2	0.1802	0.2722	0.3642	0.4262	0.4710

Table 4: Stability of reduct on Forest Type Mapping

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	0.8248	0.8843	0.8865	0.9184	0.9333
Algorithm 2	0.8608	0.9444	0.9229	0.9259	0.9407

Table 5: Stability of reduct on Ionosphere

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	0.5820	0.6897	0.6977	0.8223	0.8290
Algorithm 2	0.7329	0.8066	0.7519	0.7992	0.8263

Table 6: Stability of reduct on Parkinsons

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	0.8382	0.7280	0.7718	0.7766	0.8191
Algorithm 2	0.8032	0.7760	0.7845	0.8978	0.8748

Table 7: Stability of reduct on Wine

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	0.6102	0.7361	0.7968	0.9692	0.9692
Algorithm 2	0.7091	0.8083	0.8615	1.0000	0.9692

Table 8: Length of reduct on Climate Model Simulation Crashes

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	6.2000	7.0000	7.0000	8.0000	8.0000
Algorithm 2	6.8000	7.0000	7.8000	8.0000	8.4000

Table 9: Length of reduct on Connectionist Bench (Sonar, Mines vs. Rocks)

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	10.6000	12.0000	14.0000	17.6000	22.2000
Algorithm 2	10.6000	13.4000	16.6000	19.0000	24.2000

Table 10: Length of reduct on Forest Type Mapping

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	22.6000	24.2000	24.8000	25.8000	25.8000
Algorithm 2	23.8000	24.4000	24.8000	25.8000	26.0000

Table 11: Length of reduct on Ionosphere

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	23.0000	24.8000	23.8000	28.4000	27.6000
Algorithm 2	24.0000	27.8000	26.8000	28.6000	28.2000

Table 12: Length of reduct on Parkinsons

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	15.8000	15.8000	17.0000	19.0000	18.0000
Algorithm 2	16.4000	15.8000	17.4000	18.2000	19.4000

Table 13: Length of reduct on Wine

Radius δ	0.18	0.21	0.24	0.27	0.30
Algorithm 1	9.0000	10.4000	11.4000	12.8000	12.8000
Algorithm 2	9.2000	10.2000	12.0000	13.0000	12.8000

With a careful observation, the following conclusions can be drawn.

1. In most cases, the reducts generated from our proposed algorithm are much more stable than the reducts generated from Algorithm 1. Take Climate Model Simulation Crashes data set as an example, when the radius $\delta = 0.30$, $ReductStability = 0.3637$ in Algorithm 1. By contrast, $ReductStability = 0.5744$ in Algorithm 2.
2. The reducts generated from our proposed algorithm are only slightly longer than the reducts generated from Algorithm 1. Take Ionosphere data set as an example, when the radius $\delta = 0.18$, the average length of the reducts generated from Algorithm 1 is 23.0000. By contrast, the average length of the reducts generated from Algorithm 2 is 24.0000.

5. Conclusion

In this paper, we firstly developed an ensemble heuristic algorithm for neighborhood rough set to obtain a stable reduct. Then, a novel approach to the selection of the most significant attribute was presented in detail. Finally, its remarkable advantages over the traditional heuristic algorithm were demonstrated through experiments.

Our future research will concentrate on the following two aspects:

1. Only the stability of approximate quality reduct is explored in this paper. Whether classification based reducts such that neighborhood decision error rate reduct derived from our proposed algorithm are stable remains obscure.
2. Classification performance of the classifier based on the reduct generated from our proposed algorithm will be further investigated.

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