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Attribute Reduction of Relative Knowledge Granularity in Intuitionistic Fuzzy Ordered Decision Table

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Abstract.
For the moment, the attribute reduction algorithm of relative knowledge granularity is very important research areas. It provides a new viewpoint to simplify feature set. Based on the decision information is unchanged, fast and accurate deletion of redundant attributes, which is the meaning of attribute reduction. Distinguishing ability of attribute sets can be well described by relative knowledge granularity in domain. Therefore, how to use the information based on relative knowledge granularity to simplify the calculation of attribute reduction. It is an important direction of research. For increasing productiveness and accuracy of attribute reduction, in this paper we investigate attribute reduction method of relative knowledge granularity in intuitionistic fuzzy ordered decision table(IFODT). More precisely, we redefine the granularity of knowledge and the relative knowledge granularity by ordered relation. And their relevant properties are proved. On the premise that the decision results remain unchanged, in order to accurately calculate the relative importance of any condition attributes about the decision attribute sets, the conditional attribute of internal and external significance are designed by relative knowledge granularity. And some important properties of relative attribute significance are proved. Therefore, we determine the importance of conditional attributes based on the size of the relative attribute significance. In the aspect of computation, the corresponding algorithm is designed and time complexity of algorithm is calculated. Moreover, the attribute reduction model of relative knowledge granularity of efficiency and accuracy is proved by test. Last, the validity of algorithm is demonstrated by an case about IFODT.

1. Introduction

Rough set theory(RST) is proposed by Poland mathematician Pawlak[41], it It is used for dealing with inaccurate and incomplete information systems. RST is a significant mathematic tool in the areas of data mining[8][38]and decision theory[25]. Pawlak mainly based on the object between the indistinguishability of the theory of object clustering into basic knowledge domain. By using the basic knowledge of the upper and lower approximation[19] to describe the data object uncertainty, which derives the concept of classification or decision rule. Related researches spread many field, for instance, machine learning[15], multi-source information Fusion[26][27], cloud computing[1], knowledge discovery[30][28], decision-theoretic[29], biological information processing[20], artificial intelligence[9], neural computing[16] and so on. Attribute reduction model is an vital aspect of RST. It can be known as a kind of specific feature selection. One can

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select useful features from a given decision system based on RST. Attribute reduction keeps the separating capacity of primordial decision information system for the targets from the universe .

In 1986, the intuitionistic fuzzy set theory(IFST)[12] was presented by Atanassov. IFST is the extension of fuzzy set theory. With the improvement of IFST has been widely applied in many fields for information analysis[5][6] and pattern recognition[2][7]. IF set is compatible with the three aspects of membership and non membership and hesitation. Therefore, IF sets are more comprehensive and practical than the traditional fuzzy sets in dealing with vagueness and uncertainty. A new mathematical theory[3][23] is constituted by IFST and RST, it can be used as a tool of dealing with data set. Studies of the combination of information system and IF set theory is being accepted as a vigorous research direction to rough set theory. Based on intuitionistic fuzzy information system(IFIS)[31], there have been number of researchers focused on the theory of IF set. Xu et al[24].studied intuitionistic fuzzy ordered information systems. Recently, Zhang et al[33]. studied the generalized preponderant of a rough set model system by two new dominance relations. Moreover, based on IF decision information systems, the attribute reduction models of dominance are tested by the above two rough set models. Based on IFIS, Xu et al[17]. made use of the definite integrals of multiplicative to make decisions. On the basis of the above research, they studied the models of indefinite integrals. The basic properties of calculus are proved. The concrete formulas of calculating definite integrals are designed from different point of view, Last, the related properties of definite integral are also proved.

Attribute reduction is an important component of RST[32][4][42][13]. Many scholars have studied the attribute reduction model for many years, and found that the minimum reduction is a NP-Hard problem. Thus, Building efficient model of attribute reduction has become one of the important fields of research on RST[41]. The so-called attributes reduction is to delete the redundant attributes without weakening the classification ability of the knowledge base. Removing unnecessary attributes through knowledge reduction can simplify knowledge representation, reduce computational complexity, and do not lose necessary information.In recent years, many scholars have conducted in-depth research on attribute reduction, and have achieved a lot of results[39][40][14][22]. Based on the dynamic data environment, Jing and Li et al.[37] constructs novel attribute reduction models and related algorithms by using knowledge granularity. Wang et al[34]. proposed a generalized information system. Based on generalized information system, the criterion of attribute core and relative attribute reduction model are studied. Further extended attribute reduction model. In the context of generalized attribute concepts, Jing and Li et al.[35] extended the model of attribute reduction from knowledge granularity angles. In order to overcome that it is hard to update reduct when the large-scale data vary dynamically. Jing and Li et al.[36] developed an attribute reduction algorithm with a multi-granulation view to discover reduct of large-scale information systems. Based on ordered decision information systems, Jia[10] studied attribute reduction model of relative knowledge granulation. Ln the general decision information system, Xu et al.[11]constructed an attribute reduction model by using relative knowledge granularity. In order to solve the existing measurement problems of attribute reduction model, Teng et al.[18] proposed a new measure of attribute quality by using the discernibility. Based on incomplete data set, Wu[21] proposed plausibility reduce and belief reduce By using plausibility function and belief function. On this basis, Wu also constructed relative plausibility reduction and relative belief reduction. Based on incomplete data set, Wu further improved the attribute reduction method from the perspective of plausibility function and belief function. How to use the information based on relative knowledge granularity to simplify the calculation of features reduction. It is an important direction of research in IFODT. To overcome this problem, we defines a measure of relative knowledge granularity and its monotonicity is proved in IFODT. Then the significance of relative attribute is defines. In IFODT, the relative knowledge granulation attribute reduction algorithm is designed. And some important properties of relative attribute significance are proved. Finally, We analyze the time complexity of attribute reduction algorithms. We analyze the effectiveness of the algorithm through a case study.

Other parts of the paper is arranged as follows. Section 2 provides the basic concept of intuitionistic fuzzy (IF) sets, intuitionistic fuzzy decision table(IFDT), ordered relation of IFDT. In section 3, A new relative knowledge granulation is proposed in IFODT. And the attribute reduction model of relative knowledge granularity is proposed. Thus, two new uncertainty measures based on relative knowledge granularity are defined and its monotonicity are proved. The significance of relative attribute is redefined. And a relative knowledge granulation attribute reduction algorithm is designed in IFODT. And some important properties of relative attribute significance are proved. In section 4, the attribute reduction model of relative knowledge granularity of efficiency and accuracy is proved by test. The effectiveness of algorithm is demonstrated through an case about IFODT. And the validity of the conclusion is proved by examples. At last, we conclude our research in section 5.

2. Preliminaries

For more convenience, in this portion, some related definitions and theorems are introduced. Including intuitionistic fuzzy sets(IFS), IFDT and ordered relation of IFDT. More details can be found in[12][24].

Definition 2.1[24] Suppose O is a non empty classic objects set. The three reorganization in O like $\mathcal{A} = \{\langle x, \mu_{\mathcal{A}}(o_i), \nu_{\mathcal{A}}(o_i)\rangle | o_i \in O\}$ meets the following three points.

(1)$\mu_{\mathcal{A}} \rightarrow [0,1]$ indicates that the element of O belongs to the $\mathcal{A}$ membership degree.

(2)$\nu_{\mathcal{A}} \rightarrow [0,1]$ indicates that the element of O not belongs to the $\mathcal{A}$ membership degree.

(3)$0 \leq \mu_{\mathcal{A}}(o_i) + \nu_{\mathcal{A}}(o_i) \leq 1$.

$\mathcal{A}$ is called an IFS on the O.

Related operations of IFS.

Suppose $\mathcal{A} = \{\langle o_i, \mu_{\mathcal{A}}(o_i), \nu_{\mathcal{A}}(o_i)\rangle | o_i \in O\} \in IF(X)$, $\mathcal{B} = \{\langle o_i, \mu_{\mathcal{B}}(o_i), \nu_{\mathcal{B}}(o_i)\rangle | o_i \in O\} \in IF(O)$.

$$\mathcal{A} \subseteq \mathcal{B} \Leftrightarrow \mu_{\mathcal{A}}(o_i) \leq \mu_{\mathcal{B}}(o_i), \nu_{\mathcal{A}}(o_i) \geq \nu_{\mathcal{B}}(o_i), \forall o_i \in O;$$

$$\mathcal{A} \cap \mathcal{B} = \{\langle o_i, min\{\mu_{\mathcal{A}}(o_i), \mu_{\mathcal{B}}(o_i)\rangle\}, max\{\nu_{\mathcal{A}}(o_i), \nu_{\mathcal{B}}(o_i)\}\rangle | o_i \in O\};$$

$$\mathcal{A} \cup \mathcal{B} = \{\langle o_i, max\{\mu_{\mathcal{A}}(o_i), \mu_{\mathcal{B}}(o_i)\rangle\}, min\{\nu_{\mathcal{A}}(o_i), \nu_{\mathcal{B}}(o_i)\}\rangle | o_i \in O\};$$

$$\mathcal{A}^c = \{\langle o_i, \nu_{\mathcal{A}}(o_i), \mu_{\mathcal{A}}(o_i)\rangle | o_i \in O\}.$$

Definition 2.2[24] $I = (O, CS \cup DS, V, f)$ are called an IFDT,

$O = \{o_1, o_2, \cdots, o_n\}$ where an arbitrary $x_i \in U$ is an object, $i = 1, 2, \cdots, n$;

$CS = \{c_1, c_2, \cdots, c_p\}$ where an arbitrary $c_j \in CS$ is an condition attribute, $j = 1, 2, \cdots, p$;

$DS = \{d_1, d_2, \cdots, d_q\}$ where an arbitrary $d_k \in DT$ is a decision attribute $k = 1, 2, \cdots, q$;

$V = \bigcup\limits_{c \in CS} V_c$ and the value of the attribute c is V_c;

$f : O \times CS \rightarrow V$, the f is called a function. So $f(o, c) \in V_c$, for each $c \in CS, o \in O$, where V_c is called an IFS valued about O. This is $f(o, c) = \langle \mu_c(o), \nu_c(o)\rangle$, for every $c \in CS$.

Definition 2.3 Suppose $I^{\succeq} = (O, CS \cup DS, V, f)$ is an IFODT, $c \in CS$ be called a criteria. The ordered relation $\succeq_c$ is established on the value of c. For $o_i, o_j \in U$, $o_i \succeq_c o_j \in O$ indicates that o_i is superior to o_j about criteria c. In other words, o_j is at best as good as o_i about c. Therefore, We can say that o_i is better than o_j about criteria c. In IFODT, the ordered relations are as follows:

$$f(o_i, c) \leq f(o_j, c) \Leftrightarrow [\mu_c(o_i) \leq \mu_c(o_j), \nu_c(o_i) \geq \nu_c(o_j)],$$

$$f(o_i, c) \geq f(o_j, c) \Leftrightarrow [\mu_c(o_i) \geq \mu_c(o_j), \nu_c(o_i) \leq \nu_c(o_j)].$$

Let $A \subseteq CS$. Based on IFODT, The ordered relation $\mathcal{R}_A^{\succeq}$ of conditional attribute set A is defined as follows:

$$\mathcal{R}_A^{\succeq} = \{(o_i, o_j) \in O \times O | o_i \succeq_A o_j\} = \{(o_i, o_j) \in O \times O | \mu_c(o_i) \geq \mu_c(o_j), \nu_c(o_i) \leq \nu_c(o_j), \forall c \in A\}.$$

Based on IFODT, $[o_i]_A^{\succeq}$ is called dominance class for A by $\mathcal{R}_A^{\succeq}$.

$$[o_i]_A^{\succeq} = \{o_j \in O | (o_j, o_i) \in \mathcal{R}_A^{\succeq}\} = \{o_j \in O | \mu_c(o_j) \geq \mu_c(o_i), \nu_c(o_j) \leq \nu_c(o_i), \forall c \in A\}$$

$$O/\mathcal{R}_A^{\succeq} = \{[o]_A^{\succeq} | o_i \in O\}.$$

The $O/\mathcal{R}_A^{\succeq}$ is called a cover of O.

Proposition 2.1 Suppose $I^{\succeq} = (O, CS \cup DS, V, f)$ is an IFODT, $A, B \subseteq CS$, we have the following results.

(1) For any $A, B \subseteq CS$, $B \subseteq A \Rightarrow \mathcal{R}^{\succeq}_A \subseteq \mathcal{R}^{\succeq}_B$, $[o_i]^{\succeq}_A \subseteq [o_i]^{\succeq}_B$

(2) $o_j \in [o_i]^{\succeq}_A \Rightarrow [o_j]^{\succeq}_A \subseteq [o_i]^{\succeq}_A$, $[o_i]^{\succeq}_A = \bigcup\{[o_i]^{\succeq}_A | o_j \in [o_i]^{\succeq}_A\}$.

(3)for all $c \in A$, $\mu_c(o_i) = \mu_c(o_j)$ and $\nu_c(o_i) = \nu_c(o_j) \Rightarrow [o_i]^{\succeq}_A = [o_j]^{\succeq}_A$.

These properties are easily proved.

Proposition 2.2 Suppose $I^{\succeq} = (O, CS \cup DS, V, f)$ is an IFODT, and $A, B \subseteq CS$, then $[o]^{\succeq}_{A\cup B} = [o]^{\succeq}_A \cap [o]^{\succeq}_B$. Generally, the partial ordering on a set is defined as follows:

$$B \preceq A \Leftrightarrow \forall o \in O, [o]^{\succeq}_B \subseteq [o]^{\succeq}_A.$$

If $B \preceq A$, B is said to be finer than A; If $B \preceq A$ and $B \neq A$, $B \prec A$, B is said to be strict finer than A. In fact, $B \prec A \Leftrightarrow \forall o \in O, [o]^{\succeq}_B \subseteq [o]^{\succeq}_A$, and existence $o' \in U$, bring $[o']^{\succeq}_B \subset [o']^{\succeq}_A$.

Definition 2.4[24] Suppose $I^{\succeq} = (O, CS \cup DS, V, f)$ is an IFODT. For any $X \subseteq O$ and $A \subseteq CS$,

$$\underline{\mathcal{R}}^{\succeq}_A(X) = \{o \in O | [o]^{\succeq}_A \subseteq X\};$$

$$\overline{\mathcal{R}}^{\succeq}_A(X) = \{o \in O | [o]^{\succeq}_A \cap X \neq \emptyset\}.$$

the $\overline{\mathcal{R}}^{\succeq}_A(X)$ and the $\underline{\mathcal{R}}^{\succeq}_A(X)$ are called upper and lower approximations of X with respect to the $\mathcal{R}^{\succeq}_A$, respectively.

X is called rough about O, if $\underline{\mathcal{R}}^{\succeq}_A(X) \neq \overline{\mathcal{R}}^{\succeq}_A(X)$. If not, the X is called precise. Based on equivalence relation $\mathcal{R}$, the upper and lower approximations of X is defined as follows, respectively.

$$\underline{R}_A(X) = \{o \in O | [o]_A \subseteq X\};$$

$$\overline{R}_A(X) = \{o \in O | [o]_A \cap X \neq \emptyset\}.$$

Proposition 2.3 Suppose $I^{\succeq} = (O, CS \cup DS, V, f)$ is an IFODT. $A \subseteq CS$. For any $X \subseteq O$, the following always hold.

(1) $A \subseteq CS \Rightarrow \underline{\mathcal{R}}^{\succeq}_{CS}(X) \supseteq \underline{\mathcal{R}}^{\succeq}_A(X)$, $\overline{\mathcal{R}}^{\succeq}_{CS}(X) \subseteq \overline{\mathcal{R}}^{\succeq}_A(X)$.

(2) If $A \subseteq CS$, we can obtain that $\mathcal{R}^{\succeq}_A(X) = \mathcal{R}^{\succeq}_{CS}(X) \Rightarrow \underline{\mathcal{R}}^{\succeq}_A(X) = \underline{\mathcal{R}}^{\succeq}_{CS}(X)$, $\overline{\mathcal{R}}^{\succeq}_A(X) = \overline{\mathcal{R}}^{\succeq}_{CS}(X)$.

Definition 2.5[10] Suppose $I^{\succeq} = (O, CS \cup DS, V, f)$ is an IFODT. $A \subseteq CS$, if A is a reduction set that A relative to $d \in DS$, meet the following conditions:

(1)$\mathcal{R}^{\succeq}_A(d) = \mathcal{R}^{\succeq}_{CS}(d)$;

(2)$\forall c \in A, \mathcal{R}^{\succeq}_{A-\{c\}}(d) < \mathcal{R}^{\succeq}_A(d)$.

For $\forall c \in A$, if $\mathcal{R}^{\succeq}_{A-\{c\}}(d) = \mathcal{R}^{\succeq}_A(d)$, then c is unnecessary for d in A, otherwise c is necessary for d in A. $core_d(A) = \{c \in A | \mathcal{R}^{\succeq}_{A-\{c\}}(d) \neq \mathcal{R}^{\succeq}_A(d)\}$. $core_d(A)$ is called relative core. (1) suggests that A has the same ability to recognize knowledge compared to CS. (2) suggests that all attributes are necessary attributes in a reduction set.

3. Attribute reduction of relative knowledge granularity in IFODT

In this section, a new relative knowledge granulation is proposed in IFODT. And a new uncertainty measure of relative knowledge granularity is defined and its monotonicity is proved. The significance of relative attribute is defined. Some important properties of relative attribute significance are proved. The attribute reduction algorithm of relative knowledge granularity is designed in IFODT.

Definition 3.1 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. For $A \subseteq CS$, $O/\mathcal{R}^{\succeq}_A = \{[o_1]^{\succeq}_A, [o_2]^{\succeq}_A, \cdots [o_n]^{\succeq}_A\}$. Based on the partition a knowledge granularity of A is defined as:

$$GK(A) = \frac{1}{|O|} \sum_{i=1}^{|O|} \frac{|[o_i]^{\succeq}_A|}{|O|}$$

Among them, $\frac{|[o_i]_A^{\succeq}|}{|O|}$ represents the ratio of the dominant class $|[o_i]_A^{\succeq}|$ on the domain O. Based on IFODT, the knowledge granularity redefined by ordered relation. Some properties of knowledge granularity are shown as follows:

Proposition 3.1 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT, for any subset $A, B \subseteq CS$, $\mathcal{R}_A^{\succeq}$ is ordered relation of $I^{\succeq}$, we have the following results.

(1) If $O/\mathcal{R}_A^{\succeq} = \{[o_i]_A^{\succeq} = \{o_i\} : o_i \in O\}$, then the minimum granularity of knowledge on the A is $1/|O|$.

(2) If $O/\mathcal{R}_A^{\succeq} = \{[o_i]_A^{\succeq} = O : o_i \in O\}$, then the maximum granularity of knowledge on the A is 1.

(3) When $A = B \Rightarrow GK(A) = GK(B)$.

(4) When $A \subseteq B \Rightarrow GK(A) \geq GK(B)$.

These properties are easily proved Based on IFODT, in order to build an uncertainty measure between attribute sets, the relative knowledge granularity is redefined as follows:

Definition 3.2 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. For any $A, B \subseteq CS$, $\mathcal{R}^{\succeq}$ is dominance relation of $I^{\succeq}$, $O/\mathcal{R}_A^{\succeq} = \{[o_1]_A^{\succeq}, [o_2]_A^{\succeq}, \cdots [o_n]_A^{\succeq}\}$, $O/\mathcal{R}_B^{\succeq} = \{[o_1]_B^{\succeq}, [o_2]_B^{\succeq}, \cdots [o_m]_B^{\succeq}\}$. Based on O, a knowledge granularity of A relative to B is defined as follows:

$$RG(B|A) = \frac{1}{|O|}\Big(\sum_{i=1}^{|O|} \frac{|[o_i]_A^{\succeq}|}{|O|} - \sum_{i=1}^{|U|} \frac{|[o_i]_{A\cup B}^{\succeq}|}{|O|}\Big).$$

The relative granularity characterizes the ability to distinguish knowledge B from knowledge A on the domain O. The smaller the $RG(B|A)$, the weaker the ability to distinguish the knowledge B from the knowledge A on the O, Conversely, the larger the $RG(B|A)$, the stronger the ability to distinguish the knowledge B from the knowledge A on the O. Some properties of RG are shown as follows:

Proposition 3.2 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. For any $A, B \subseteq CS$, and $\mathcal{R}_A^{\succeq}$ is ordered relation of $I^{\succeq}$, if $B \leq A$, then $RG(d|B) \leq RG(d|A)$.

Proof. Since $B \leq A$, we get $\forall o_i \in O (i = 1, 2, \cdots, |O|)$, $[o_i]_B^{\succeq} \subseteq [o_i]_A^{\succeq}$.

On the basis of **Definition 3.2**

$RG(d|B) - RG(d|A)$

$= GK(B) - GK(d \cup B) - GK(A) + GK(d \cup A)$

$= \frac{1}{|O|}\sum_{i=1}^{|O|} \frac{|[o_i]_B^{\succeq}|}{|O|} - \frac{1}{|O|}\sum_{i=1}^{|O|} \frac{|[o_i]_{d\cup B}^{\succeq}|}{|O|} - \sum_{i=1}^{|O|} \frac{|[o_i]_A^{\succeq}|}{|O|} + \sum_{i=1}^{|O|} \frac{|[o_i]_{d\cup A}^{\succeq}|}{|O|}.$

$= \frac{1}{|O|^2}\sum_{i=1}^{|O|}((|[o_i]_B^{\succeq}| - |[o_i]_A^{\succeq}|) - (|[o_i]_B^{\succeq} \cap [o_i]_d^{\succeq}| - |[o_i]_A^{\succeq} \cap [o_i]_d^{\succeq}|)) \leq 0$

Therefore, $RG(d|B) \leq RG(d|A)$ is clearly established. □

Proposition 3.3 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. $\forall c_i \in CS (i = 1, 2, \cdots, |CS|)$, then $RG(d|CS) \leq RG(d|CS \backslash \{c_m\}) \leq RG(d|CS \backslash \{c_{|CS|}, a_{|CS|-1}, \cdots, c_i\}) \leq \cdots \leq RG(d|\{c_1\})$

Proof. On the basis of **Proposition 2.2** and **Proposition 3.2** can be proved. □

Based on a IFODT, how to judge necessity of each attributes under the premise that the results of the decision remain unchanged. We propose **Theorem 3.1** to solve this problem by using relative knowledge granularity.

Theorem 3.1 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. $A \subseteq CS$, $\forall c \in A$,

$$RG(d|A) = RG(d|A \backslash \{c\})$$

The c is called not necessary condition attribute about the d, or else c is called necessary condition attribute.

Proposition 3.4 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. $A \subseteq CS$, then

$$\mathcal{R}_A^{\succeq}(d) = \mathcal{R}_{CS}^{\succeq}(d) \Leftrightarrow RG(d|A) = RG(d|CS).$$

Proof. (1) First, prove the adequacy. Because $\mathcal{R}^{\succeq}_{A}(d) = \mathcal{R}^{\succeq}_{CS}(d)$, then $\forall o_i \in O$, $\{o_i | [o]^{\succeq}_{A} \subseteq d^{\succeq}_{i}\} = \{o_i | [o]^{\succeq}_{CS} \subseteq d^{\succeq}_{i}\} \Rightarrow [o_i]^{\succeq}_{A} = [o_i]^{\succeq}_{CS}$, on the basis of **Proposition 3.2**, we can obtain $RG(d|A) = RG(d|CS)$.

(2) Secondly, prove the necessity. Because $RG(d|A) = RG(d|CS) \Rightarrow RG(d|A) - RG(d|CS) = \frac{1}{|O|^2}\sum_{i=1}^{|O|}((|[o_i]^{\succeq}_{A}| - |[o_i]^{\succeq}_{CS}|) - (|[o_i]^{\succeq}_{A} \cap [o_i]^{\succeq}_{\{d\}}| - |[o_i]^{\succeq}_{A} \cap [o_i]^{\succeq}_{\{d\}}|)) = 0$. In addition, $[o_i]^{\succeq}_{A} \subseteq [o_i]^{\succeq}_{CS} \Rightarrow [o_i]^{\succeq}_{A} = [o_i]^{\succeq}_{CS}$ or $[o_i]^{\succeq}_{CS} \subseteq [o_i]^{\succeq}_{A} \subseteq [o_i]^{\succeq}_{\{d\}}$. Thus, the $\mathcal{R}^{\succeq}_{A}(d) = \mathcal{R}^{\succeq}_{CS}(d)$ can be get. □

Based on a IFSDT, in order to accurately describe the importance of attributes, the inner and outer attribute significance measure based on relative knowledge granularity are defined as follows, respectively.
Definition 3.3 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. $A \subseteq CS$, $\forall c \in A$, the inner significance of c in A is defined as follow:

$$sig^{\succeq}_{inner}(c, A, d) = RG(d|(A - \{c\})) - RG(d|A)$$

If $sig^{\succeq}_{inner}(c, A, d) > 0$, this show that the c is necessary for a core attribute of $I^{\succeq}$. Therefore, one can obtain that relative core attribute of $I^{\succeq}$. And one can obtain that two conclusions as follows:

(1) $0 \leq sig^{\succeq}_{inner}(c, A, d) \leq 1 - 1/|O|$;

(2) $Rcore = \{c \in A | sig^{\succeq}_{inner}(c, A, d) > 0\}$ is called relative core attribute of $I^{\succeq}$.

Proof. (1) Because $A \preceq A - \{c\}$, on the basis of **Proposition 3.1**, we can obtain: $sig^{\succeq}_{inner}(c, A, d) \geq 0$. $sig^{\succeq}_{inner}(c, A, d) = \frac{1}{|O|^2}\sum_{i=1}^{|O|}((|[o_i]^{\succeq}_{A-\{c\}}| - |[o_i]^{\succeq}_{A}|) - (|[o_i]^{\succeq}_{A-\{c\}} \cap [o_i]^{\succeq}_{\{d\}}| - |[o_i]^{\succeq}_{A} \cap [o_i]^{\succeq}_{\{d\}}|)).\forall o_i \in O$, when $[o_i]^{\succeq}_{\{d\}} = \{o_i\}$, we can obtain: $sig^{\succeq}_{inner}(c, A, d) = \frac{1}{|O|^2}\sum_{i=1}^{|O|}(|[o_i]^{\succeq}_{A-\{c\}}| - |[o_i]^{\succeq}_{A}|)$. On the basis of **Proposition 2.2**, we can obtain: $[o_i]^{\succeq}_{A} = [o_i]^{\succeq}_{A-\{c\}} \cap [o_i]^{\succeq}_{\{c\}}$; if $[o_i]^{\succeq}_{A-\{c\}} = O, [o_i]^{\succeq}_{\{c\}} = \{o_i\}$, then the maximal $sig^{\succeq}_{inner}(c, A, d) = 1 - 1/|O|$.

(2) It can be directly inferred. □

The $RG(d|CS) \neq RG(d|Rcore)$ will be obtained, if we regard $Rcore$ as final attribute reduction result. Obviously, the result is not what we want. It is because of such a phenomenon that just consider the significant measure of certain attributes, not think over significant measure of other attributes in IFODT. Therefore, in order to make up for the shortcoming of the above problems, the **Definition 3.4** is defined.
Definition 3.4 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. $Rcore = A \subseteq CS$, $\forall c \in A^c$, the outer significance of c based on RG is shown as follow:

$$sig^{\succeq}_{outer}(c, A, d) = RG(d|A) - RG(d|(A \cup \{c\}))$$

The **Definition 3.4** shows that the relative granularity been changed by adding attribute c in A, the outer significance measure of new attribute c in A for d is gauged by above changes.

For any an IFODT, the attribute core is unique. We can easily obtain the $Rcore$ of $I^{\succeq}$ by **Definition 3.4**. On the begin of the $Rcore$ of $I^{\succeq}$, the attributes reduction can be get by constantly increase attributes with maximum $sig^{\succeq}_{outer}(c, A, d)$ of c to the $Rcore$.
Definition 3.5 Suppose $I^{\succeq} = (O, CS \cup \{d\}, V, f)$ is an IFODT. $red \subseteq CS$, red is a final attribute reduction of $I^{\succeq}$, if red meet the following conditions:

(1) $RG(d|red) = RG(d|CS)$;

(2) $\forall a \in red, RG(d|red) \neq RG(d|red - \{a\})$.

The red is obtained by $sig^{\succeq}_{inter}(c, A, d)$ and $sig^{\succeq}_{outer}(c, A, d)$.

Firstly, the $Rcore$ is get by $sig^{\succeq}_{inter}(c, A, d) > 0$. On the begin of the $Rcore$ of $I^{\succeq}$, the $sig^{\succeq}_{inter}(c, A, d)$ is calculated for each $c \in A^c$.

Algorithm 1: Based on IFODIS attribute reduction algorithm of relative knowledge granularity

begin
  Input: A IFODIS $I^{\succeq} = (O, CS \cup \{d\}, V, f)$
  Output: A reduction *red* on O
  for $1 \leq i \leq |O|$ **do** /* the $|O|$ is the number of object */
    $[o_i]^{\succeq} \leftarrow []$;
    /* the $[o_i]^{\succeq}$ is dominant class on O */
    for $1 \leq j \leq |O|$ **do**
      if $(lsd(j,:) \geq lsd(i,:)$ *and* $flsd(j,:) \leq flsd(i,:))$ **then**
        /* the *lsd* is membership of object */
        /* the *lsd* is membership of object */
        $[o_i]^{\succeq} = [o_i, j]$;
      end
    end
  end
  Let $Rcore \leftarrow \emptyset$; /* The $Rcore$ is relative core of $I^{\succeq}$ */
  for $m = 1 : |A|$ **do** /* The $|A|$ is attribute number of A */
    Compute $sig^{\succeq}_{inter}(c_m, A, d)$;
    if $sig^{\succeq}_{inter}(c_m, A, d) > 0$ **then**
      $Rcore \leftarrow (Rcore \cup \{c_m\})$;
    end
  end
  Let $red \leftarrow \emptyset$; /* The *red* is reduction of $I^{\succeq}$ */
  while $RG(d|Rcore) \neq RG(d|CS)$ **do**
    for *each* $c_n \in (CS - Rcore)$ **do**
      Compute $sig^{\succeq}_{outer}(c_n, Rcore, d)$;
      $c_0 = max\{sig^{\succeq}_{outer}(c_n, Rcore, d), c_n \in (CS - Rcore)\}$;
      $red \leftarrow (Rcore \cup \{c_n\})$;
      if $RG(d|red) = RG(d|CS)$ **then**
        return reduction *red*;
      end
    end
  end
  for *each* $c_k \in red$ **do**
    if $RG(d|(red - \{c_k\})) = RG(d|CS)$ **then**
      $red \leftarrow (red - \{c_k\})$;
    end
  end
  return reduction *red*
end

Second, when the $max\{sig^{\succeq}_{outer}(c, A, d)\}$ is discovered, we can obtain that the $red = Rcore \cup \{c\}$. Eventually we got *red* by repeating the above process.

Finally, the *red* is examined by **Definition 3.5**.

For verify the feasibility and accuracy of attribute reduction method based on relative knowledge granularity in IFODT, the **Algorithm 1** is designed by the above procedurerelated definitions and theorems.

And the time complexity of **Algorithm 1** is analyzed.

The time complexity of **Algorithm 1** is analysed. A IFODT is given, on the basis of **Definition 2.3**, the dominant class and decision class is calculated of IFODT, and the time complexity is $O(|O|^2|CS|)$. Next, the value of relative knowledge granularity is calculated, the time complexity being $O(|CS||O|)$. Then $sig^{\geq}_{inter}$ and $sig^{\geq}_{outer}$ are calculated, respectively. Their time complexity is $O(|CS||O| + |n|)$. Thus, the time complexity of steps 2-10 is $O(|O|^2|CS|)$, the time complexity of steps 12-18 is $O(|CS||O|^2)$, the time complexity of steps 20-30 is $O(|CS - Rcore||O|^2)$, the time complexity of steps31-37 is $O(|red||O|^2)$. Therefore, based on IFODT the time complexity of attribute reduction algorithm of relative knowledge granularity is $O(|O|^2|CS| + |CS||O|^2 + |CS - Rcore||O|^2 + |red||O|^2)$.

4. Case study

Set 8 investment projects, from the perspective of risk factors for their assessment, risk factors for 5 categories: credit risk, operational risk, legal risk, environmental risk and production risk. Table 1 is the risk assessment form of investment, among $O = \{o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8\}$, AT={credit risk, operational risk, legal risk, environmental risk, production risk}. For the sake of simplicity, Using c_1, c_2, c_3, c_4, c_5 said credit risk, operational risk, legal risk, environmental risk and production risk. Decision class is $d = \{accept, reject\}$. $CS = \{c_1, c_2, c_3, c_4, c_5\}$. 1 stands for this investment project can be invested, 0 stands for this investment project can not be invested.

Table 1: The IFODIS of venture capital

U	c_1	c_2	c_3	c_4	c_5	d
o_1	$\langle 0.5, 0.4\rangle$	$\langle 0.5, 0.0\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.5, 0.4\rangle$	1
o_2	$\langle 0.7, 0.0\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.6, 0.1\rangle$	0
o_3	$\langle 0.6, 0.4\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.4, 0.5\rangle$	1
o_4	$\langle 0.3, 0.6\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0.3, 0.7\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0.3, 0.6\rangle$	1
o_5	$\langle 0.2, 0.0\rangle$	$\langle 0.7, 0.0\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.2, 0.2\rangle$	$\langle 0.2, 0.7\rangle$	1
o_6	$\langle 0.5, 0.3\rangle$	$\langle 0.5, 0.0\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.2, 0.0\rangle$	$\langle 0.6, 0.3\rangle$	0
o_7	$\langle 0.5, 0.4\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0.4, 0.0\rangle$	$\langle 0.4, 0.1\rangle$	$\langle 0.5, 0.4\rangle$	0
o_8	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.2\rangle$	$\langle 0.6, 0.0\rangle$	$\langle 0.6, 0.0\rangle$	$\langle 0.6, 0.1\rangle$	0

The dominant class for every $o_i \in O$ are computed on CS, as follows (by **Definition 2.3**):

$[o_1]^{\geq}_{CS} = \{o_1\}$, $[o_2]^{\geq}_{CS} = \{o_2\}$, $[o_3]^{\geq}_{CS} = \{o_2, o_3, o_8\}$, $[o_4]^{\geq}_{CS} = \{o_1, o_2, o_3, o_4, o_7, o_8\}$, $[o_5]^{\geq}_{CS} = \{o_5\}$, $[o_6]^{\geq}_{CS} = \{o_6\}$, $[o_7]^{\geq}_{CS} = \{o_7\}$, $[o_8]^{\geq}_{CS} = \{o_8\}$.

Let's take $A = \{c_1, c_2, c_3\}$ as attribute subset of CS. On the basis of **Algorithm 1** of step 11-18, we can obtain that

$$sig^{\geq}_{inter}(c_1, A, d) = 0, sig^{\geq}_{inter}(c_2, A, d) = 3/64 > 0, sig^{\geq}_{inter}(c_3, A, d) = 0$$

Therefore, the $Rcore = \{c_2\}$.

On the basis of **Algorithm 1** of step 19-30, we can obtain that

$$sig^{\geq}_{outer}(c_4, Rcore, d) = 2/64, sig^{\geq}_{outer}(c_5, Rcore, d) = 0$$

Thus, the $red = \{c_2, c_4\}$.

Finally, On the basis of **Algorithm 1** of step 31-37, the red is examined by $RG(d|CS) = RG(d|red) = 5/64$. So, on the premise that the decision results of IFODT remain unchanged, the final reduction result is $\{c_2, c_4\}$. In order to make decisions quickly and accurately, we only need to consider operational risk and environmental risk.

5. Conclusions

In the field of the development and application of RST, extending the classical RST to intuitionistic fuzzy front is an important direction. For the moment, the representative and hot research is attribute reduction algorithm of relative knowledge granularity which provides a new viewpoint to simplify feature set. In this paper, based on IFODT, we redefine the granularity of knowledge and the relative knowledge granularity by ordered relation. And their relevant properties are proved. The conditional attribute internal and external significance are designed by relative knowledge granularity. And some important properties of relative attribute significance are proved. In the aspect of computation, the corresponding algorithm is designed and time complexity of algorithm is calculated. Moreover, the attribute reduction model of relative knowledge granularity of efficiency and accuracy is proved by test. Finally, the availability and accuracy of algorithm is demonstrated by an case about IFODT.

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