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Estimation for inverse Gaussian Distribution Under First-failure Progressive Hybird Censored Samples

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Abstract. In this paper, a first-failure progressive hybird censoring scheme is introduced that combines progressive first-failure censoring and Type-I censoring. We obtain the maximum likelihood estimators (MLEs) and the Bayes estimators of the unknown parameters from the inverse Gaussian distribution based on the first-failure progressive hybird censoring scheme. The Bayes estimates are computed under squared error, Linex and general entropy loss functions. The asymptotic confidence intervals and coverage probabilities for the parameters are obtained based on the observed Fisher's information matrix. Also, highest posterior density credible intervals for the parameters are computed using Gibbs sampling procedure. A Monte Carlo simulation study is conducted in order to compare the Bayes estimators with the MLEs. Real life data sets are provided to illustration purposes.

1. Introduction

The probability density function (PDF) and the cumulative distribution function (CDF) of inverse Gaussian (IG) distribution are given by

$$f(x,\mu,\lambda) = (\frac{\lambda}{2\pi x^3})^{1/2} e^{\frac{-\lambda(x-\mu)^2}{2\mu^2 x}}, x>0, \mu, \lambda>0, \tag{1}$$

$$F(x,\mu,\lambda) = \Phi(\sqrt{\frac{\lambda}{x}}(\frac{x}{\mu}-1)) + e^{\frac{2\lambda}{\mu}}\Phi(-\sqrt{\frac{\lambda}{x}}(\frac{x}{\mu}+1)), x>0, \mu, \lambda>0. \tag{2}$$

where μ is the location parameter, λ is the shape parameter and $\Phi(\cdot)$ denotes the standard normal CDF.

Since the review article of Folks and Chhikara [5] was published, the inverse Gaussian (IG) distribution as a useful modeling tool to model and analyse right skewed data has received a lot of attention in many different fields, such as finance, lifetime testing, demography, etc. The IG distribution was first obtained by Schrödinger [9] as first passage time distribution of Brownian. Tweedi [10] has investigated the properties of IG distribution in detail. Koutrouvelis and Karagrigorious [7] provided a complete review on existing tests

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for the IG distribution. Basak and Balakrishnan [1] developed estimation methods based on progressive Type-II censored samples from three-parameter IG distribution.

Censoring schemes are common in life tests because of time limits and other restrictions on data collection. There are several censoring methods available to experimenters, for Type-I, Type-II and first-failure censoring. The first-failure censored sampling plan has an advantage in terms of shorter test time and a saving resources. Wu and Kus [12] introduced the progressively first-failure scheme. In this article, we develop a new life test scheme that combines progressive first-failure censoring and Type-I censoring, name the first-failure progressive hybird censoring scheme.

The life-testing experiment of first-failure progressive hybird censoring scheme can be described as follows: assume that $k \times n$ items are put on test in n independent groups with k items in each group. The integer $m <= n$ is fixed at the beginning of the experiment and the progressive censoring scheme $R = (R_1, R_2, \cdots, R_m)$ is prefixed. The time point T is also a fixed constant before the experiment. At the first-failure of unit (say $X^R_{1:m:n:k}$), we remove that group in which first failure occurred and R_1 additional groups randomly from the remaining $n-1$ groups in the experiment. As soon as second failure (say $X^R_{2:m:n:k}$) takes place we remove that group and additional R_2 groups randomly from remaining $n - R_1 - 2$ groups and so on. This process continues until, the m th failure of unit (say $X^R_{m:m:n:k}$) observed or time point T, all the remaining surviving units are removed and the test is terminated. If $X^R_{m:m:n:k} < T$, the experiment proceeds with the pre-specified progressive censoring scheme $R = (R_1, R_2, \cdots, R_m)$ and stops at the time $X^R_{m:m:n:k}$. On the contrary, if $X^R_{m:m:n:k} > T$ and only J failures occur before the time point T, where $1 \le J < m$, then, at the time point T, all remaining surviving groups are removed and the experiment is terminated. Obviously, we have $R^*_J = n - J - (R_1 + R_2 + \cdots + R_J)$. Denote the above two cases as case I and case II respectively. This censoring scheme is so-called first-failure progressive hybird censoring scheme. Specifically, under first-failure progressive hybird censoring scheme, we only have one case of the following two types of observations:

Case I: $X^R_{1:m:n:k}, X^R_{2:m:n:k}, \cdots, X^R_{m:m:n:k}, \quad X^R_{m:m:n:k} < T$,
Case II: $X^R_{1:m:n:k}, X^R_{2:m:n:k}, \cdots, X^R_{J:m:n:k}, \quad X^R_{J:m:n:k} < T < X^R_{J+1:m:n:k}$.

Supposed we denote the number of progressively censored ordered failures occur before time T by d, then the likelihood function can be expressed as

$$L(\mu, \lambda|x) = \begin{cases} C_m k^m \prod_{i=1}^{m} f(x^R_{i:m:n:k})(\bar{F}(x^R_{i:m:n:k}))^{k(R_i+1)-1}, \text{if } x^R_{m:m:n:k} < T, \\ C_d k^d \prod_{i=1}^{d} f(x^R_{i:m:n:k})\bar{F}(x^R_{i:m:n:k})^{k(R_i+1)-1}\bar{F}(T)^{kR^*}, \text{if } x^R_{m:m:n:k} > T, \end{cases} \tag{3}$$

where $C_m = n(n - R_1 - 1)(n - R_1 - R_2 - 2) \cdots (n - R_1 - \cdots - R_{m-1} - m + 1)$, $\bar{F}(\cdot) = 1 - F(\cdot)$ denotes the survival function and C_d can be written similarly taking $m = d$.

The rest of this paper is organized as follows: The MLEs and Bayes estimators unknown parameters of IG distribution, based on first-failure progressively hybird censoring scheme are investigated in Section 2 and 3. For illustration, a set of real data is introduced and analyzed to show that the IG distribution is a suitable distribution for these data in Section 4. In Section 5, Monte Carlo simulation results are presented. Finally, we conclude the paper in Section 6.

2. Maximum Likelihood Estimation

Let $X^R_{1:m:n:k}, X^R_{2:m:n:k}, \cdots, X^R_{D:m:n:k}$, be the first-failure progressive hybird censored sample from IG distribution, with a censoring scheme R. Where D denotes the number of the observed failures up to the end of the experiment. For simplicity, we used X_i instead of $X^R_{i:m:n:k}$. Based on Equation (3), the likelihood function is given by

$$L(\lambda,\mu|x) = C_D k^D \prod_{i=1}^{D} \Big[\Phi(\sqrt{\frac{\lambda}{x_i}}(1-\frac{x_i}{\mu})) - e^{\frac{2\lambda}{\mu}}\Phi(-\sqrt{\frac{\lambda}{x_i}}(\frac{x_i}{\mu}+1))\Big]^{k(R_i+1)-1}$$
$$\Big[\Phi(\sqrt{\frac{\lambda}{T}}(1-\frac{T}{\mu})) - e^{\frac{2\lambda}{\mu}}\Phi(-\sqrt{\frac{\lambda}{T}}(\frac{T}{\mu}+1))\Big]^{kR^*}(\frac{\lambda}{2\pi x_i^3})^{1/2} e^{-\frac{\lambda(x_i-\mu)^2}{2\mu^2 x_i}} \tag{4}$$

Then, the log-likelihood in function (4) can be combined as follows:

$$l(\mu,\lambda|x) = \log(C) + \frac{D}{2}\log(\lambda) - \frac{3}{2}\sum_{i=1}^{D}\log(x_i) - \sum_{i=1}^{D}\frac{\lambda}{2x_i}(\frac{x_i}{\mu}-1)^2 + k\sum_{i=1}^{D}((R_i+1)-1)$$
$$\log[\Phi(\sqrt{\frac{\lambda}{x_i}}(1-\frac{x_i}{\mu})) - e^{\frac{2\lambda}{\mu}}\Phi(-\sqrt{\frac{\lambda}{x_i}}(\frac{x_i}{\mu}+1))] + kR^*\log[\Phi(\sqrt{\frac{\lambda}{T}}(1-\frac{T}{\mu})) - e^{\frac{2\lambda}{\mu}}\Phi(-\sqrt{\frac{\lambda}{T}}(\frac{T}{\mu}+1))], \tag{5}$$

where $R^* = 0, D = m$ for case I and $R^* = R_J^*, D = J$ for case II. The first derivative of Eq (5) with respect to λ, μ and putting them equal zero we obtain

$$\frac{\partial l}{\partial \lambda} = \frac{D}{2\lambda} + \sum_{i=1}^{D}(k(R_i+1)-1)\frac{\varphi(\psi_{1i})L_{1i} - \frac{2}{\mu}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i}) - e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{1i}}{\bar{F}(x_i,\lambda,\mu)}$$
$$+kR^*\frac{\varphi(\psi_{1T})L_{1T} - \frac{2}{\mu}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T}) - e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{1T}}{\bar{F}(T,\lambda,\mu)} - \sum_{i=1}^{D}\frac{1}{2x_i}(\frac{x_i}{\mu}-1)^2 = 0, \tag{6}$$

$$\frac{\partial l}{\partial \mu} = -\frac{\lambda D}{\mu^2} + \sum_{i=1}^{D}(k(R_i+1)-1)\frac{\varphi(\psi_{1i})L_{2i} + \frac{2\lambda}{\mu^2}\Phi(\psi_{2i}) - e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{2i}}{\bar{F}(x_i,\lambda,\mu)}$$
$$+kR^*\frac{\varphi(\psi_{1T})L_{2T} + \frac{2\lambda}{\mu^2}\Phi(\psi_{2T}) - e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{2T}}{\bar{F}(T,\lambda,\mu)} + \frac{\lambda}{\mu^3}\sum_{i=1}^{D}x_i = 0, \tag{7}$$

where $\varphi(\cdot)$ denotes the standard normal PDF and $\varphi'(y) = -y\varphi(y), \psi_{1i} = \sqrt{\frac{\lambda}{x_i}}(1-\frac{x_i}{\mu}), \psi_{2i} = -\sqrt{\frac{\lambda}{x_i}}(1+\frac{x_i}{\mu}), \psi_{1T} = \sqrt{\frac{\lambda}{T}}(1-\frac{T}{\mu}), \psi_{2T} = -\sqrt{\frac{\lambda}{T}}(1+\frac{T}{\mu}), L_{1i} = \frac{\partial\psi_{1i}}{\partial\lambda} = \frac{1}{2\sqrt{x_i\lambda}}(1-\frac{x_i}{\mu}), L_{2i} = \frac{\partial\psi_{1i}}{\partial\mu} = \frac{\sqrt{\lambda x_i}}{\mu^2}, M_{1i} = \frac{\partial\psi_{2i}}{\partial\lambda} = -\frac{1}{2\sqrt{x_i\lambda}}(1+\frac{x_i}{\mu}), M_{2i} = \frac{\partial\psi_{2i}}{\partial\mu} = L_{2i}$. We observe that the exact solution of the likelihood Eq (6) and (7) for both the above cases is not possible. Therefore, we intend to evaluate the MLEs by solving the likelihood equations numerically using iteration method such as Newton-Raphson method.

2.1. Asymptotic confidence interval estimation

The second derivatives in Equations (5) are as follows

$$\frac{\partial^2 l}{\partial\lambda^2} = -\frac{D}{2\lambda^2} + \sum_{i=1}^{D}(k(R_i+1)-1)[\frac{\varphi(\psi_{1i})L_{11i} - \psi_{1i}\varphi(\psi_{1i})L_{1i}^2 - (\frac{2}{\mu})^2 e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i}) - \frac{4}{\mu}e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{1i} - e^{\frac{2\lambda}{\mu}}\psi_{2i}M_{1i}^2\varphi(\psi_{2i})}{\bar{F}(x_i,\lambda,\mu)}$$
$$-\frac{(\varphi(\psi_{1i})L_{1i} - \frac{2}{\mu}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i}) - e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{1i})^2}{\bar{F}^2(x_i,\lambda,\mu)}] + kR^*[\frac{\varphi(\psi_{1T})L_{11T} - \psi_{1T}\varphi(\psi_{1T})L_{1T}^2 - (\frac{2}{\mu})^2 e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T})}{\bar{F}(T,\lambda,\mu)} +$$
$$\frac{e^{\frac{2\lambda}{\mu}}\psi_{2T}\varphi(\psi_{2T})M_{1T}^2 - e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{11T} - \frac{4}{\mu}e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{1T}}{\bar{F}(T,\lambda,\mu)} - \frac{(\varphi(\psi_{1T})L_{1T} - \frac{2}{\mu}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T}) - \varphi(\psi_{2i})e^{\frac{2\lambda}{\mu}}M_{1T})^2}{\bar{F}^2(T,\lambda,\mu)}] \tag{8}$$

$$\frac{\partial^2 l}{\partial \mu^2}=\lambda\sum_{i=1}^{D}(\frac{2}{\mu^3}-\frac{3x_i}{\mu^4})+\sum_{i=1}^{D}(k(R_i+1)-1)[\frac{\varphi(\psi_{1i})L_{22i}-\psi_{1i}\varphi(\psi_{1i})L_{2i}^2-\frac{4\lambda}{\mu^3}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i})-(\frac{2\lambda}{\mu})^2e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i})}{\bar{F}(x_i,\lambda,\mu)}$$
$$+\frac{\frac{4\lambda}{\mu^2}e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{2i}-e^{\frac{2\lambda}{\mu}}\psi_{2i}\varphi(\psi_{2i})M_{2i}^2+e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{22i}}{\bar{F}(x_i,\lambda,\mu)}-\frac{(\varphi(\psi_{1i})L_{2i}+\frac{2\lambda}{\mu^2}\Phi(\psi_{2i})-e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{2i})^2}{\bar{F}^2(x_i,\lambda,\mu)}]+$$
$$kR^*[\frac{\varphi(\psi_{1T})L_{22T}-\psi_{1T}\varphi(\psi_{1T})L_{2T}^2-(\frac{2\lambda}{\mu})^2e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T})-\frac{4\lambda}{\mu^3}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T})+\frac{4\lambda}{\mu^2}e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{2T}}{\bar{F}(T,\lambda,\mu)}$$
$$+\frac{e^{\frac{2\lambda}{\mu}}\psi_{2T}\varphi(\psi_{2T})M_{2T}^2-e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{22T}}{\bar{F}(T,\lambda,\mu)}-\frac{(\varphi(\psi_{1T})L_{2T}+\frac{2\lambda}{\mu^2}\Phi(\psi_{2T})-e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{2T})^2}{\bar{F}^2(T,\lambda,\mu)}] \quad (9)$$

$$\frac{\partial^2 l}{\partial\lambda\partial\mu}=\sum_{i=1}^{D}(\frac{1}{\mu^2}-\frac{x_i}{\mu^3})+\sum_{i=1}^{D}(k(R_i+1)-1)[\frac{\varphi(\psi_{1i})L_{2i}L_{21i}-\psi_{1i}\varphi(\psi_{1i})L_{2i}L_{1i}+\frac{2}{\mu^2}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i})+\frac{2\lambda}{\mu^2}e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{1i}}{\bar{F}(x_i,\lambda,\mu)}+$$
$$\frac{\frac{4\lambda}{\mu^3}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i})-\frac{2}{\mu}e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{2i}+e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})\psi_{2i}M_{1i}M_{2i}-e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{1i}M_{2i}}{\bar{F}(x_i,\lambda,\mu)}-$$
$$\frac{(\varphi(\psi_{1i})L_{2i}+\frac{2\lambda}{\mu^2}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i})-e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{2i})(\varphi(\psi_{1i})L_{1i}-\frac{2}{\mu}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2i})-e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2i})M_{1i})}{\bar{F}^2(x_i,\lambda,\mu)}]+kR^*[\frac{e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})\psi_{2T}M_{1T}M_{2T}}{\bar{F}(T,\lambda,\mu)}$$
$$+\frac{\varphi(\psi_{1T})L_{2T}L_{21T}-\psi_{1T}\varphi(\psi_{1T})L_{2T}L_{1T}+\frac{2}{\mu^2}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T})+\frac{4\lambda}{\mu^3}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T})+\frac{2\lambda}{\mu^2}e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{1T}-\frac{2}{\mu}e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{2T}}{\bar{F}(T,\lambda,\mu)}-$$
$$\frac{e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{1T}M_{2T}}{\bar{F}(T,\lambda,\mu)}-\frac{(\varphi(\psi_{1T})L_{2T}+\frac{2\lambda}{\mu^2}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T})-e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{2T})(\varphi(\psi_{1T})L_{1T}-\frac{2}{\mu}e^{\frac{2\lambda}{\mu}}\Phi(\psi_{2T})-e^{\frac{2\lambda}{\mu}}\varphi(\psi_{2T})M_{1T})}{\bar{F}^2(T,\lambda,\mu)}] \quad (10)$$

where $L_{11i}=\frac{\partial^2\psi_{1i}}{\partial\lambda^2}=-\frac{1}{4\sqrt{x_i}}\lambda^{-3/2}(1-\frac{x_i}{\mu})$, $L_{22i}=\frac{\partial^2\psi_{1i}}{\partial\mu^2}=-2\frac{\sqrt{\lambda x_i}}{\mu^3}$, $L_{12i}=\frac{\partial^2\psi_{1i}}{\partial\lambda\partial\mu}=\frac{1}{2\sqrt{x_i\lambda}}\frac{x_i}{\mu^2}$, $M_{11i}=\frac{\partial^2\psi_{2i}}{\partial\lambda^2}=\frac{1}{4\sqrt{x_i}}\lambda^{-3/2}(1+\frac{x_i}{\mu})$, $M_{12i}=\frac{\partial^2\psi_{2i}}{\partial\lambda\partial\mu}=\frac{\sqrt{x_i}}{2\sqrt{\lambda}\mu^2}$, $M_{22i}=\frac{\partial^2\psi_{2i}}{\partial\mu^2}=L_{22i}$. The asymptotic variance and covariance of the MLEs for parameters for λ and μ are given by the elements of the inverse of the Fisher's information matrix

$$I_{sk}=-E(\frac{\partial^2 l(\theta_1,\theta_2;x)}{\partial\theta_s\partial\theta_k}),\quad s,k=1,2.$$

Unfortunately, to get the exact mathematical expressions for the above expectations are very difficulty. Therefore, we will take the approximate asymptotic variance-covariance matrix for MLEs, it is given by

$$I_0(\hat{\lambda},\hat{\mu})=\begin{bmatrix}-\frac{\partial^2 l}{\partial\lambda^2} & -\frac{\partial^2 l}{\partial\lambda\partial\mu}\\ -\frac{\partial^2 l}{\partial\mu\partial\lambda} & -\frac{\partial^2 l}{\partial\mu^2}\end{bmatrix}^{-1}_{(\hat{\lambda},\hat{\mu})}=\begin{bmatrix}var(\hat{\lambda}) & cov(\hat{\lambda},\hat{\mu})\\ cov(\hat{\mu},\hat{\lambda}) & var(\hat{\mu})\end{bmatrix}.$$

Approximate confidence interval for λ and μ can be found by taking $(\hat{\lambda},\hat{\mu})$ to be bivariate normal distribution with mean (λ,μ) and covariance matrix $I_0^{-1}(\hat{\lambda},\hat{\mu})$. Namely, $(\hat{\lambda},\hat{\mu})^T\sim N_2((\lambda,\mu)^T,I_0^{-1}(\hat{\lambda},\hat{\mu}))$. Thus, the 100(1-$\alpha$)% approximate confidence intervals for the parameters λ and μ become

$$\hat{\lambda}\pm Z_{\alpha/2}\sqrt{var(\hat{\lambda})},\ \hat{\mu}\pm Z_{\alpha/2}\sqrt{var(\hat{\mu})},$$

the coverage probabilities (CPs) of λ and μ can be found by the Monte Carlo simulations

$$CP_\lambda=P(|\frac{(\hat{\lambda}-\lambda)}{\sqrt{var(\hat{\lambda})}}|\le Z_{\alpha/2})\text{ and }CP_\mu=P(|\frac{(\hat{\mu}-\mu)}{\sqrt{var(\hat{\mu})}}|\le Z_{\alpha/2}),$$

where $Z_{\alpha/2}$ is the percentile of the standard normal distribution with right-tail probability $\alpha/2$.

3. Bsyesian Estimation

In this section we consider the Bayesian estimations of unknown parameters of IG distribution based on a first-failure progressive hybrid censored sample. It is assumed that λ and μ have independent gamma priors:

$$\lambda \sim \pi_1(\lambda) = \frac{b^a \lambda^{a-1} e^{-b\lambda}}{\Gamma(a)}, \quad \mu \sim \pi_2(\mu) = \frac{d^c \mu^{c-1} e^{-d\mu}}{\Gamma(c)},$$

where a, b, c and d are assumed to be known and non-negative. The joint prior for λ and μ is $\pi(\lambda, \mu) \propto \lambda^{a-1} e^{-b\lambda} \mu^{c-1} e^{-d\mu}$. Thus, the joint posterior distribution of λ and μ be written as

$$\pi(\lambda, \mu | x) = \frac{L(x|\lambda, \mu)\pi(\lambda, \mu)}{\int_0^\infty \int_0^\infty L(x|\lambda, \mu)\pi(\lambda, \mu)\,\mathrm{d}\mu\mathrm{d}\lambda}, \tag{11}$$

where the conditional posterior distributions $\pi_1(\mu|\lambda, x)$ and $\pi_2(\lambda|\mu, x)$ of parameters μ and λ can be computed, respectively, as

$$\pi_1(\mu|\lambda, x) \propto (\bar{F}(T|\lambda, \mu))^{kR^*} \mu^{c-1} \prod_{i=1}^{D} (\bar{F}(x_i|\lambda, \mu))^{k(R_i+1)-1} e^{-\lambda(\frac{\sum_{i=1}^{D} x_i}{2\mu^2} - \frac{D}{\mu}) - d\mu}, \tag{12}$$

$$\pi_2(\lambda|\mu, x) \propto \bar{F}(T|\lambda, \mu)^{kR^*} \lambda^{D/2+a-1} \prod_{i=1}^{D} \bar{F}(x_i|\lambda, \mu)^{k(R_i+1)-1} e^{-\lambda(\sum_{i=1}^{D}(\frac{x_i}{2\mu^2} + \frac{1}{2x_i}) + b - \frac{D}{\mu})}. \tag{13}$$

The Bayes estimates of unknown parameters depend on the form of loss function. In this paper, the Bayes estimates have been obtained under three different loss functions, the squared error, the Linex and the general entropy loss functions.

The squared error loss function (SELF) is one of the most popular loss function. This loss is symmetric, and its use is very popular due to its computational simplicity. Under the SELF, the Bayes estimator of the parameter is the posterior mean. Therefore, the Bayesian estimation of any function of λ and μ, say $\phi(\lambda, \mu)$, under SELF is given by

$$\hat{\phi}_{BS}(\lambda, \mu|x) = E(\phi(\lambda, \mu|x)) = \frac{\int_0^\infty \int_0^\infty \phi(\lambda, \mu|x) L(x|\lambda, \mu)\pi(\lambda, \mu)\mathrm{d}\mu\mathrm{d}\lambda}{\int_0^\infty \int_0^\infty L(x|\lambda, \mu)\pi(\lambda, \mu)\mathrm{d}\mu\mathrm{d}\lambda}. \tag{14}$$

In life-testing and reliability analysis problem, the essence of losses is not always symmetric and hence the use of SELF is not appropriate in some circumstances. To resolve such situation, Varian [4] introduced the asymmetric Linex loss function. The Linex loss function is defined as follows:

$$L(\hat{\theta}, \theta) = e^{\beta(\hat{\theta}-\theta)} - \beta(\hat{\theta} - \theta) - 1, \beta \neq 0,$$

where $\hat{\theta}$ is an estimate of θ. The sign of the constant β represents the direction and its magnitude represents the degree of asymmetry. Under the Linex loss function, the Bayes estimate of $\phi(\mu, \lambda)$ is given by

$$\hat{\phi}_{BL}(\lambda, \mu|x) = -\frac{1}{\beta} \log[E(e^{-\beta,\phi(\lambda,\mu)}|x)] = -\frac{1}{\beta} \log[\frac{\int_0^\infty \int_0^\infty e^{-\beta\phi(\lambda,\mu)} L(x|\lambda, \mu)\pi(\lambda, \mu)\mathrm{d}\mu\mathrm{d}\lambda}{\int_0^\infty \int_0^\infty L(x|\lambda, \mu)\pi(\lambda, \mu)\mathrm{d}\mu\mathrm{d}\lambda}]. \tag{15}$$

The Linex loss function is suitable for situations where overestimate may lead to serious consequence. Basu and Ebrahimi [9] and Parsian and Farsipour [2] found that the Linex loss function is unsuitable for estimating the scale parameter and other quantities. Hence, Calabria and Pulcini [3] proposed a suitable alternative to the modified Linex loss function that is, the general entropy loss function.

The general entropy loss function is defined as

$$L(\hat{\theta}, \theta) = (\frac{\hat{\theta}}{\theta})^q - q \log(\frac{\hat{\theta}}{\theta}) - 1, q \neq 0.$$

Under the general entropy loss function, the Bayes estimate of $\phi(\lambda,\mu)$ is given by

$$\hat{\phi}_{BGE}(\lambda,\mu|x) = [E(\phi(\lambda,\mu)^{-q}|x)]^{-\frac{1}{q}} = [\frac{\int_0^\infty\int_0^\infty \phi(\lambda,\mu)^{-q}L(x|\lambda,\mu)\pi(\lambda,\mu)\mathrm{d}\mu\mathrm{d}\lambda}{\int_0^\infty\int_0^\infty L(x|\lambda,\mu)\pi(\lambda,\mu)\mathrm{d}\mu\mathrm{d}\lambda}]^{-\frac{1}{q}}. \tag{16}$$

It is not possible to compute (14), (15) and (16) analytically, we recommend approximating it by using the Gibbs sampling technique. The following steps are used for computation purpose:
Step 1. Start with initial value λ_0. Set $i = 1$.
Step 2. Generate μ_i from $\pi_1(\mu,|\lambda_{i-1},x)$.
Step 3. Generate λ_i from $\pi_2(\lambda,|\mu_i,x)$. Set $i = i+1$.
Step 4. Repeat steps 2–3 N times. Obtain the Bayes estimates of $\phi = \lambda$ and μ with respect to squared error, Linex and general entropy loss functions as

$$\hat{\phi}_{BS} = \frac{1}{N-M}\sum_{j=M+1}^{N}\phi_j,\ \hat{\phi}_{BL} = -\frac{1}{\beta}\log[\frac{1}{N-M}\sum_{j=M+1}^{N}e^{-\beta\phi_j}],\ \hat{\phi}_{BGE} = [\frac{1}{N-M}\sum_{j=M+1}^{N}\phi_j^{-q}]^{-\frac{1}{q}},$$

where M is burn in period, and BS,BL and BGE denote respectively the Bayes estimates under squared error, Linex and general entropy loss functions.

3.1. HPD Credible Interval Estimation

Now, we construct the highest posterior density (HPD) credible interval of λ and μ using Gibbs sampling procedure. Let $\lambda_{(1)},\lambda_{(2)},\cdots,\lambda_{(M)}$ denote the order values of $\lambda_1,\lambda_2,\cdots,\lambda_M$. Then using the algorithm proposed by Chen and Shao [7], the 100(1-α)% HPD credible interval for λ is given by $(\lambda_{(j)},\lambda_{(j+[(1-\alpha)M])})$, $[a]$ is the integer part of a. Similarly, we can construct the 100(1-α)% HPD credible interval for μ.

4. Real Life Data

In this section, we consider a real life data to illustrate the proposed method in the previous sections and verity how our estimators work in practice. The data set is from Kimber [6], and it represents the lifetimes of steel specimens tested at stress level of 32 MPa. There were 24 observations listed: 1144, 231, 523, 474, 4510, 3107, 815, 6297, 1580, 605, 1786, 206, 1943, 935, 283, 1336, 727, 370, 1056, 413, 619, 2214, 1826, 597.

Before progressing further, we first fit the IG to the complete data set and compare its fitting with some well-known lifetime distributions, namely normal, Weibull, Lindly and exponential distributions.To test the goodness of fit of above models, we have used the (i) Akaikes information criterion (AIC),(ii) Bayesian information criterion (BIC) and (iii) Kolmogrov-Smirnov (K-S) values for date set has been presented in Table 1. This Table shows that the IG is the best choice among the competing reliability models in the literature for fitting lifetime data, since it has the smallest $-\log(L)$, AIC, BIC, K-S statistic values and highest $p-$value.

Next, we generate a first-failure progressively hybird censored samples with $n = 24$, $k = 1$, $m = 20$ under the two following censoring schemes (CS):
Scheme 1: $R_1 = (4,0,\cdots,0)$; and Scheme 2: $R_2 = (0,0,\cdots,4)$.

Because we have no prior in formation about the unknown parameters, the Bayesian is done under the non-informative prior assuming the value of hyper-parameters to be $(a = b = c = d = 0)$. The estimates of λ and μ under two schemes for $T = 2000$ and $T = 4000$ based on different methods are provided in Table 2.

Table 1 Fitting summary of various models for above data set

Model	MLEs		$-\log(L)$	AIC	BIC	K-S	p-value
Normal	1400	1424	208.3	420.7	423	0.2	0.3
IG	1400	1243	194.8	393.6	396	0.081	0.99
Weibull	1	0.0007147	197.9	399.7	402.1	0.14	0.7
Lindley	0.001427		198.3	398.5	399.7	0.19	0.3
Exponential	0.0007143		197.9	397.7	398.9	0.14	0.7

Table 2 The MLEs and Bayes estimates of the parameters for the real data set

				Bayes				
				BS	B L		B GE	
T	CS	parameter	MLE	q=-1	β=-0.5	β=0.5	q=-0.5	q=0.5
2000	1	μ	1654	1567	1631	1510	1534	1473
		λ	1320	1299	1339	1262	1270	1212
	2	μ	1284	1506	1589	1453	1472	1414
		λ	1334	1290	1327	1255	1262	1207
4000	1	μ	1798	1710	1834	1633	1666	1591
		λ	1218	1220	1245	1189	1194	1142
	2	μ	1284	1507	1591	1453	1472	1414
		λ	1334	1289	1326	1254	1261	1206

5. Simulations

In this section, we report the obtained results of a simulation study, which was carried out by software R, to compare the performance of the MLEs and Bayes estimates based on first-failure progressive hybrid censoring schemes. This simulation has done by considering different values of n, m, k and T, and by choosing $\mu = 1.5$ and $\lambda = 1.5$ in all cases. We have used three different censoring schemes:
Scheme I: $R_1 = n - m, \quad R_i = 0$ for $i \neq 1$.
Scheme II: $R_{m/2} = n - m, \quad R_i = 0$ for $i \neq m/2$.
Scheme III: $R_m = n - m, \quad R_i = 0$ for $i \neq m$.

In the Bayes estimates, we have chosen the hyper-parameters in such a way that the prior mean became the expected value of the corresponding parameter($a = 1.5, b = 1, c = 1.5$ and $d = 1$). We simulate the whole process 1000 times and compute biases and MSEs of different estimates. Also, we obtain the average lengths of 95% confidence/HPD credible intervals and the CPs of the parameters based on simulation.

To save space, we only present part of results. The results of the Monte Carlo simulation study are given in Tables 3-5. From these tables the following conclusions are made:

(1) From the reported values, we observe that the biases and MSEs of the parameters decrease as the value of T increases, and the average lengths of the confidence/HPD credible interval decrease when the value of T increases. This can be explained by the fact that more failures can be observed on average for larger value of T. It is also observed that when (n, m) increases, the biases and MSEs decrease for both MLEs and Bayesian estimators. The biases and MSEs for the all estimates based on first-failure progressive hybrid censoring schemes with $k = 2$ are similar to those for first-failure progressive hybrid censoring with $k = 3$.

(2) In most simulations, the Bayes estimates outperform the MLEs for estimation of λ, however, the MLEs outperform the Bayes estimates for estimation of μ. We can easily notice that scheme II gives the smallest bias and MSEs among the other schemes for estimation of λ, but the scheme I gives the smallest bias and MSEs among the other schemes for estimation of μ. Tables 3-4 show that Bayes estimates of λ, μ based on asymmetric loss function (Linex and general entropy) are sensitive to the value of the scale parameters β and q. The Bayes estimates based on symmetric and asymmetric loss functions are all perform well.

(3) For interval estimation, we can see that CPs for λ, μ based on HPD intervals are always close to the desired level of 95%. The CP for λ based on the approximate confidence intervals is close to the desired level of 95%, however, the CP for μ based on the approximate confidence intervals is always less than the desired level. HPD credible interval are better than confidence intervals in respect of average length. As k increases, the length of confidence interval and HPD credible intervals for λ narrow down. For fixed n, m, as T increases, the average interval lengths decrease and the corresponding CPs increase. The average lengths of all intervals become shorter as (n, m) increases.

It can be conclude that the asymmetric loss function make the Bayes estimates attractive for use in really, the scale parameters β and q of the Linex loss function and general entropy function make one to estimate the unknown parameters with more flexibility. So, we would recommend to use the Bayes estimate of the unknown parameters of IG distribution based on first-failure progressive hybird censoring scheme.

Table 3 Biases and MSEs (in the parenthese) of μ for different methods

						Bayes				
						BS	B	L	B	GE
n	m	k	T	CS	MLE	q=-1	β =-0.5	β =0.5	q=-0.5	q=0.5
40	30	2	1	I	0.2709	0.2845	0.3599	0.2175	0.2512	0.1872
					(0.6328)	(0.4083)	(0.5389)	(0.3113)	(0.3698)	(0.303)
				II	0.2055	0.3408	0.4292	0.2615	0.3031	0.2302
					(0.9459)	(0.5259)	(0.7056)	(0.3845)	(0.4741)	(0.3822)
				III	0.2055	0.2776	0.3509	0.2133	0.2459	0.1854
					(0.176)	(0.3897)	(0.5192)	(0.2954)	(0.3519)	(0.2866)
			2	I	0.04144	0.21104	0.26831	0.16062	0.18527	0.13625
					(0.1854)	(0.2642)	(0.3512)	(0.1988)	(0.2384)	(0.1939)
				II	0.08206	0.24947	0.31310	0.19355	0.22149	0.16823
					(0.3374)	(0.3544)	(0.4605)	(0.274)	(0.324)	(0.271)
				III	0.02034	0.23187	0.29859	0.17336	0.20230	0.14600
					(0.1935)	(0.3231)	(0.4288)	(0.2443)	(0.2919)	(0.2381)
		3	1	I	0.09018	0.2877	0.3659	0.2177	0.2536	0.1875
					(0.1374)	(0.4156)	(0.5579)	(0.3061)	(0.3729)	(0.298)
				II	0.1943	0.3104	0.3960	0.2346	0.2735	0.2025
					(0.506)	(0.4554)	(0.6153)	(0.3372)	(0.4089)	(0.3278)
				III	0.09718	0.2633	0.3418	0.1946	0.2288	0.1628
					(0.1623)	(0.4011)	(0.5372)	(0.304)	(0.3626)	(0.2975)
			2	I	0.02422	0.2278	0.2926	0.1707	0.1989	0.1440
					(0.1346)	(0.3107)	(0.4128)	(0.2354)	(0.2801)	(0.2277)
				II	0.04144	0.23370	0.30236	0.17314	0.20309	0.14452
					(0.3327)	(0.3272)	(0.4397)	(0.2462)	(0.2942)	(0.238)
				III	0.04616	0.24599	0.32237	0.17878	0.21177	0.14649
					(0.4457)	(0.3533)	(0.4725)	(0.2654)	(0.3178)	(0.2575)
60	50	2	1	I	0.06693	0.22463	0.28334	0.17290	0.19845	0.14849
					(0.2538)	(0.269)	(0.3562)	(0.2048)	(0.2417)	(0.1949)
				II	0.06395	0.24085	0.30249	0.18649	0.21361	0.16163
					(0.246)	(0.3065)	(0.4037)	(0.2342)	(0.2769)	(0.2261)
				III	0.05139	0.18951	0.24015	0.14550	0.16669	0.12355
					(0.2509)	(0.2191)	(0.2861)	(0.1706)	(0.1984)	(0.1632)
			2	I	0.03379	0.16811	0.20590	0.13510	0.15049	0.11714
					(0.09412)	(0.1509)	(0.1924)	(0.1209)	(0.1373)	(0.1142)
				II	0.02335	0.17319	0.21428	0.13743	0.15416	0.11824
					(0.08948)	(0.1714)	(0.2195)	(0.1369)	(0.1564)	(0.1312)
				III	-0.008958	0.132423	0.170326	0.099387	0.114333	0.080278
					(0.08664)	(0.1377)	(0.1747)	(0.1113)	(0.1258)	(0.106)
		3	1	I	0.05719	0.22907	0.30916	0.16816	0.19810	0.14200
					(0.2208)	(0.2456)	(0.3749)	(0.176)	(0.2142)	(0.1657)
				II	0.07521	0.24521	0.32955	0.18026	0.21262	0.15335
					(0.3215)	(0.2829)	(0.4209)	(0.2018)	(0.2482)	(0.1928)
				III	0.01328	0.19026	0.27598	0.13362	0.16034	0.10806
					(0.1928)	(0.1932)	(0.297)	(0.1434)	(0.1702)	(0.1356)
			2	I	0.01735	0.16561	0.21627	0.12554	0.14403	0.10460
					(0.1009)	(0.1508)	(0.2079)	(0.1176)	(0.1353)	(0.111)
				II	0.00813	0.04053	0.04268	0.03839	0.03927	0.03672
					(0.1249)	(0.03246)	(0.03359)	(0.03137)	(0.03195)	(0.03096)
				III	0.006098	0.140777	0.166915	0.115411	0.128056	0.102456
					(0.142)	(0.2325)	(0.2809)	(0.1911)	(0.2183)	(0.1913)

Table 4 Biases and MSEs (in the parenthese) of λ for different methods

						Bayes				
						BS	B	L	B	GE
n	m	k	T	CS	MLE	q=-1	β =-0.5	β =0.5	q=-0.5	q=0.5
40	30	2	1	I	0.124889	0.064259	0.098332	0.032945	0.045129	0.007443
					(0.19)	(0.1162)	(0.134)	(0.1029)	(0.1104)	(0.1015)

				II	0.09435	0.03549	0.06608	0.00722	0.01804	-0.01631
					(0.1836)	(0.1147)	(0.1299)	(0.1033)	(0.1101)	(0.103)
				III	0.13067	0.06666	0.09741	0.03806	0.04925	0.01483
					(0.1761)	(0.1144)	(0.1306)	(0.1018)	(0.1089)	(0.1002)
			2	I	0.109563	0.055906	0.084470	0.029051	0.039224	0.006073
					(0.1474)	(0.1055)	(0.1181)	(0.09567)	(0.1014)	(0.09488)
				II	0.101993	0.044833	0.071249	0.019918	0.029365	-0.001385
					(0.1516)	(0.1046)	(0.1167)	(0.09516)	(0.1008)	(0.09462)
				III	0.14044	0.06540	0.09610	0.03682	0.04808	0.01378
					(0.1981)	(0.1243)	(0.1421)	(0.1105)	(0.1186)	(0.1092)
		3	1	I	0.0896004	0.0439543	0.0695469	0.0200317	0.0291171	-0.001002
					(0.1375)	(0.09331)	(0.104)	(0.08503)	(0.08979)	(0.08423)
				II	0.092799	0.043116	0.066954	0.020763	0.029254	0.001965
					(0.1287)	(0.08316)	(0.09261)	(0.07577)	(0.0799)	(0.07468)
				III	0.14001	0.06822	0.09482	0.04346	0.05323	0.02377
					(0.1625)	(0.1007)	(0.1141)	(0.09002)	(0.09592)	(0.088)
			2	I	0.11545	0.06016	0.08422	0.03744	0.04615	0.01841
					(0.1349)	(0.09438)	(0.1048)	(0.086)	(0.09072)	(0.0847)
				II	0.102255	0.045081	0.067029	0.024303	0.032184	0.006657
					(0.1244)	(0.08527)	(0.0942)	(0.07814)	(0.08221)	(0.0772)
				III	0.14395	0.07281	0.09962	0.04788	0.05778	0.02827
					(0.1705)	(0.105)	(0.1192)	(0.0938)	(0.09996)	(0.09159)
60	50	2	1	I	0.07509	0.03101	0.05253	0.01061	0.01814	-0.00735
					(0.1095)	(0.08114)	(0.08839)	(0.07542)	(0.07879)	(0.07515)
				II	0.080868	0.031766	0.052350	0.012187	0.019430	-0.005011
					(0.1053)	(0.07598)	(0.08284)	(0.07052)	(0.07367)	(0.07002)
				III	0.076843	0.033620	0.053026	0.015065	0.021857	-0.001507
					(0.09073)	(0.06803)	(0.07376)	(0.06345)	(0.06604)	(0.06294)
			2	I	0.073288	0.034072	0.051847	0.016953	0.023170	0.001435
					(0.08798)	(0.06954)	(0.07463)	(0.06538)	(0.06782)	(0.05791)
				II	0.0562112	0.0155255	0.0319461	-0.0033	0.0052780	-0.015163
					(0.07239)	(0.05868)	(0.06234)	(0.05576)	(0.05754)	(0.05591)
				III	0.09183	0.04672	0.06513	0.02903	0.03560	0.01345
					(0.0959)	(0.07446)	(0.08067)	(0.06933)	(0.07224)	(0.06858)
		3	1	I	0.065902	0.026608	0.043546	0.010350	0.016256	-0.004284
					(0.08306)	(0.06317)	(0.06771)	(0.05948)	(0.06161)	(0.05917)
				II	0.0710955	0.0302549	0.0464949	0.0146406	0.0203738	0.0007553
					(0.08636)	(0.06381)	(0.06852)	(0.05992)	(0.06212)	(0.05937)
				III	0.09771	0.04949	0.06622	0.03336	0.03937	0.01923
					(0.08714)	(0.06328)	(0.0686)	(0.05882)	(0.06123)	(0.05778)
			2	I	0.0639854	0.0280843	0.0428328	0.0138029	0.0189305	0.0006881
					(0.06564)	(0.05271)	(0.0561)	(0.04991)	(0.05149)	(0.04958)
				II	0.073318	0.015080	0.022072	0.008150	0.010586	0.001569
					(0.0666)	(0.04237)	(0.04357)	(0.04131)	(0.04192)	(0.04119)
				III	0.08514	0.03890	0.05211	0.02618	0.03089	0.01501
					(0.08934)	(0.06394)	(0.06777)	(0.06069)	(0.06252)	(0.0601)

Table 5 The average confidence/credible lengths and CPs (in the parenthese) of μ and λ for different methods

					μ		λ	
n	m	k	T	CS	MLE	Bayes	MLE	Bayes
40	30	2	1	I	1.85(0.87)	1.73(0.939)	1.595(0.956)	1.331(0.956)
				II	2.742(0.89)	1.85(0.951)	1.526(0.955)	1.26(0.944)
				III	2.34(0.869)	1.677(0.942)	1.492(0.963)	1.268(0.962)
			2	I	1.602(0.88)	1.502(0.959)	1.391(0.96)	1.238(0.962)
				II	1.927(0.872)	1.58(0.937)	1.345(0.944)	1.189(0.95)
				III	1.742(0.864)	1.605(0.953)	1.47(0.956)	1.262(0.958)
		3	1	I	2.815(0.885)	1.738(0.946)	1.371(0.954)	1.165(0.95)
				II	1.757(0.852)	1.745(0.95)	1.356(0.966)	1.137(0.958)
				III	1.865(0.857)	1.732(0.94)	1.409(0.96)	1.174(0.96)
			2	I	1.686(0.868)	1.576(0.947)	1.286(0.956)	1.134(0.956)
				II	1.973(0.862)	1.634(0.937)	1.245(0.965)	1.085(0.958)
				III	2.333(0.846)	1.712(0.946)	1.413(0.953)	1.174(0.951)

60	50	2	1	I	1.812(0.9)	1.511(0.945)	1.227(0.948)	1.081(0.951)
				II	1.821(0.881)	1.54(0.952)	1.203(0.953)	1.056(0.956)
				III	1.623(0.907)	1.404(0.949)	1.153(0.949)	1.033(0.954)
			2	I	1.269(0.91)	1.243(0.956)	1.076(0.965)	0.9957(0.958)
				II	1.205(0.917)	1.285(0.956)	1.039(0.97)	0.9594(0.958)
				III	1.155(0.89)	1.247(0.95)	1.099(0.952)	1.008(0.946)
		3	1	I	1.675(0.899)	1.681(0.958)	1.068(0.946)	0.9645(0.948)
				II	1.871(0.888)	1.744(0.96)	1.051(0.948)	0.9426(0.96)
				III	1.48(0.874)	1.616(0.956)	1.054(0.96)	0.9623(0.959)
			2	I	1.237(0.902)	1.396(0.961)	0.9812(0.962)	0.9094(0.961)
				II	1.298(0.874)	1.2903(0.95)	0.9683(0.96)	0.9137(0.96)
				III	1.412(0.888)	1.304(0.951)	1.042(0.946)	0.8636(0.934)

6. Conclusions

In this paper, we combines the concepts of progressive first-failure censoring and Type-I censoring to develop a new life plan called a first-failure progressive hybird censoring scheme. The Bayes and classical estimates the unknown parameters of IG distribution have been obtained based on this new censoring scheme. We computed Bayes estimators of the unknown parameters under square error, Linex and general entropy loss functions. The MLEs and Bayes estimates cannot be obtained in closed form, but can be derived numerically. The asymptotic confidence intervals and coverage probabilities for the parameters are obtained based on the observed Fisher's information matrix. Also, highest posterior density credible intervals for the parameters are computed using Gibbs sampling procedure. From our study, we find Bayes estimates for λ are better in term of biases and MSEs. For intervals estimation of λ and μ, HPD credible intervals is recommended.

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