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Fixed Point of Single-Valued Cyclic Weak φ_F-Contraction Mappings

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Abstract. Fixed point results are presented for single-valued cyclic weakly φ_F-contractive mappings on complete metric spaces (X, d), where $\varphi : [0, +\infty) \longrightarrow [0, +\infty)$ is a function with $\varphi^{-1}(0) = \{0\}$, $\varphi(t) < t$ for all $t > 0$ and $\varphi(t_n) \to 0$ implies $t_n \to 0$, and $F : [0, +\infty) \longrightarrow [0, +\infty)$ is continuous with $F^{-1}(0) = \{0\}$ and $F(t_n) \to 0$ implies $t_n \to 0$. Our results extend previous results given by Rhoades (2001)[20], Moradi and Beiranvand (2010)[13], Amini-Harandi (2010)[2] and Karapinar (2011)[11].

1. Introduction

Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be a $\varphi-$weak contraction if there exists a map $\varphi[0, +\infty) \longrightarrow [0, +\infty)$ with $\varphi^{-1}(0) = \{0\}$ such that

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) \tag{1}$$

for all $x, y \in X$.
The concept of the $\varphi-$weak contraction was defined by Alber and Guerre-Delabriere [1] in 1977. Rhoades [20, Theorem 2] proved the following fixed point theorem for $\varphi-$weak contraction single-valued mappings, giving another generalization of the Banach contraction principle.

Theorem 1.1. *Let (X, d) be a complete metric space and let $T : X \to X$ be a mapping such that*

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)) \tag{2}$$

for all $x, y \in X$(i.e. it is $\varphi-$weakly contractive), where $\varphi[0, +\infty) \longrightarrow [0, +\infty)$ is a continuous and nondecreasing function with $\varphi^{-1}(0) = \{0\}$. Then, T has a unique fixed point.

By choosing $\psi(t) = t - \varphi(t)$, $\varphi-$weak contractions become mappings of Boyd and Wong type [4], and on defining $k(t) = \frac{1-\varphi(t)}{t}$ for $t > 0$ and $k(0) = 0$, then $\varphi-$weak contractions become mappings of Reich [21].
In fixed point theory, $\varphi-$weak contraction has been studied by many authors, see for example [6], [11]-[18], [22, 23], and the references therein.
In (2010) Amini-Harandi [2] proved the following theorem on the existence of a fixed point for a single-valued mapping.

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Theorem 1.2. *Let (X,d) be a complete metric space and let $T : X \to X$ be a mapping satisfies*

$$d(Tx,Ty) \leq \psi(d(x,y)) \tag{3}$$

for each $x,y \in X$, where $\psi[0,+\infty) \longrightarrow [0,+\infty)$ is upper semicontinuous, $\psi(t) < t$ for each $t > 0$ and satisfies $\liminf_{t\to\infty}(t - \psi(t)) > 0$. Then, T has a fixed point.

In (2010) Păcurar [19] presented the following definitions.

Definition 1.3. *Let X be a non-empty set, m a positive integer and $T : X \to X$ an operator. By definition, $X = \cup_{i=1}^{m} X_i$ is a cyclic representation on X with respect to T if*

(1) *$X_i, i = 1, ..., m$ are non-empty sets;*

(2) *$T(X_1) \subseteq X_2, T(X_2) \subseteq X_3, ..., T(X_{m-1}) \subseteq X_m, T(X_m) \subseteq X_1$.*

Definition 1.4. *Let (X,d) be a metric space, m a positive integer, $A_1, A_2, ..., A_m$ closed non-empty subsets of X and $Y = \cup_{i=1}^{m} A_i$. An operator $T : Y \to Y$ is called a cyclic weak $\varphi-$contraction if*

(1) *$\cup_{i=1}^{m} A_i$ is a cyclic representation of Y with respect to T, and*

(2) *there exists a continuous, non-decreasing function $\varphi : [0,+\infty) \to [0,+\infty)$ with $\varphi(t) > 0$ for $t > 0$ and $\varphi(0) = 0$, such that*

$$d(Tx,Ty) \leq d(x,y) - \varphi(d(x,y)) \tag{4}$$

for any $x \in A_i, y \in A_{i+1}, i = 1, 2, ..., m$, where $A_{m+1} = A_1$.

Recently, Karapinar [11] proved the following theorem on the existence of fixed point for cyclic weak $\varphi-$contraction mappings.

Theorem 1.5. *Let (X,d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, ..., A_m$ closed non-empty subsets of X and $Y = \bigcup_{i=1}^{m} A_i$. Let $T : Y \to Y$ be a cyclic weak $\phi-$ contractive mapping, where $\phi[0,+\infty) \longrightarrow [0,+\infty)$ with $\phi(t) > 0$ is a continuous function for $t \in (0,+\infty)$, and $\phi(0) = 0$. Then, T has a unique fixed point $z \in \bigcap_{i=1}^{m} A_i$.*

There are another results on the existence of fixed point for cyclic mappings, see for example [3], [7], [8], [9] and [10].

In Section 3, we extend Rhoades, Moradi and Beiranvand, Amini-Harandi and Karapinar' results.

2. Preliminaries

In this work, (X,d) denote a complete metric space. We introduce the notation $\mathcal{F}$ for all continuous mappings $F : [0,+\infty) \longrightarrow [0,+\infty)$ with $F^{-1}(0) = \{0\}$, and satisfies the following condition:

$$F(t_n) \to 0 \ \textit{implies} \ t_n \to 0. \tag{5}$$

Let Ψ be the class of all nondecreasing mapping $\psi : [0,+\infty) \longrightarrow [0,+\infty)$ with $\psi^{-1}(0) = \{0\}$ and $\psi(t) < t$ for all $t > 0$.
Also we introduce the notation Φ for all mappings $\varphi : [0,+\infty) \longrightarrow [0,+\infty)$ with $\varphi^{-1}(0) = \{0\}$ and $\varphi(t) < t$ for all $t > 0$ and satisfies the following condition:

$$\varphi(t_n) \to 0 \ \textit{implies} \ t_n \to 0. \tag{6}$$

Obviously $\Psi \subset \Phi$. Also, every l.s.c. mapping $\varphi : [0,+\infty) \longrightarrow [0,+\infty)$ with $\varphi^{-1}(0) = \{0\}$, $\varphi(t) < t$ for all $t > 0$ and $\liminf_{t\to\infty} \varphi(t) > 0$ belong to Φ.
At last, suppose Ω be the class of all mappings $\varphi : [0,+\infty) \longrightarrow [0,+\infty)$ with $\varphi^{-1}(0) = \{0\}$ and satisfies the following condition:
"for every interval $[a,b] \subset (0,+\infty)$, there exists $\alpha \in (0,1)$ such that $t - \varphi(t) \leq \alpha t$ for all $t \in [a,b]$."

In Section 3 we show that $\Phi \subset \Omega$.

Definition 2.1. *Let (X, d) be a metric space, m a positive integer, $A_1, A_2, ..., A_m$ closed non-empty subsets of X and $Y = \cup_{i=1}^{m} A_i$. An operator $T : Y \to Y$ is called a cyclic weak φ_F-contraction if*

(1) *$\cup_{i=1}^{m} A_i$ is a cyclic representation of Y with respect to T, and*

(2) *there exist two mappings $\varphi, F : [0, +\infty) \to [0, +\infty)$ with $F^{-1}(0) = \varphi^{-1}(0) = \{0\}$ and $\varphi(t) < t$ for all $t > 0$ such that*

$$F(d(Tx, Ty)) \leq F(d(x, y)) - \varphi(F(d(x, y))) \tag{7}$$

for any $x \in A_i, y \in A_{i+1}, i = 1, 2, ..., m$, where $A_{m+1} = A_1$.

3. Main Results

At first we prove the following useful lemma.

Lemma 3.1. *Let $\varphi \in \Phi$. Then for every closed interval $[a, b] \subset (0, +\infty)$ there exists $\alpha \in (0, 1)$ such that*

$$t - \varphi(t) \leq \alpha t \tag{8}$$

for all $t \in [a, b]$.

Proof. Suppose for every $\alpha \in (0, 1)$ there exists $t \in [a, b]$ such that $t - \varphi(t) > \alpha t$. Hence for a sequence $\{\alpha_n\}_{n=1}^{\infty} \subset (0, 1)$ with $\lim_{n\to\infty} \alpha_n = 1$, there exists a sequence $\{t_n\}_{n=1}^{\infty} \subset [a, b]$ such that $t_n - \varphi(t_n) > \alpha_n t_n$, for all $n \in \mathbb{N}$. Therefore, $0 \leq \varphi(t_n) < (1 - \alpha_n)t_n$, for all $n \in \mathbb{N}$. Since $\lim_{n\to\infty} \alpha_n = 1$ and $\{t_n\}_{n=1}^{\infty} \subset [a, b]$, $\lim_{n\to\infty}(1 - \alpha_n)t_n = 0$. Therefore, $\lim_{n\to\infty} \varphi(t_n) = 0$. Since $\varphi \in \Phi$, then $\lim_{n\to\infty} t_n = 0$ and this is a contradiction. □

The following theorem extends Rhoades [20], Amini-Harandi [2], Karapinar [11], Moradi and Beiranvand [13] and Branciari's results [5].

Theorem 3.2. *Let (X, d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, ..., A_m$ closed non-empty subsets of X and $Y = \bigcup_{i=1}^{m} A_i$. Suppose that $\varphi \in \Omega$ and $F \in \mathcal{F}$. Let $T : Y \to Y$ be a cyclic weak ϕ_F- contractive mapping. Then, T has a unique fixed point $x \in \bigcap_{i=1}^{m} A_i$.*

Proof. Let $x_1 \in Y$, and set $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. We may assume that $x_1 \in A_1$. Notice that for any n, there exists $i_n \in \{1, 2, \cdots, m\}$ such that $x_n \in A_{i_n}$ and $x_{n+1} \in A_{i_n+1}$. So $x_1 \in A_1, x_2 \in A_2, \cdots, x_m \in A_m, x_{m+1} \in A_1, x_{m+2} \in A_2, \cdots, x_{2m} \in A_m, x_{2m+1} \in A_1, \cdots$.
At first we show that $\lim\limits_{n\to\infty} d(x_{n+1}, x_n) = 0$. Using (6), for all $n \in \mathbb{N}$

$$F(d(x_{n+2}, x_{n+1})) \leq F(d(x_{n+1}, x_n)) - \varphi(F(d(x_{n+1}, x_n))). \tag{9}$$

So the sequence $\{F(d(x_{n+1}, x_n))\}$ is monotone nonincreasing and bounded below. Hence, there exists $r \geq 0$ such that

$$\lim_{n\to\infty} F(d(x_{n+1}, x_n)) = r. \tag{10}$$

If $r > 0$, then there exists $\varepsilon > 0$ such that $r - \varepsilon > 0$. From (10), there exists $N_0 \in \mathbb{N}$ such that for all $n \geq N_0$, $F(d(x_{n+1}, x_n)) \in [r - \varepsilon, r + \varepsilon]$. Since $\varphi \in \Omega$, there exists $\alpha \in (0, 1)$ such that

$$t - \varphi(t) \leq \alpha t \tag{11}$$

for all $t \in [r - \varepsilon, r + \varepsilon]$. Hence for all $n \geq N_0$, from (9)

$$F(d(x_{n+2}, x_{n+1})) \leq \alpha F(d(x_{n+1}, x_n)). \tag{12}$$

Since $F \in \mathcal{F}$, letting $n \to \infty$ in (12) we get $F(r) \le \alpha F(r)$. Since $\alpha \in (0,1)$, then $F(r) = 0$ and hence $r = 0$. So from $F \in \mathcal{F}$ and (10) we conclude that

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = 0. \tag{13}$$

Using triangular inequality and above inequality

$$\lim_{n\to\infty} d(x_{n+l}, x_n) = 0, \tag{14}$$

for all $l \in \{1, 2, \cdots, m\}$.
Now we show that $\{x_n\}$ is a Cauchy sequence.
Suppose that $\{x_n\}$ is not Cauchy. So there exists $a > 0$ and sequence $\{n(k)\}$ such that $n(k+1) > n(k)$ is minimal in the sense that $d(x_{n(k+1)}, x_{n(k)}) > a$. Obviously, $n(k) \ge k$ for all $k \in \mathbb{N}$. Using (13), there exists $N_0 \in \mathbb{N}$ such that for all $k \ge N_0$, $d(x_{k+1}, x_k) > \frac{a}{3}$. So for all $k \ge N_0$, $n(k+1) - n(k) \ge 2$ and

$$\begin{aligned} a &< d(x_{n(k+1)}, x_{n(k)}) \\ &\le d(x_{n(k+1)}, x_{n(k+1)-1}) + d(x_{n(k+1)-1}, x_{n(k)}) \\ &\le d(x_{n(k+1)}, x_{n(k+1)-1}) + a. \end{aligned} \tag{15}$$

Letting $k \to \infty$ in above inequality, we get

$$\lim_{k\to\infty} d(x_{n(k+1)}, x_{n(k)}) = a. \tag{16}$$

Suppose that $m(1) = n(1), m(2) = n(2) + l_2$, where $l_2 \in \{0, 1, \cdots, m-1\}$ such that $m(2) \equiv m(1) + 1(mod\ m)$, $m(3) = n(3), m(4) = n(4) + l_4$, where $l_4 \in \{0, 1, \cdots, m-1\}$ such that $m(4) \equiv m(3) + 1(mod\ m)$, $\cdots$, $m(2k-1) = n(2k-1), m(2k) = n(2k) + l_{2k}$, where $l_{2k} \in \{0, 1, \cdots, m-1\}$ such that $m(2k) \equiv m(2k-1) + 1(mod\ m)$ and $\cdots$.
For all $k \in \mathbb{N}$

$$\begin{aligned} a &\le d(x_{n(2k)}, x_{n(2k-1)}) \\ &\le d(x_{n(2k)}, x_{n(2k)+l_{2k}}) + d(x_{n(2k)+l_{2k}}, x_{n(2k-1)}) \\ &= d(x_{n(2k)}, x_{n(2k)+l_{2k}}) + d(x_{m(2k)}, x_{m(2k-1)}) \\ &\le 2d(x_{n(2k)}, x_{n(2k)+l_{2k}}) + d(x_{n(2k)}, x_{n(2k-1)}). \end{aligned} \tag{17}$$

Using (14), (16) and above inequality, we conclude that

$$\lim_{k\to\infty} d(x_{m(2k)}, x_{m(2k-1)}) = a. \tag{18}$$

Since $F \in \mathcal{F}$ and (18) holds, then

$$\lim_{k\to\infty} F(d(x_{m(2k)}, x_{m(2k-1)})) = F(a). \tag{19}$$

Also,

$$\begin{aligned} d(x_{m(2k)}, x_{m(2k-1)}) &\le d(x_{m(2k)}, x_{m(2k)-1}) + d(x_{m(2k)-1}, x_{m(2k-1)-1}) + d(x_{m(2k-1)-1}, x_{m(2k-1)}) \\ &\le 2d(x_{m(2k)}, x_{m(2k)-1}) + d(x_{m(2k)}, x_{m(2k-1)}) + 2d(x_{m(2k-1)-1}, x_{m(2k-1)}). \end{aligned} \tag{20}$$

From (13), (18) and above inequality

$$\lim_{k\to\infty} d(x_{m(2k)-1}, x_{m(2k-1)-1}) = a. \tag{21}$$

Hence,

$$\lim_{k\to\infty} F(d(x_{m(2k)-1}, x_{m(2k-1)-1})) = F(a). \tag{22}$$

From (6) and $m(2k) \equiv m(2k-1) + 1 (mod\ m)$ for all $k \in \mathbb{N}$, we have

$$F(d(x_{m(2k)}, x_{m(2k-1)})) \leq F(d(x_{m(2k)-1}, x_{m(2k-1)-1})) - \varphi(F(d(x_{m(2k)-1}, x_{m(2k-1)-1}))). \tag{23}$$

If $F(a) > 0$ then for some $\varepsilon > 0$, $F(a) - \varepsilon > 0$. From (19) and (20), there exists $N_0 \in \mathbb{N}$ such that for all $n \geq N_0$, $F(d(x_{m(2k)}, x_{m(2k-1)})), F(d(x_{m(2k)-1}, x_{m(2k-1)-1})) \in [F(a) - \varepsilon, F(a) + \varepsilon]$. Since $\varphi \in \Omega$, there exists $\alpha \in (0, 1)$ such that

$$t - \varphi(t) \leq \alpha t \tag{24}$$

for all $t \in [F(a) - \varepsilon, F(a) + \varepsilon]$. Hence for all $k \geq N_0$, from (23)

$$F(d(x_{m(2k)}, x_{m(2k-1)})) \leq \alpha F(d(x_{m(2k)-1}, x_{m(2k-1)-1})). \tag{25}$$

Letting $k \to \infty$ in above inequality, we get, $F(a) \leq \alpha F(a)$. Since $\alpha \in (0, 1)$, then $F(a) = 0$ and hence $a = 0$ and this is a contradiction.
Therefore $\{x_n\}$ is Cauchy.
Since (X, d) is complete and $\{x_n\}$ is Cauchy, there exists $x \in X$ such that $\lim_{n\to\infty} x_n = x$. From $\lim_{n\to\infty} x_{nm+i} = x$, $\{x_{nm+i} : n \in \mathbb{N}\} \subseteq A_i$ and A_i is closed, we conclude that $x \in A_i$ for $i = 1, 2, \cdots, m$. Therefore $x \in \bigcap_{i=1}^m A_i$.
For all $n \in \mathbb{N}$, from (6) and $x \in \bigcap_{i=1}^m A_i$

$$\begin{aligned} F(d(x_{n+1}, Tx)) &= F(d(Tx_n, Tx)) \\ &\leq F(d(x_n, x)) - \varphi(F(d(x_n, x))) \\ &\leq F(d(x_n, x)). \end{aligned} \tag{26}$$

Letting $n \to \infty$ in above inequality, we get

$$F(d(x, Tx)) \leq \alpha F(d(x, x)) = 0. \tag{27}$$

Therefore $F(d(x, Tx)) = 0$. So $d(x, Tx) = 0$ and hence, $Tx = x$. Thus T has a fixed point $x \in \bigcap_{i=1}^m A_i$.
Uniqueness of the fixed point in $\bigcap_{i=1}^m A_i$ follows from (8) and this completes the proof. □

Remark 3.3. *By taking $A_1 = A_2 = \cdots = A_m = X$ and define $\varphi(t) = t - \psi(t)$, we can generalized Theorem 1.2.*

Theorem 3.4. *Let $T : Y \to Y$ be a mapping as in Theorem 3.1 and $F(t) = t$. Then the fixed point problem for T is well-posed, that is, if there exists a sequence $\{y_n\}$ in Y with $\lim_{n\to\infty} d(y_n, Ty_n) = 0$, then $\lim_{n\to\infty} y_n = x$ (x is fixed point of T in $x \in \bigcap_{i=1}^m A_i$).*

Proof. Since $x \in \bigcap_{i=1}^m A_i$ and $y_n \in Y$, from (6)

$$d(y_n, x) \leq d(y_n, Ty_n) + d(Ty_n, Tx) \leq d(y_n, Ty_n) + d(y_n, x) - \varphi(d(y_n, x)), \tag{28}$$

Therefore $\lim_{n\to\infty} \varphi(d(y_n, x)) = 0$. Since $\varphi \in \Phi$, then $\lim_{n\to\infty} d(y_n, x) = 0$ and this completes the proof. □

Theorem 3.5. *Let $T : Y \to Y$ be a mapping as in Theorem 3.1 and $F(t) = t$. Then T has the limit shadowing property, that is, if there exists a convergent sequence $\{y_n\}$ in Y with $\lim_{n\to\infty} d(y_{n+1}, Ty_n) = 0$, then there exists $x \in Y$ such that $\lim_{n\to\infty} d(y_n, T^n x) = 0$.*

Proof. Let $x \in \bigcap_{i=1}^m A_i$ be the fixed point of T. With a method similar to that in Theorem 3.3 we can conclude this theorem. □

The following theorem is a direct result of Theorem 3.2, where extends Theorem 1.2.

Theorem 3.6. *Let (X, d) be a complete metric space and let $T : X \to X$ be a mapping satisfies*

$$F(d(Tx, Ty)) \leq \psi(F(d(x, y))) \tag{29}$$

for all $x, y \in X$, where $F \in \Psi$ and $\psi : [0, +\infty) \to [0, +\infty)$ is upper semi-continuous with $\psi(t) < t$ for all $t > 0$ and satisfies $\liminf_{t\to\infty}(t - \psi(t)) > 0$. Then T has a unique fixed point.

Proof. Let $\phi(t) = t - \psi(t)$ and apply Theorem 3.2. $\square$

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