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Integral type Fixed point Theorems for α-admissible Mappings Satisfying α-ψ-ϕ-Contractive Inequality

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Abstract. In this paper, we establishe some new fixed point theorems by α-admissible mappings satisfying α-ψ-ϕ-contractive inequality of integral in complete metric spaces. Presented results can be considered as an extension of the theorems of Banach-Cacciopoli and Branciari.

1. Introduction and Preliminaries

The first well known result on fixed points for contractive mapping was Banach-Cacciopoli theorem(1922)[1], as follow:

Theorem 1.1. *Let (X, d) be a complete metric space, $c \in (0, 1)$, and let $f : X \to X$ be a mapping such that for each $x, y \in X$,*

$$d(fx, fy) \leq cd(x, y) \tag{1}$$

Then, f has a unique fixed point $a \in X$ such that for each $x \in X$, $lim_{n\to\infty} f^n x = a$.

Branciari(2002)[2], proved the following theorem:

Theorem 1.2. *Let (X, d) be a complete metric space, $c \in (0, 1)$, and let $f : X \to X$ be a mapping such that for each $x, y \in X$,*

$$\int_0^{d(fx,fy)} \phi(t)dt \leq c \int_0^{d(x,y)} \phi(t)dt \tag{2}$$

where $\phi : [0, \infty) \to [0, \infty)$ is a Lebesgue-integrable map which is summable, (i.e., with finite integral) on each compact subset of $[0, \infty)$, nonnegative, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \phi(t)dt > 0$; then f has a unique fixed point $a \in X$ such that for each $x \in X$, $lim_{n\to\infty} f^n x = a$.

Samet et al. (2012)[7] have introduced α-ψ-contractive type mappings and also α-admissible functions in complete metric spaces. In this paper, we introduce α-admissible mappings satisfying α-ψ-ϕ-contractive integral type inequality and we establish some fixed point theorems in complete metric spaces. Our results can be considered as an extension of Banach-Cacciopoli and Branciari theorems.

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Definition 1.3. *[7] Let $\alpha : X \times X \to [0,\infty)$ be a function, we say $f : X \to X$, is α-admissible if for all $x, y \in X$,*

$$\alpha(x,y) \geq 1 \Longrightarrow \alpha(f(x), f(y)) \geq 1 \tag{3}$$

Definition 1.4. *[7] Let Ψ be a family of nondecreasing functions $\psi : [0,\infty) \to [0,\infty)$ such that for each $\psi \in \Psi$ and $t > 0$, $\sum_{n=1}^{\infty} \psi^n(t) < +\infty$. where ψ^n is the n-th iterate of ψ.*

Lemma 1.5. *([7]) If $\psi : [0,\infty) \to [0,\infty)$ is nondecreasing function and for each $t > 0$, $\lim_{n\to\infty} \psi^n(t) = 0$ then $\psi(t) < t$.*

Definition 1.6. *Let Φ be a collection of mappings $\phi : [0,\infty) \to [0,\infty)$ which are Lebesgue-integrable, summable on each compact subset of $[0,\infty)$ and satisfying following condition:*

$$\int_0^{\epsilon} \phi(t)dt > 0, \quad for\ each\ \epsilon > 0.$$

Lemma 1.7. *([3]) Let $\phi \in \Phi$ and $\{r_n\}_{n\in\mathbb{N}}$ be a nonnegative sequence with $\lim_{n\to\infty} r_n = a$, then*

$$\lim_{n\to\infty} \int_0^{r_n} \phi(t)dt = \int_0^{a} \phi(t)dt.$$

Lemma 1.8. *([3]) Let $\phi \in \Phi$ and $\{r_n\}_{n\in\mathbb{N}}$ be a nonnegative sequence. Then*

$$\lim_{n\to\infty} \int_0^{r_n} \phi(t)dt = 0 \Longleftrightarrow \lim_{n\to\infty} r_n = 0.$$

2. Fixed Point Theorems

Definition 2.1. *Let (X,d) be a metric space and $f : X \to X$ be a given mapping. We say that f is an α-ψ-ϕ-contractive integral type mapping if there exist three functions $\alpha : X \times X \to [0,+\infty)$, $\phi \in \Phi$ and $\psi \in \Psi$ such that*

$$\int_0^{\alpha(x,y)d(fx,fy)} \phi(t)dt \leq \psi\left(\int_0^{d(x,y)} \phi(t)dt\right), \tag{4}$$

for all $x, y \in X$ and all $t \in [0,+\infty)$.

Theorem 2.2. *Let (X,d) be a complete metric space and $f : X \to X$ be a mapping satisfying following conditions:*

(i) f is α-admissible;
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$;
(iii) f is an α-ψ-ϕ-contractive integral type mapping,

then f has a unique fixed point $a \in X$ such that for $x_0 \in X$, $lim_{n\to\infty} f^n x_0 = a$.

Proof. First, we show that

$$\int_0^{d(f^n x_0, f^{n+1} x_0)} \phi(t)dt \leq \psi^n\left(\int_0^{d(x_0, fx_0)} \phi(t)dt\right). \tag{5}$$

Since f is α-admissible, from (ii) we get

$$\alpha(x_0, fx_0) \geq 1 \to \alpha(fx_0, f^2x_0) \geq 1 \to \ldots \to \alpha(f^n x_0, f^{n+1} x_0) \geq 1,$$

then for all $n \in \mathbb{N}$,

$$\alpha(f^n x_0, f^{n+1} x_0) \geq 1. \tag{6}$$

To prove (5), by induction and (i), we have for $n = 1$

$$\int_0^{d(fx_0,f^2x_0)} \phi(t)dt \le \int_0^{\alpha(x_0,fx_0)d(fx_0,f^2x_0)} \phi(t)dt \le \psi\left(\int_0^{d(x_0,fx_0)} \phi(t)dt\right).$$

Now, assuming that (5) holds for an $n \in \mathbb{N}$, then from (6) we get

$$\begin{aligned}\int_0^{d(f^{n+1}x_0,f^{n+2}x_0)} \phi(t)dt &\le \int_0^{\alpha(f^nx_0,f^{n+1}x_0)d(f^{n+1}x_0,f^{n+2}x_0)} \phi(t)dt \\ &\le \psi\left(\int_0^{d(f^nx_0,f^{n+1}x_0)} \phi(t)dt\right) \\ &\le \psi\left(\psi^n\left(\int_0^{d(x_0,fx_0)} \phi(t)dt\right)\right) \\ &= \psi^{n+1}\left(\int_0^{d(x_0,fx_0)} \phi(t)dt\right),\end{aligned}$$

therefore (5) holds for all $n \in \mathbb{N}$. Now, if $n \to +\infty$ we obtain

$$\int_0^{d(f^nx_0,f^{n+1}x_0)} \phi(t)dt \longrightarrow 0,$$

then, from Lemma 1.8 we have

$$d(f^nx_0, f^{n+1}x_0) \longrightarrow 0.$$

In the following, we show that $\{f^nx_0\}_{n=1}^{\infty}$ is a Cauchy sequence. Suppose that there exists an $\epsilon > 0$ such that for each $k \in \mathbb{N}$ there are $m_k, n_k \in \mathbb{N}$ with $m_k > n_k > k$, such that

$$d(f^{m_k}x_0, f^{n_k}x_0) \ge \epsilon, \quad d(f^{m_k-1}x_0, f^{n_k}x_0) < \epsilon. \tag{7}$$

Hence

$$\begin{aligned}\epsilon \le d(f^{m_k}x_0, f^{n_k}x_0) &\le d(f^{m_k}x_0, f^{m_k-1}x_0) + d(f^{m_k-1}x_0, f^{n_k}x_0) \\ &< d(f^{m_k}x_0, f^{m_k-1}x_0) + \epsilon.\end{aligned}$$

Letting $k \to +\infty$ then we have

$$d(f^{m_k}x_0, f^{n_k}x_0) \longrightarrow \epsilon^+, \tag{8}$$

this implies that there exists $l \in \mathbb{N}$, such that

$$k > l \Longrightarrow d(f^{m_l+1}x_0, f^{n_l+1}x_0) < \epsilon. \tag{9}$$

Actually, if there exists a subsequence $\{k_l\} \subset \mathbb{N}$, $k_l > l$,

$$d(f^{m_{k_l+1}}x_0, f^{n_{k_l+1}}x_0) \ge \epsilon,$$

if $l \to +\infty$ we get

$$\begin{aligned}\epsilon &\le d(f^{m_{k_l+1}}x_0, f^{n_{k_l+1}}x_0) \\ &\le d(f^{m_{k_l+1}}x_0, f^{m_{k_l}}x_0) + d(f^{m_{k_l}}x_0, f^{n_{k_l}}x_0) + d(f^{n_{k_l}}x_0, f^{n_{k_l+1}}x_0) \\ &< \epsilon\end{aligned}$$

therefore, from (4) and lemma 1.5 we have

$$\begin{aligned}\int_0^{d(f^{m_{k_l}+1}x_0,f^{n_{k_l}+1}x_0)}\phi(t)dt &\le \psi\left(\int_0^{d(f^{m_{k_l}}x_0,f^{n_{k_l}}x_0)}\phi(t)dt\right)\\ &< \int_0^{d(f^{m_{k_l}}x_0,f^{n_{k_l}}x_0)}\phi(t)dt,\end{aligned}$$

letting $l \to +\infty$, then we obtain

$$\int_0^{\epsilon}\phi(t)dt < \int_0^{\epsilon}\phi(t)dt,$$

which is a contradiction. Then, (9) holds. Now, we prove that there exists $\sigma_\epsilon \in (0,\epsilon), k_\epsilon \in \mathbb{N}$ such that

$$k > k_\epsilon \Longrightarrow d(f^{m_k+1}x_0, f^{n_k+1}x_0) < \epsilon - \sigma_\epsilon. \tag{10}$$

Assume that (10) is not true, from (9) there exists a subsequence $\{k_l\} \subset \mathbb{N}$,

$$d(f^{m_{k_l+1}}x_0, f^{n_{k_l+1}}x_0) \to \epsilon^-, \ \ as \ \ l \to +\infty.$$

Then from (4) and lemma 1.5, we get

$$\int_0^{d(f^{m_{k_l}+1}x_0,f^{n_{k_l}+1}x_0)}\phi(t)dt \le \psi\left(\int_0^{d(f^{m_{k_l}}x_0,f^{n_{k_l}}x_0)}\phi(t)dt\right) < \int_0^{d(f^{m_{k_l}}x_0,f^{n_{k_l}}x_0)}\phi(t)dt.$$

Suppose that $l \to +\infty$ then we obtain

$$\int_0^{\epsilon}\phi(t)dt < \int_0^{\epsilon}\phi(t)dt,$$

which is a contradiction. So, (10) holds. Therefore, $\{f^n x_0\}_{n=1}^{\infty}$ is a Cauchy sequence. In fact, for each $k > k_\epsilon$ we have

$$\begin{aligned}\epsilon \le d(f^{m_k}x_0, f^{n_k}x_0) &\le d(f^{m_k}x_0, f^{m_k+1}x_0) + d(f^{m_k+1}x_0, f^{n_k+1}x_0) + d(f^{n_k+1}x_0, f^{n_k}x_0)\\ &< d(f^{m_k}x_0, f^{m_k+1}x_0) + (\epsilon - \sigma_\epsilon) + d(f^{n_k+1}x_0, f^{n_k}x_0) \to \epsilon - \sigma_\epsilon \ \ as \ \ k \to +\infty.\end{aligned}$$

which is a contradiction. Since (X,d) is a complete metric space, then there exists $a \in X$ such that $\lim_{n\to\infty} f^n x_0 = a$. Now, we show that a is a fixed point of f. We claim that

$$d(fa,a) \le d(fa, f(f^n x_0)) + d(f^{n+1}x_0, a) \longrightarrow 0$$

to prove our claim it is sufficient to show that, $d(fa, f(f^n x_0)) \longrightarrow 0$. We have

$$\begin{aligned}0 < \int_0^{d(fa,f(f^n x_0))}\phi(t)dt &\le \int_0^{\alpha(a,f^n x_0)d(fa,f(f^n x_0))}\phi(t)dt\\ &\le \psi\left(\int_0^{d(a,f^n x_0)}\phi(t)dt\right)\\ &< \int_0^{d(a,f^n x_0)}\phi(t)dt,\end{aligned}$$

if $n \longrightarrow +\infty$ then

$$\int_0^{d(fa,f(f^n x_0))}\phi(t)dt \longrightarrow 0,$$

consequently, from lemma 1.8 we obtain

$$d(fa, f(f^n x_0)) \longrightarrow 0.$$

Therefore, $d(fa, a) = 0$ and so $fa = a$, which means a is a fixed point of f. a is a unique fixed point. Let b another fixed point of f, then we have

$$\begin{aligned}\int_0^{d(a,b)} \phi(t)dt &= \int_0^{d(fa,fb)} \phi(t)dt \\ &\le \int_0^{\alpha(a,b)d(fa,fb)} \phi(t)dt \\ &\le \psi\left(\int_0^{d(a,b)} \phi(t)dt\right) \\ &< \int_0^{d(a,b)} \phi(t)dt\end{aligned}$$

which is impassible. Then, $d(a, b) = 0$ and so $a = b$. □

In the following, we define subclass of integrals and we prove the existence of fixed point by applying these integrals.

Definition 2.3. *Let Γ be a collection of mappings $\gamma : [0, \infty) \to [0, \infty)$ which are Lebesgue-integrable, summable on each compact subset of $[0, \infty)$ and satisfying following conditions:*

(1) $\int_0^{\epsilon} \gamma(t)dt > 0$, *for each* $\epsilon > 0$

(2) $\int_0^{a+b} \gamma(t)dt \le \int_0^{a} \gamma(t)dt + \int_0^{b} \gamma(t)dt$, *for each* $a, b \ge 0$

Definition 2.4. *Let (X, d) be a metric space and $f : X \to X$ be a given mapping. We say that f is an α-ψ-γ-contractive integral type mapping if there exist three functions $\alpha : X \times X \to [0, +\infty)$, $\gamma \in \Gamma$ and $\psi \in \Psi$ such that*

$$\int_0^{\alpha(x,y)d(fx,fy)} \gamma(t)dt \le \psi\left(\int_0^{d(x,y)} \gamma(t)dt\right), \tag{11}$$

for all $x, y \in X$ and all $t \in [0, +\infty)$.

Theorem 2.5. *Let (X, d) be a complete metric space and $f : X \to X$ be a mapping satisfying following conditions:*

(i) f is α-admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \ge 1$;

(iii) f is an α-ψ-γ-contractive integral type mapping,

then f has a unique fixed point $a \in X$ such that for $x_0 \in X$, $lim_{n\to\infty} f^n x_0 = a$.

Proof. First, we show that

$$\int_0^{d(f^n x_0, f^{n+1} x_0)} \gamma(t)dt \le \psi^n\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right). \tag{12}$$

Since f is α-admissible then from (ii) we get

$$\alpha(x_0, fx_0) \ge 1 \to \alpha(fx_0, f^2x_0) \ge 1 \to \ldots \to \alpha(f^n x_0, f^{n+1} x_0) \ge 1.$$

then for all $n \in \mathbb{N}$,

$$\alpha(f^n x_0, f^{n+1} x_0) \ge 1. \tag{13}$$

To prove (12),by induction and (i), we have for $n = 1$

$$\int_0^{d(fx_0,f^2x_0)} \gamma(t)dt \leq \int_0^{\alpha(x_0,fx_0)d(fx_0,f^2x_0)} \gamma(t)dt$$
$$\leq \psi\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right).$$

Now, assuming that (12) holds for an $n \in \mathbb{N}$, then from (13) we get

$$\begin{aligned}\int_0^{d(f^{n+1}x_0,f^{n+2}x_0)} \gamma(t)dt &\leq \psi\left(\int_0^{\alpha(f^nx_0,f^{n+1}x_0)d(f^{n+1}x_0,f^{n+2}x_0)} \gamma(t)dt\right)\\ &\leq \psi\left(\int_0^{d(f^nx_0,f^{n+1}x_0)} \gamma(t)dt\right)\\ &\leq \psi\left(\psi^n\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right)\right)\\ &= \psi^{n+1}\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right),\end{aligned}$$

therefore (12) holds for all $n \in \mathbb{N}$. In the following, we show that $\{f^nx_0\}_{n=1}^{\infty}$ is a Cauchy sequence. Fix $\epsilon > 0$ and let $n(\epsilon) \in \mathbb{N}$ such that

$$\sum_{n \geq n(\epsilon)} \psi^n\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right) < \epsilon.$$

Let $m, n \in \mathbb{N}$ with $m > n > n(\epsilon)$, we get

$$\begin{aligned}\int_0^{d(f^nx_0,f^mx_0)} \gamma(t)dt &\leq \int_0^{d(f^nx_0,f^{n+1}x_0)+\cdots+d(f^{m-1}x_0,f^mx_0)} \gamma(t)dt\\ &\leq \int_0^{d(f^nx_0,f^{n+1}x_0)} \gamma(t)dt + \cdots + \int_0^{d(f^{m-1}x_0,f^mx_0)} \gamma(t)dt\\ &\leq \psi^n\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right) + \cdots + \psi^{m-1}\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right)\\ &= \sum_{k=n}^{m-1} \psi^k\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right)\\ &\leq \sum_{n \geq n(\epsilon)} \psi^n\left(\int_0^{d(x_0,fx_0)} \gamma(t)dt\right) < \epsilon,\end{aligned}$$

which means

$$\int_0^{d(f^nx_0,f^mx_0)} \gamma(t)dt \longrightarrow 0,$$

therefore, by lemma 1.8 we have

$$d(f^nx_0, f^mx_0) \longrightarrow 0.$$

Then $\{f^nx_0\}_{n=1}^{\infty}$ is a Cauchy sequence. Since (X, d) is a complete metric space, then there exists $a \in X$ such that, $\lim_{n\to\infty} f^nx_0 = a$. Now, we show that a is a fixed point of f. We claim that

$$d(fa, a) \leq d(fa, f(f^nx_0)) + d(f^{n+1}x_0, a) \longrightarrow 0.$$

To prove our claim, it is sufficient to show that $d(fa, f(f^n x_0)) \longrightarrow 0$. We have

$$
\begin{aligned}
0 < \int_0^{d(fa,f(f^n x_0))} \gamma(t)dt &\leq \int_0^{\alpha(a,f^n x_0)d(fa,f(f^n x_0))} \gamma(t)dt \\
&\leq \psi\left(\int_0^{d(a,f^n x_0)} \gamma(t)dt\right) \\
&\leq \int_0^{d(a,f^n x_0)} \gamma(t)dt,
\end{aligned}
$$

if $n \longrightarrow +\infty$ then

$$\int_0^{d(fa,f(f^n x_0))} \gamma(t)dt \longrightarrow 0,$$

consequently, from lemma 1.8 we obtain

$$d(fa, f(f^n x_0)) \longrightarrow 0.$$

Therefore, $d(fa, a) = 0$ and so $fa = a$, which means a is a fixed point of f. The proof of the uniqueness of fixed point is similar to theorem 2.2. □

3. Examples and Remarks

Example 3.1. *Let $X = \mathbb{R}$ and $d(x, y) = |x - y|$ for all $x, y \in \mathbb{R}$, then (X, d) is a complete metric space. Define $f : \mathbb{R} \to \mathbb{R}$ by $fx = \frac{x}{2}$ and $\alpha : \mathbb{R} \times \mathbb{R} \to [0, +\infty)$ by*

$$\alpha(x, y) = \begin{cases} \frac{4}{3} & \text{if } x, y \in [0, 1]; \\ 0 & o.w. \end{cases}$$

f is α-admissible, since $\alpha(x, y) \geq 1$ implies that

$$\alpha(fx, fy) = \alpha\left(\frac{x}{2}, \frac{y}{2}\right) \geq 1.$$

Define $\phi : [0, \infty) \to [0, \infty)$ by $\phi(t) = t$ then $\phi \in \Phi$. There exists $x_0 \in \mathbb{R}$ such that $\alpha(x_0, fx_0) = \alpha(x_0, \frac{x_0}{2}) \geq 1$. Let $\psi(t) = \frac{t}{2}$ where $\psi : [0, \infty) \to [0, \infty)$ then, for all $x, y \in \mathbb{R}$ we have

$$
\begin{aligned}
\int_0^{\alpha(x,y)d(fx,fy)} \phi(t)dt &= \int_0^{\frac{4}{3}d(\frac{x}{2},\frac{y}{2})} tdt \\
&\leq \frac{\int_0^{d(x,y)} tdt}{2} \\
&= \psi\left(\int_0^{d(x,y)} \phi(t)dt\right).
\end{aligned}
$$

Then all the hypotheses of Theorem 2.2 are satisfied, consequently f has a unique fixed point. Here, 0 is a fixed point of $f(x)$.

Example 3.2. *Let $X = \mathbb{R}^+$ and $d(x, y) = |x - y|$ for all $x, y \in \mathbb{R}^+$, then (X, d) is a complete metric space. Define $f : \mathbb{R}^+ \to \mathbb{R}^+$ by $fx = \frac{x}{3} + 1$ and $\alpha : \mathbb{R}^+ \times \mathbb{R}^+ \to [0, +\infty)$ by*

$$\alpha(x, y) = \begin{cases} \frac{5}{4} & \text{if } x, y \in [0, 2]; \\ 0 & o.w \end{cases}$$

f is α-admissible, since $\alpha(x,y) \geq 1$ implies that

$$\alpha(fx, fy) = \alpha\left(\frac{x}{3}+1, \frac{y}{3}+1\right) \geq 1.$$

Define $\gamma : [0,\infty) \to [0,\infty)$ by $\gamma(t) = 2$ then $\gamma \in \Gamma$. There exists $x_0 \in X$ such that $\alpha(x_0, fx_0) = \alpha(x_0, \frac{x_0}{3}+1) \geq 1$, for instance let $x_0 = 0$. Let $\gamma(t) = \frac{t}{2}$ where $\gamma : [0,\infty) \to [0,\infty)$ then for all $x, y \in X$ we have

$$\begin{aligned}\int_0^{\alpha(x,y)d(fx,fy)} \gamma(t)dt &= \int_0^{\frac{5}{4}d(\frac{x}{3}+1,\frac{y}{3}+1)} 2dt \\ &= \frac{5}{6}d(x,y) \\ &\leq \frac{\int_0^{d(x,y)} 2dt}{2} \\ &= d(x,y) \\ &= \psi\left(\int_0^{d(x,y)} \gamma(t)dt\right).\end{aligned}$$

Then all the hypotheses of Theorem 2.8 are satisfied, consequently f has a unique fixed point. Here, $\frac{3}{2}$ is a fixed point of $f(x)$.

Remark 3.3. *In theorem 2.2, if we consider $f : X \to X$ which is α-admissible where $\alpha(x,y) = 1$ for all $x, y \in X$ and $\psi(t) = ct$ for all $t > 0$ and some $c \in (0,1)$ then we get Branciari theorem.*

Remark 3.4. *Moreover, by above conditions if $\phi(t) = 1$ or $\gamma(t) = 1$ in theorem 2.5 then we conclude Banach-Caccioppoli principle. In fact, we have*

$$\int_0^{d(fx,fy)} 1dt = d(fx,fy) \leq cd(x,y) = c\int_0^{d(a,b)} 1dt.$$

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