

Certain properties of generalized Einstein spaces

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Abstract. In the present paper are introduced generalized Einstein spaces. Einstein type tensors are represented in the generalized Einstein spaces. Some relations of Einstein type tensors of the first and the second kind in the generalized Riemannian space are obtained. Also, geodesic mappings of T -connected generalized Einstein spaces onto Riemannian space are considered.

1. Introduction

In 1922 Cartan was put forward a modification of General Relativity Theory (GRT), by relaxing the assumption that the affine connection has vanishing the antisymmetric part (torsion tensor), and relating the torsion to the density of intrinsic angular momentum. Also, the torsion is implicit in the 1928 Einstein theory of gravitation with teleparallelism. Afterwards, several mathematicians dealt with non-symmetric affine connection, for example, Eisenhart [6], [7], Prvanović [24], Minčić [14]-[20], Zlatanović [9], [21], [32]-[39].

Geodesic and almost geodesic lines, play an important role in geometry and physics. Sinyukov [25] introduced the concept of geodesic mappings between affine connected spaces without torsion. Mikeš [1], [8]-[13], [26], [31] gave some significant contributions to the study of geodesic and almost geodesic mappings of affine connected, Riemannian and Einstein spaces. Contribution to the theory of geodesic and almost geodesic mappings of spaces with non-symmetric affine connection and generalized Riemannian spaces gave Stanković [19], [20], [27]-[30], [35].

2. Notation and preliminaries

A generalized Riemannian space GR_N in the sense of Eisenhart's definition [5] is a differentiable N -dimensional manifold, equipped with a nonsymmetric basic tensor g_{ij} . Connection coefficients of this space are generalized Cristoffel's symbols of the second kind.

Generalized Cristoffel's symbols of the first kind of the space GR_N are given by the formula

$$\Gamma_{i,jk} = \frac{1}{2}(g_{ji,k} - g_{jk,i} + g_{ik,j}), \quad (1)$$

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where, for example, $g_{i,jk} = \frac{\partial g_{ij}}{\partial x^k}$. Connection coefficients of the space GR_N are the generalized Cristoffel's symbols of the second kind

$$\Gamma_{jk}^i = g^{ip} \Gamma_{p,jk}, \quad (2)$$

where $\|g_{ij}^{-1}\| = \|g_{ij}\|^{-1}$, and $g_{ij} = \frac{1}{2}(g_{ij} + g_{ji})$. We suppose that $\det \|g_{ij}\| \neq 0$, $\det \|g_{ij}^{-1}\| \neq 0$. Generally, we have $\Gamma_{jk}^i \neq \Gamma_{kj}^i$. The symmetric and anti-symmetric part of Γ_{jk}^i are given by the formulas

$$\Gamma_{jk}^i = \frac{1}{2}(\Gamma_{jk}^i + \Gamma_{kj}^i) = S_{jk}^i, \quad \Gamma_{jk}^i = \frac{1}{2}(\Gamma_{jk}^i - \Gamma_{kj}^i) = T_{jk}^i \quad (3)$$

The magnitude T_{jk}^i is the torsion tensor of the space GR_N . Obviously,

$$\Gamma_{jk}^i = S_{jk}^i + T_{jk}^i. \quad (4)$$

The use of non-symmetric basic tensor and non-symmetric connection became especially actual after appearance the works of A. Einstein [2]-[5] related to create the Unified Field Theory (UFT). Remark that at UFT the symmetric part g_{ij} of the basic tensor g_{ij} is related to the gravitation, and anisymmetric one g_{ij} to the electromagnetism.

In a generalized Riemannian space one can define four kinds of covariant derivatives [14], [17]. In this paper, we consider only the first two kinds of covariant differentiation. For example, for a tensor a_j^i in GR_N we have

$$a_{j|m}^i = a_{j,m}^i + \Gamma_{pm}^i a_j^p - \Gamma_{jm}^p a_p^i, \quad a_{j|_2m}^i = a_{j,m}^i + \Gamma_{mp}^i a_j^p - \Gamma_{mj}^p a_p^i, \quad (5)$$

where $|$ ($\theta = 1, 2$) denotes a covariant derivative of the kind θ and $a_{j,m}^i = \frac{\partial a_j^i}{\partial x^m}$.

In the case of the space GR_N we have five independent curvature tensors [14]. In this paper we will consider only the first two curvature tensors:

$$\begin{aligned} R_{1jmn}^i &= \Gamma_{jm,n}^i - \Gamma_{jn,m}^i + \Gamma_{jm}^p \Gamma_{pn}^i - \Gamma_{jn}^p \Gamma_{pm}^i, \\ R_{2jmn}^i &= \Gamma_{mj,n}^i - \Gamma_{nj,m}^i + \Gamma_{mj}^p \Gamma_{np}^i - \Gamma_{nj}^p \Gamma_{mp}^i. \end{aligned} \quad (6)$$

Designating by semicolon (;) covariant derivative with respect to S_{jm}^i , we have [5]:

$$\begin{aligned} R_{1jmn}^i &= R_{jmn}^i + T_{jm;n}^i - T_{jn;m}^i + T_{jm}^p T_{pn}^i - T_{jn}^p T_{pm}^i, \\ R_{2jmn}^i &= R_{jmn}^i + T_{jn;m}^i - T_{jm;n}^i + T_{jm}^p T_{pn}^i - T_{jn}^p T_{pm}^i, \end{aligned} \quad (7)$$

where R_{jmn}^i is the curvature tensor with respect to the symmetric connection S_{jm}^i .

Contracting by indices i and n in curvature tensor of the first kind R_{1jmn}^i in the space GR_N we have the first type Ricci tensor

$$R_{jm} = R_{1jmp}^p = R_{jm} + T_{jm;p}^p + T_{jq}^p T_{mp}^q, \quad (8)$$

where $R_{jm} = R_{1jmp}^p$ is Ricci tensor with respect to the symmetric connection S_{jm}^i .

Now, contracting by indices i and n in curvature tensor of the second kind R_{2jmn}^i we get the second type Ricci tensor

$$R_{jm} = R_{2jmp}^p = R_{jm} - T_{jm;p}^p + T_{jq}^p T_{mp}^q. \quad (9)$$

3. Geodesic mappings of generalized Riemannian spaces

One says that reciprocal one valued mapping $f : GR_N \rightarrow G\bar{R}_N$ is *geodesic* [19], [20], if geodesics of the generalized Riemannian space GR_N pass to geodesics of the generalized Riemannian space $G\bar{R}_N$. We can consider these spaces in the common by this mapping system of local coordinates. In the corresponding points $M(x)$ and $\bar{M}(x)$ we can put

$$\bar{\Gamma}_{jk}^i(x) = \Gamma_{jk}^i(x) + P_{jk}^i(x), \quad (i, j, k = 1, \dots, N), \quad (10)$$

where $P_{jk}^i(x)$ is the *deformation tensor* of the connection Γ of GR_N according to the mapping $f : GR_N \rightarrow G\bar{R}_N$.

A necessary and sufficient condition that the mapping $f : GR_N \rightarrow G\bar{R}_N$ be geodesic (see [19], [20]) is that the deformation tensor P_{jk}^i in (10) at the mapping f has the form

$$P_{jk}^i(x) = \delta_j^i \psi_k(x) + \delta_k^i \psi_j(x) + \xi_{jk}^i(x), \quad (11)$$

where

$$\psi_i(x) = \frac{1}{N+1} (\bar{\Gamma}_{ia}^\alpha(x) - \Gamma_{ia}^\alpha(x)), \quad \xi_{jk}^i(x) = P_{jk}^i = \frac{1}{2} (P_{jk}^i - P_{kj}^i). \quad (12)$$

4. Some relations of Einstein type tensors

4.1. Einstein type tensors of the first kind

Starting from the Bianchi type identity (see [18])

$$Cicl R_{jmn|v}^i = 2 Cicl T_{mnv}^p R_{jpv}^i, \quad (13)$$

i.e.

$$R_{jmn|v}^i + R_{jnv|m}^i + R_{jvm|n}^i = 2(T_{mn}^p R_{jpv}^i + T_{nv}^p R_{jpm}^i + T_{vm}^p R_{jpn}^i), \quad (14)$$

composing by g_{ih} in (14) and using property of antisymmetry

$$R_{ijmn} = -R_{jimn}, \quad R_{ijmn} = -R_{ijnm},$$

we get

$$R_{hjmn|v} + R_{hjnvm} + R_{hjvnm} = 2(T_{mn}^p R_{hjp} + T_{nv}^p R_{hjpm} + T_{vm}^p R_{hjpn}). \quad (15)$$

Now, composing by $g^{hm}g^{jm}$ from (15) we have

$$(R_v^m - \frac{1}{2} \delta_v^m R)_{|m} = \bar{S}_v - \frac{1}{2} S_v, \quad (16)$$

where we denote

$$S_v = g^{mj} S_{mjv}, \quad \bar{S}_v = g^{mj} S_{vjm} \text{ and } S_{mjv} = 2T_{mq}^p R_{jpv}^q. \quad (17)$$

Definition 4.1. A tensor $E_v^m = R_v^m - \frac{1}{2}\delta_v^m R$ is called the Einstein tensor of the first kind.

In this way, the following theorem is proven

Theorem 4.2. The Einstein tensor of the first kind E_v^m satisfied the relation

$$E_{1v|1}^m = \bar{S}_v - \frac{1}{2}S_v, \quad (18)$$

where vectors S_v and \bar{S}_v are given by relations (17).

Analogously, starting from identity (see [18])

$$Cicl R_{mnv}^i = 2Cicl (T_{jm}^p R_{1pnv}^i + T_{mn}^p R_{1jvp}^i + T_{mp}^i R_{1jnv}^p), \quad (19)$$

we obtain that the following theorem is valid:

Theorem 4.3. The Einstein tensor of the first kind E_v^m satisfied the relation

$$E_{1v|2}^m = -\frac{1}{2}(P_v + \bar{P}_v + \bar{S}_v - Q_v), \quad (20)$$

where vector \bar{S}_v is given by relation (17) and

$$P_v = 2g^{jm} T_{jn}^p R_{1pvm}^n, \quad \bar{P}_v = 2g^{jm} T_{jv}^p R_{1pmm}^n, \quad Q_v = 2g^{jm} T_{jm}^p R_{1pvn}^n.$$

4.2. Einstein type tensor of the second kind

Starting from the Bianchi type identity (see [18])

$$Cicl R_{mnv}^i = 2Cicl (T_{mj}^p R_{2pnv}^i + T_{mn}^p R_{2jpv}^i + T_{mp}^i R_{2jvn}^p), \quad (21)$$

we obtain

$$(R_{2v}^m - \frac{1}{2}\delta_v^m R)_1 = \frac{1}{2}(P_v + \bar{P}_v + \bar{S}_v - Q_v), \quad (22)$$

where

$$\begin{aligned} S_v &= g^{mj} S_{mjv}, & \bar{S}_v &= g^{mj} S_{vjm}, & S_{mjv} &= 2T_{mq}^p R_{jpv}^q, \\ P_v &= 2g^{jm} T_{jn}^p R_{2pvm}^n, & \bar{P}_v &= 2g^{jm} T_{jv}^p R_{2pmm}^n, & Q_v &= 2g^{jm} T_{jm}^p R_{2pvn}^n. \end{aligned} \quad (23)$$

Definition 4.4. A tensor $E_v^m = R_v^m - \frac{1}{2}\delta_v^m R$ is called the Einstein tensor of the second kind.

Therefore, the following theorem is valid

Theorem 4.5. *The Einstein tensor of the second kind E_{2v}^m satisfied the relation*

$$E_{2v|m}^m = \frac{1}{2}(P_{2v} + \bar{P}_{2v} + \bar{S}_{2v} - Q_{2v}), \tag{24}$$

where \bar{S}_{2v} , P_{2v} , \bar{P}_{2v} and Q_{2v} are given by relations (23)

Starting from the Bianchi type identity (see [18])

$$Cicl_{mnv} R^i{}_{jmn|v} = 2 Cicl_{mnv} T_{mn}^p R^i{}_{jvp}, \tag{25}$$

i.e.

$$R^i{}_{jmn|v} + R^i{}_{jnv|m} + R^i{}_{jvm|n} = 2(T_{mn}^p R^i{}_{jvp} + T_{nv}^p R^i{}_{jmp} + T_{vm}^p R^i{}_{jnp}). \tag{26}$$

Composing by g_{ih} in (26) and using property of antisymmetry

$$R_{ijmn} = -R_{jimn}, \quad R_{ijmn} = -R_{ijnm},$$

we get

$$R_{hijm|v} + R_{hijnv|m} + R_{hijvm|n} = 2(T_{mn}^p R_{hjvp} + T_{nv}^p R_{hjmp} + T_{vm}^p R_{hjnp}). \tag{27}$$

Now, composing by $g^{hm}g^{jm}$ from (27) we have

$$(R_{2v}^m - \frac{1}{2}\delta_{2v}^m R)_{|m} = \frac{1}{2}S_{2v} - \bar{S}_{2v}, \tag{28}$$

where we denote

$$S_{2v} = g^{mj}S_{mjv}, \quad \bar{S}_{2v} = g^{mj}S_{vjm} \text{ and } S_{mjv} = 2T_{mq}^p R^q{}_{jpv}. \tag{29}$$

In this way, the following theorem is proven

Theorem 4.6. *The Einstein tensor of the second kind E_{2v}^m satisfied the relation*

$$E_{2v|m}^m = \frac{1}{2}S_{2v} - \bar{S}_{2v}, \tag{30}$$

where vectors S_{2v} and \bar{S}_{2v} are given by relations (29).

5. Geodesic mappings of T-connected generalized Einstein spaces

Einstein space V_N is Riemannian space, with symmetric basic metric tensor g_{jm} , where Ricci tensor R_{jm} satisfies the relation

$$R_{jm} = K \cdot g_{jm}, \quad K = \text{const.} \tag{31}$$

After contraction of (31) we obtain $K = \frac{R}{N}$.

Einstein spaces provide simple, highly symmetric cosmological models. In general relativity the Einstein equations relate the curvature of space-time to the energy and momentum of all the matter present in following way

$$R_{jm} - \frac{1}{2}Rg_{jm} + \Lambda g_{jm} = GT_{jm}. \tag{32}$$

The first two summands on the left side of the relation (32) represent so-called Einstein tensor, Λ is the cosmological constant, T_{jm} is energy-momentum tensor and G is the gravitational constant.

In the case of generalized Riemannian spaces we can define *generalized Einstein spaces of the kind θ* in the following way:

Definition 5.1. *Generalized Einstein space of the kind θ ($\theta = 1, 2$) is generalized Riemannian space if Ricci tensor of the kind θ satisfies the condition*

$$R_{\theta jm} = K_{\theta} \cdot g_{jm} \quad (\theta = 1, 2), \tag{33}$$

where K_{θ} are constants. Generalized Einstein space of the kind θ ($\theta = 1, 2$) denoted by $GV_{\theta N}$.

From (8) for symmetric and antisymmetric part of Ricci tensor R_{jm} we obtain

$$R_{\underline{jm}} = R_{jm} + T_{jq}^p T_{mp}^q \quad \text{and} \quad R_{\overset{p}{j}\underset{m}{p}} = T_{jm;p}^p. \tag{34}$$

In this way the following theorem is satisfied:

Theorem 5.2. *In generalized Einstein space GV_{1N} the next conditions are valid:*

$$R_{jm} + T_{jq}^p T_{mp}^q = K_1 g_{\underline{jm}} \quad \text{and} \quad T_{jm;p}^p = K_1 g_{\overset{p}{j}\underset{m}{p}}, \tag{35}$$

where R_{jm} is Ricci tensor with respect to the symmetric part $g_{\underline{jm}}$ of g_{jm} , T_{jk}^i is the torsion tensor, $g_{\overset{p}{j}\underset{m}{p}}$ is an antisymmetric part of g_{jm} and K_1 is a constant.

Analogously, for generalized Einstein space of the second kind the next theorem is valid:

Theorem 5.3. *In generalized Einstein space GV_{2N} the next conditions are valid:*

$$R_{jm} + T_{jq}^p T_{mp}^q = K_2 g_{\underline{jm}} \quad \text{and} \quad T_{jm;p}^p = -K_2 g_{\overset{p}{j}\underset{m}{p}}, \tag{36}$$

where R_{jm} is Ricci tensor with respect to the symmetric part $g_{\underline{jm}}$ of g_{jm} , T_{jk}^i is a torsion tensor, $g_{\overset{p}{j}\underset{m}{p}}$ is an antisymmetric part of g_{jm} and K_2 is a constant.

In the case of Riemannian space, when the basic metric tensor g_{ij} is a symmetric, the torsion tensor T_{jk}^i is a zero and the conditions (35) and (36) reduce to the condition $R_{jm} = K g_{jm}$.

Definition 5.4. Generalized Riemannian space GR_n is T -connected if the torsion tensor T_{jm}^i satisfies the condition:

$$T_{jq}^p T_{mp}^q = \mu g_{jm}, \quad \mu = \text{const.} \quad (37)$$

It is not difficult to conclude that the following assertion is true

Lemma 5.5. If the generalized Einstein space GV_N , ($\theta = 1, 2$) is T -connected, then:

$$R_{jm} = \mu g_{jm}, \quad (38)$$

where R_{jm} is Ricci tensor with respect to g_{jm} and $\mu = K - \mu$ are constants.

Therefore the following assertion holds:

Theorem 5.6. For generalized Einstein tensors E_v^m of the kind v ($v = 1, 2$) in generalized Einstein spaces GV_N of the type θ ($\theta = 1, 2$) the next relations are valid:

$$E_v^m = \left(K - \frac{1}{2} \right) \delta_v^m + K g_{pv}^{pm}, \quad (\theta = 1, 2; v = 1, 2), \quad (39)$$

where R are scalar curvatures.

The Einstein spaces consists a closed class in relation to geodesic mappings, i.e. satisfied the following theorem (see [10], [13]):

Theorem 5.7. If Einstein space V_N permits nontrivial geodesic mapping onto \bar{V}_N , then \bar{V}_N is an Einstein space.

In general case associate space, with the symmetric basic metric tensor g_{jm} , at the generalized Einstein space of the kind θ ($\theta = 1, 2$), is not an Einstein space. According to the Lemma 5.5 we conclude that the associate space, at the T -connected generalized Einstein space of the kind θ ($\theta = 1, 2$), is an Einstein space.

In this way we have proved the following theorem:

Theorem 5.8. If T -connected generalized Einstein space GV_N of the kind θ ($\theta = 1, 2$) permits nontrivial geodesic mapping onto Riemannian space \bar{V}_N , then \bar{V}_N is an Einstein space.

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