



## Study of Nurses' Roster by Coloring Algorithm Parsus Manually

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**Abstract.** Scheduling shifts is a tiresome and time consuming task in any business, and particularly in hospitals where errors are costly, rules are plentiful and changes are rapid. The person performing this function (Head Nurse) will have to keep track of all the employees concerned, distributing hours fairly and avoiding collisions. Rules regulating working hours and breaks have to be followed and the qualifications of individual employees need to be considered. Hours are spent every day on this task in every ward.

The goal of this paper is to solve Nurses Scheduling Problem (NSP) and initialize a fair roster for Pediatric Department (PD) in a local hospital in Riyadh Saudi Arabia.

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### 1. Introduction

#### 1.1. Background of study

Graphs are mathematical structures used to model pairwise relations between objects. The definition of graph is given by Shekhar [9] "a graph is finite set of vertices (or nodes) and a finite collection of edges". A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another; Graphs are one of the prime objects of study in discrete mathematics [10, 12, 13].

Graphs can be used to model many types of relations and processes in physical, biological, social and information systems. Many practical problems can be represented by graphs [15, 17, 19]. Proper coloring of a graph in the simplest form is an assignment of colors either to vertices of the graphs, or to its edges, in such a way that adjacent vertices/edges are colored differently [21, 22].

In any organization that operates continuously such as a hospital, there are two different kind of duties. Daily work, and parallel work which is divided into shifts to carry on necessary tasks in a smooth way. For any department in a hospital the goal is complete these duties through 'fair' scheduling of shifts.

One of the definitions of scheduling is given by Wren, who stated that "Scheduling is the arrangement of objects into pattern in time or space in such a way that some goals are achieved or nearly achieved". Wren has described the rostering problem also. Rostering is the placing of resources into slots in a pattern.

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### 1.2. Comparison with Other Algorithms

Bafghi (2016), studied a project which has been applied to design a program based on data analyzed by converting the process of planning of scheduler to mathematical function of Simulated Annealing (SA) Algorithm using the graph coloring and C programming language. Finally, the performance of the proposed algorithm was compared with a genetic algorithm [1]. This program was able to plan in all cases whereas the program of the genetic algorithm is taken in 92 of scheduling cases ONLY [1, 2, 7]. Results of SA algorithm project indicate that SA has a high performance for solving the nurses scheduling problems compared with the genetic algorithm. Arumawadu, Tharanga, and Seneviratna (2016), conducted a mathematical method by using the graph coloring and C.net programming language to handle database management system [2]. Some work that have been done [3] and [4] is not applicable for different reasons to present problem. In [5] their results aim to automate roster scheduling approach developed under two-stages. Beginning with Enhanced Greedy Optimization algorithm to optimize the hot-line roster and compared with other optimization algorithms (SA algorithm and genetic algorithm)[7, 8].

### 1.3. Problem Definition

In this paper we shall consider a scheduling problem for Pediatric Department (PD) at a local hospital in Riyadh/ Saudi Arabia, which it is considered one of the top governmental hospitals with almost twenty-seven departments. There are more than five hundred doctors and three thousand nurses.

The head nurse (HN) of PD has explained that monthly rosters are made manually before the end of each month. Therefore, HN has the responsibility to publish next month's roster. Even though making monthly rosters manually required great effort, it did not resolve all conflicts. Instead it has created more tedious adjustments to accomplish needed duties.

This paper is concerned with scheduling shifts for nurses in PD in such away that conflicts are resolved without any adjustments for one block (four weeks) only.

### 1.4. Objective of the Study

The purpose of this paper is to apply graph coloring techniques to make a nurses/roster which satisfies hospital requirements as much as possible. Such as equating between nurses in their days off, as well as number of shifts (Day shift/Night shift). In a similar manner executing needed shifts with specified number of different nurses (See 3.1).

### 1.5. Methodology

As described above, to solve Nurse Scheduling Problem (NSP), first nurses should be assigned into shifts. In the proposed solution for the NSP, nurses are distributed randomly into sets (fulfilling hospital requirements). Second an incidence matrix containing nurses data is inputted into Java Program [15] (JP), as well as number of vertices (based on the first step). JP output of colored vertices is used to construct shift groups depending on colors. So, each color represents a group assigned to one shift. Lastly the final fair nurses/roster is reached manually [18].

### 1.6. Scope and Limitation of the Study

This paper considers NSP focusing primarily on shifts for nurses of PD in a local hospital. The roster will be made for all different nurses specified by department administration (See 3.1).

## 2. Formulation And Solution of The Problem

In this section the given data of nurses and restrictions of the hospital are detailed and used to reach constraints as part of formulation of the problem. In 2.2 graph theory and coloring is applied to make needed roster.

2.1. Nurses Data and Restrictions

There are five types of nurses in this department (PD). There is one Head Nurse (HN), and one vice HN (called Charge Nurse/CN). In addition there are: Staff Nurse1(SN1) who is a senior nurse, and Staff Nurse2(SN2) who is a junior nurse. This PD has thirteen wards, and thirty six nurses, only twenty nine nurses are available for solution of NSP.

In terms of shifts there are three shifts: Morning shift, Day shift, and Night shift ( $M, D, N$ ) respectively.

In addition there is an annual leave (AL) of seven weeks allocated for each nurse, this will be pointed out for some nurses in the final roster in 4.1

Moreover the given restrictions were used to reach the following constraints:

1. Each Period of Duty Schedule (PDS) is four weeks (twenty eight days).
2. Duration of each shift is twelve hours, where  $M$  is from 7:00 a.m. to 4:00 p.m.,  $D$  is from 7:00 a.m. to 19:00 p.m. and  $N$  is from 19:00 p.m. to 7:00 a.m..
3. Each of  $HN$  and  $CN$  only takes  $M$ .
4. During each PDS maximum number of shifts for each nurse is sixteen. However nurses who get less than sixteen days in one PDS, will be given priority for a total of sixteen days in the following PDS. So that average of shifts per nurse is sixteen days per year.
5. In a similar manner, minimum number of days off for each nurse is twelve during each PDS. However nurses who get more than twelve days in one PDS, will have less days off in the following PDS. So that average of her days off per is twelve days per year.
6. Maximum allowance of consecutive  $N$  is three for each nurse.
7. Maximum allowance of consecutive  $D$  is three for each nurse.
8. Maximum allowance of consecutive days off is four for each nurse.
9. The pediatric department needs up to two from  $SN1$  and up to six from  $SN2$  each shift.
10. If any nurse has a vacation for two weeks which will be counted from her annual leave, then total of her shifts must be eight (for the remaining two weeks).
11. Each day, the department needs 6-8 nurses total for  $N$  and  $D$  shifts, and 7 is the best.
12. Each nurse can have a  $D$  followed by a  $N$ . The reverse is not allowed.
13. For purpose of leavenating fatigue, each nurse will have two weeks  $N$  followed two weeks  $D$  on the opposite.

The following list of names and abbreviations were given to us by  $HN$  to be used in executing needed shifts, note there are six  $SN1$  and twenty three  $SN2$ .

Name	Rank	Name	Rank	Name	Rank	Name	Rank
Nour	HN	Rejie	SN <sub>2</sub> 1	Vicy	SN <sub>2</sub> 9	Jayne	SN <sub>2</sub> 17
Marissa	CN	Vaness	SN <sub>2</sub> 2	Annie	SN <sub>2</sub> 10	Karla	SN <sub>2</sub> 18
Teresite	SN <sub>1</sub> 1	Josie	SN <sub>2</sub> 3	Antonia	SN <sub>2</sub> 11	Jamilah	SN <sub>2</sub> 19
Lala	SN <sub>1</sub> 2	Jisha	SN <sub>2</sub> 4	Mae Ann	SN <sub>2</sub> 12	Jackline	SN <sub>2</sub> 20
Conchite	SN <sub>1</sub> 3	Mary	SN <sub>2</sub> 5	Rhea	SN <sub>2</sub> 13	Lalaine	SN <sub>2</sub> 21
Ma.Cecilia	SN <sub>1</sub> 4	Glecilda	SN <sub>2</sub> 6	Jane	SN <sub>2</sub> 14	Florda	SN <sub>2</sub> 22
Elena	SN <sub>1</sub> 5	Laarni	SN <sub>2</sub> 7	Mae	SN <sub>2</sub> 15	Aiza	SN <sub>2</sub> 23
Charmi	SN <sub>1</sub> 6	Cristina	SN <sub>2</sub> 8	Cristana	SN <sub>2</sub> 16		

Table 1. List of Nurses

2.2. Applying Graph Theory to Solve the Problem

Based on the data which was given by PD, we chose 2 from  $SN1$  and 5 from  $SN2$  to implement one day shift during PDS. An incident matrix  $29 \times 29$  is initiated. If any two nurses are in same set then  $ij - th$  entry is 1 otherwise it is 0.

Accordingly, when we used this matrix for JP we get colors of one PDS, a problem has appeared. The average number of shifts in PDS for each  $SN1$  and  $SN2$  did not satisfy the required constraints, (this lead to unfair organization of shifts and disturbed the nurses who are dealing with patients).

So we tried to change our choice to be 3 from SN1 and 5 from SN2 to solve the problem, yet this still caused the same problem in the average number of shifts for SN2 in PDS.

To avoid resulting faults each time, we kept changing chosen number of SN2, which result in changing number of daily shifts for SN2. Then input incident matrix and get the colors to reach best choice which executes given constraints.

Finally, it seemed that choosing 3 from SN1 and 8 from SN2 is the best, since it satisfied all required constrains for PDS.

Accordingly, we get 46 sets for nurses each of which contains 3 from SN1 and 8 from SN2, (S1, S2, S3,..., S46). The reason for this number (46 sets) is the fair distribution for nurses. i.e. each nurse from SN1 was repeated the same number in 46 sets, and similar thing happened for each nurse in SN2. In the following table, members of each set will be listed for all 46 sets.

S1	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 1	SN <sub>2</sub> 2	SN <sub>2</sub> 3
	SN <sub>2</sub> 4	SN <sub>2</sub> 5	SN <sub>2</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8	
S2	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 9	SN <sub>2</sub> 10	SN <sub>2</sub> 11
	SN <sub>2</sub> 12	SN <sub>2</sub> 13	SN <sub>2</sub> 14	SN <sub>2</sub> 15	SN <sub>2</sub> 16	
S3	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 17	SN <sub>2</sub> 18	SN <sub>2</sub> 19
	SN <sub>2</sub> 20	SN <sub>2</sub> 21	SN <sub>2</sub> 22	SN <sub>2</sub> 23	SN <sub>2</sub> 1	
S4	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 2	SN <sub>2</sub> 3	SN <sub>2</sub> 4
	SN <sub>2</sub> 5	SN <sub>2</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8	SN <sub>2</sub> 9	
S5	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 10	SN <sub>2</sub> 11	SN <sub>2</sub> 12
	SN <sub>2</sub> 13	SN <sub>2</sub> 14	SN <sub>2</sub> 15	SN <sub>2</sub> 16	SN <sub>2</sub> 17	
S6	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 18	SN <sub>2</sub> 19	SN <sub>2</sub> 20
	SN <sub>2</sub> 21	SN <sub>2</sub> 22	SN <sub>2</sub> 23	SN <sub>2</sub> 1	SN <sub>2</sub> 2	
S7	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5
	SN <sub>2</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8	SN <sub>2</sub> 9	SN <sub>2</sub> 10	
S8	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 11	SN <sub>2</sub> 12	SN <sub>2</sub> 13
	SN <sub>2</sub> 14	SN <sub>2</sub> 15	SN <sub>2</sub> 16	SN <sub>2</sub> 17	SN <sub>2</sub> 18	
S9	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 19	SN <sub>2</sub> 20	SN <sub>2</sub> 21
	SN <sub>2</sub> 22	SN <sub>2</sub> 23	SN <sub>2</sub> 1	SN <sub>2</sub> 2	SN <sub>2</sub> 3	
S10	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 4	SN <sub>2</sub> 5	SN <sub>2</sub> 6
	SN <sub>2</sub> 7	SN <sub>2</sub> 8	SN <sub>2</sub> 9	SN <sub>2</sub> 10	SN <sub>2</sub> 11	
S11	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 12	SN <sub>2</sub> 13	SN <sub>2</sub> 14
	SN <sub>2</sub> 15	SN <sub>2</sub> 16	SN <sub>2</sub> 17	SN <sub>2</sub> 18	SN <sub>2</sub> 19	
S12	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 20	SN <sub>2</sub> 21	SN <sub>2</sub> 22
	SN <sub>2</sub> 23	SN <sub>2</sub> 1	SN <sub>2</sub> 2	SN <sub>2</sub> 3	SN <sub>2</sub> 4	
S13	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 5	SN <sub>2</sub> 6	SN <sub>2</sub> 7
	SN <sub>2</sub> 8	SN <sub>2</sub> 9	SN <sub>2</sub> 10	SN <sub>2</sub> 11	SN <sub>2</sub> 12	
S14	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 13	SN <sub>2</sub> 14	SN <sub>2</sub> 15
	SN <sub>2</sub> 16	SN <sub>2</sub> 17	SN <sub>2</sub> 18	SN <sub>2</sub> 19	SN <sub>2</sub> 20	
S15	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 21	SN <sub>2</sub> 22	SN <sub>2</sub> 23
	SN <sub>2</sub> 1	SN <sub>2</sub> 2	SN <sub>2</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5	
S16	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8
	SN <sub>2</sub> 9	SN <sub>2</sub> 10	SN <sub>2</sub> 11	SN <sub>2</sub> 12	SN <sub>2</sub> 13	
S17	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 14	SN <sub>2</sub> 15	SN <sub>2</sub> 16
	SN <sub>2</sub> 17	SN <sub>2</sub> 18	SN <sub>2</sub> 19	SN <sub>2</sub> 20	SN <sub>2</sub> 21	
S18	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 22	SN <sub>2</sub> 23	SN <sub>2</sub> 1
	SN <sub>2</sub> 2	SN <sub>2</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5	SN <sub>2</sub> 6	
S19	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 7	SN <sub>2</sub> 8	SN <sub>2</sub> 9
	SN <sub>2</sub> 10	SN <sub>2</sub> 11	SN <sub>2</sub> 12	SN <sub>2</sub> 13	SN <sub>2</sub> 14	

S20	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 15	SN <sub>2</sub> 16	SN <sub>2</sub> 17
	SN <sub>2</sub> 18	SN <sub>2</sub> 19	SN <sub>2</sub> 20	SN <sub>2</sub> 21	SN <sub>2</sub> 22	
S21	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 23	SN <sub>2</sub> 1	SN <sub>2</sub> 2
	SN <sub>2</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5	SN <sub>2</sub> 6	SN <sub>2</sub> 7	
S22	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 8	SN <sub>2</sub> 9	SN <sub>2</sub> 10
	SN <sub>2</sub> 11	SN <sub>2</sub> 12	SN <sub>2</sub> 13	SN <sub>2</sub> 14	SN <sub>2</sub> 15	
S23	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 16	SN <sub>2</sub> 17	SN <sub>2</sub> 18
	SN <sub>2</sub> 19	SN <sub>2</sub> 20	SN <sub>2</sub> 21	SN <sub>2</sub> 22	SN <sub>2</sub> 23	
S24	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 1	SN <sub>2</sub> 2	SN <sub>2</sub> 3
	SN <sub>2</sub> 4	SN <sub>2</sub> 5	SN <sub>2</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8	
S25	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 9	SN <sub>2</sub> 10	SN <sub>2</sub> 11
	SN <sub>2</sub> 12	SN <sub>2</sub> 13	SN <sub>2</sub> 14	SN <sub>2</sub> 15	SN <sub>2</sub> 16	
S26	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 17	SN <sub>2</sub> 18	SN <sub>2</sub> 19
	SN <sub>2</sub> 20	SN <sub>2</sub> 21	SN <sub>2</sub> 22	SN <sub>2</sub> 23	SN <sub>2</sub> 1	
S27	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 2	SN <sub>2</sub> 3	SN <sub>2</sub> 4
	SN <sub>2</sub> 5	SN <sub>2</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8	SN <sub>2</sub> 9	
S28	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 10	SN <sub>2</sub> 11	SN <sub>2</sub> 12
	SN <sub>2</sub> 13	SN <sub>2</sub> 14	SN <sub>2</sub> 15	SN <sub>2</sub> 16	SN <sub>2</sub> 17	
S29	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 18	SN <sub>2</sub> 19	SN <sub>2</sub> 20
	SN <sub>2</sub> 21	SN <sub>2</sub> 22	SN <sub>2</sub> 23	SN <sub>2</sub> 1	SN <sub>2</sub> 2	
S30	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5
	SN <sub>2</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8	SN <sub>2</sub> 9	SN <sub>2</sub> 10	
S31	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 11	SN <sub>2</sub> 12	SN <sub>2</sub> 13
	SN <sub>2</sub> 14	SN <sub>2</sub> 15	SN <sub>2</sub> 16	SN <sub>2</sub> 17	SN <sub>2</sub> 18	
S32	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 19	SN <sub>2</sub> 20	SN <sub>2</sub> 21
	SN <sub>2</sub> 22	SN <sub>2</sub> 23	SN <sub>2</sub> 1	SN <sub>2</sub> 2	SN <sub>2</sub> 3	
S33	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5	SN <sub>2</sub> 6
	SN <sub>2</sub> 7	SN <sub>2</sub> 8	SN <sub>2</sub> 9	SN <sub>2</sub> 10	SN <sub>2</sub> 11	
S34	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 12	SN <sub>2</sub> 13	SN <sub>2</sub> 14
	SN <sub>2</sub> 15	SN <sub>2</sub> 16	SN <sub>2</sub> 17	SN <sub>2</sub> 18	SN <sub>2</sub> 19	
S35	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 20	SN <sub>2</sub> 21	SN <sub>2</sub> 22
	SN <sub>2</sub> 23	SN <sub>2</sub> 1	SN <sub>2</sub> 2	SN <sub>2</sub> 3	SN <sub>2</sub> 4	
S36	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 5	SN <sub>2</sub> 6	SN <sub>2</sub> 7
	SN <sub>2</sub> 8	SN <sub>2</sub> 9	SN <sub>2</sub> 10	SN <sub>2</sub> 11	SN <sub>2</sub> 12	
S37	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 13	SN <sub>2</sub> 14	SN <sub>2</sub> 15
	SN <sub>2</sub> 16	SN <sub>2</sub> 17	SN <sub>2</sub> 18	SN <sub>2</sub> 19	SN <sub>2</sub> 20	
S38	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 21	SN <sub>2</sub> 22	SN <sub>2</sub> 23
	SN <sub>2</sub> 1	SN <sub>2</sub> 2	SN <sub>2</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5	
S39	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8
	SN <sub>2</sub> 9	SN <sub>2</sub> 10	SN <sub>2</sub> 11	SN <sub>2</sub> 12	SN <sub>2</sub> 13	
S40	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 14	SN <sub>2</sub> 15	SN <sub>2</sub> 16
	SN <sub>2</sub> 17	SN <sub>2</sub> 18	SN <sub>2</sub> 19	SN <sub>2</sub> 20	SN <sub>2</sub> 21	
S41	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 22	SN <sub>2</sub> 23	SN <sub>2</sub> 1
	SN <sub>2</sub> 2	SN <sub>2</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5	SN <sub>2</sub> 6	
S42	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 7	SN <sub>2</sub> 8	SN <sub>2</sub> 9
	SN <sub>2</sub> 10	SN <sub>2</sub> 11	SN <sub>2</sub> 12	SN <sub>2</sub> 13	SN <sub>2</sub> 14	
S43	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 15	SN <sub>2</sub> 16	SN <sub>2</sub> 17
	SN <sub>2</sub> 18	SN <sub>2</sub> 19	SN <sub>2</sub> 20	SN <sub>2</sub> 21	SN <sub>2</sub> 22	
S44	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 23	SN <sub>2</sub> 1	SN <sub>2</sub> 2
	SN <sub>2</sub> 3	SN <sub>2</sub> 4	SN <sub>2</sub> 5	SN <sub>2</sub> 6	SN <sub>2</sub> 7	
S45	SN <sub>1</sub> 1	SN <sub>1</sub> 2	SN <sub>1</sub> 3	SN <sub>2</sub> 8	SN <sub>2</sub> 9	SN <sub>2</sub> 10
	SN <sub>2</sub> 11	SN <sub>2</sub> 12	SN <sub>2</sub> 13	SN <sub>2</sub> 14	SN <sub>2</sub> 15	
S46	SN <sub>1</sub> 4	SN <sub>1</sub> 5	SN <sub>1</sub> 6	SN <sub>2</sub> 16	SN <sub>2</sub> 17	SN <sub>2</sub> 18
	SN <sub>2</sub> 19	SN <sub>2</sub> 20	SN <sub>2</sub> 21	SN <sub>2</sub> 22	SN <sub>2</sub> 23	

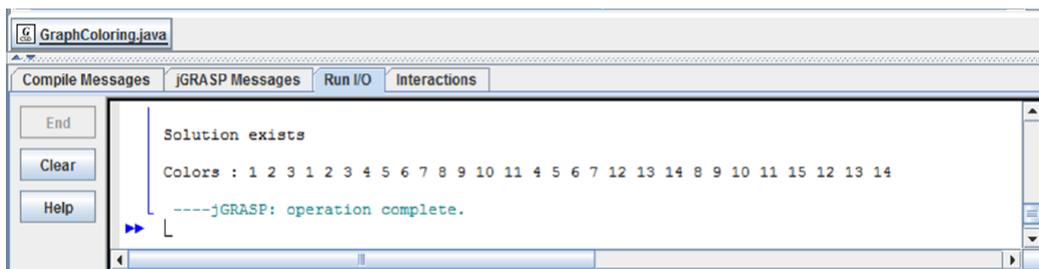
Table 2. Sets of Nurses For Incidence Matrix

By using Table 2 above, we get the incidence matrix in Table 3.

	SN <sub>1</sub> ,1	2	3	4	5	6	SN <sub>2</sub> ,1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	
SN <sub>1</sub> ,1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
2	1	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
3	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
4	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
5	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
6	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SN <sub>2</sub> ,1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
2	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
3	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
4	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	
6	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	
7	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	
8	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0

Table 3. Incidence Matrix For Nurses

Next step, JP was used to find number of colors that's reached to number of colors and the vertices which is colored.



In the figure above, fifteen colors are used to color twenty nine vertices (representing six  $SN_1$  , and twenty three  $SN_2$ ). In the following manner:

Colors	Nurses	Group
1	$SN_11, SN_14.$	G1
2	$SN_12, SN_15.$	G2
3	$SN_13, SN_16.$	G3
4	$SN_21, SN_29.$	G4
5	$SN_22, SN_210.$	G5
6	$SN_23, SN_211.$	G6
7	$SN_24, SN_212.$	G7
8	$SN_25, SN_216.$	G8
9	$SN_26, SN_217.$	G9
10	$SN_27, SN_218.$	G10
11	$SN_28, SN_219.$	G11
12	$SN_213, SN_221.$	G12
13	$SN_214, SN_222.$	G13
14	$SN_215, SN_223.$	G14
15	$SN_220.$	G15

Table 4. Shift Groups of Nurses

In Table 4 vertices with the same color are in the same group. This means nurse in each group could be in same shift and have the same roster (for the specific block). This means  $SN_11$  and  $SN_14$  can be in same shift and can work together because they are all members G1. On the other hand  $SN_11$  and  $SN_12$  can not work together because they are in different groups G1 and G2.

### 3. Conclusions

#### 3.1. Result

Based on restrictions specified in Section 3 concerning NSP for one block (four weeks) in PD of a local hospital in Riyadh, Saudi Arabia, we have divided nurses into sets.

Then using the incidence matrix of these sets, JP was applied to reach best colors, see Subsection 3.2 for details.

After coding, we used JP to color. We used fifteen colors for twenty nine vertices.

In JP nurses with same color are in the same group. Since they will be have the same roster for specific block and they will be work in the same shift.

The final roster for nurses is in Table 5.



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