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Univalence Conditions for an Integral Operator Defined by a Generalization of the Srivastava-Attiya Operator

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Abstract. The main object of this paper is to introduce and study systematically the univalence criteria of a new family of integral operators by using a substantially general form of the widely-investigated Srivastava-Attiya operator. In particular, we derive several new sufficient conditions of univalence for this generalized Srivastava-Attiya operator. Relevant connections with other related earlier works are also pointed out.

1. Introduction, Definitions and Preliminaries

Let $\mathcal{A}$ denote the class of functions $f(z)$ of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic and univalent in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

If the function $g \in \mathcal{A}$ is given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n\, z^n, \tag{2}$$

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then the *Hadamard product* (or *convolution*) of $f(z)$ and $g(z)$ is defined by (see also [27])

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n\, b_n\, z^n = (g * f)(z). \tag{3}$$

In the year 2007, Srivastava and Attiya (see [21]) defined the operator $\mathcal{J}_{s,a}$ by

$$\mathcal{J}_{s,a}(f)(z) = z + \sum_{n=2}^{\infty} \left(\frac{1+a}{n+a}\right)^s a_n\, z^n \tag{4}$$

$$(z \in \mathbb{U};\ a \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ \mathbb{Z}_0^- = \{0, 1, 2, \cdots\};\ s \in \mathbb{C}).$$

In fact, in terms of the *Hadamard product* (or *convolution*), the linear Srivastava-Attiya operator $\mathcal{J}_{s,a}(f)$ defined by (4) can be written as follows (see also the recent works [8], [25] and [28]):

$$\mathcal{J}_{s,b} f(z) = G_{s,a}(z) * f(z),$$

where $G_{s,a}(z)$ is given by

$$G_{s,a}(z) = (1+a)^s \left[\Phi(z, s, a) - a^{-s}\right] \qquad (z \in \mathbb{U}) \tag{5}$$

and the function $\Phi(z, s, a)$ involved in the right-hand side of (5) is the well-known Hurwitz-Lerch zeta function defined by (see [22])

$$\Phi(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{(n+a)^s} \tag{6}$$

$$\Big(z \in \mathbb{U};\ a \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ s \in \mathbb{C} \quad \text{when} \quad |z| < 1;\ \Re(s) > 1 \quad \text{when} \quad |z| > 1\Big).$$

Recently, a new family of λ-generalized Hurwitz-Lerch zeta functions was investigated by Srivastava (see [20]) who introduced this λ-generalized Hurwitz-Lerch zeta function

$$\Phi_{\lambda_1,\cdots,\lambda_p;\mu_1,\cdots,\mu_q}^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}(z, s, a; b, \lambda)$$

as well as gave the following explicit series representation for it (see [20, p. 1489, Eq. (2.1)]):

$$\Phi_{\lambda_1,\cdots,\lambda_p;\mu_1,\cdots,\mu_q}^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}(z, s, a; b, \lambda) = \frac{1}{\lambda \Gamma(s)} \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p} (\lambda_j)_{n\rho_j}}{(a+n)^s \cdot \prod_{j=1}^{q} (\mu_j)_{n\sigma_j}}$$

$$\cdot H_{0,2}^{2,0}\left[(a+n) b^{\frac{1}{\lambda}} \,\middle|\, \begin{matrix} \overline{\qquad\qquad} \\ (s,1), \left(0, \frac{1}{\lambda}\right) \end{matrix}\right] \frac{z^n}{n!} \qquad (\lambda > 0) \tag{7}$$

$$\Big(\lambda > 0;\ \lambda_j \in \mathbb{C}\ \ (j = 1, \cdots, p);\ \mu_j \in \mathbb{C} \setminus \mathbb{Z}_0^-\ \ (j = 1, \cdots, q);$$

$$\rho_j > 0\ \ (j = 1, \cdots, p);\ \sigma_j > 0\ \ (j = 1, \cdots, q)$$

$$1 + \sum_{j=1}^{q} \sigma_j - \sum_{j=1}^{p} \rho_j \geqq 0;\ \min\{\Re(a), \Re(b)\} > 0\Big),$$

where the equality in the convergence condition holds true for suitably bounded values of $|z|$ given by

$$|z| < \nabla := \left(\prod_{j=1}^{p} \rho_j^{-\rho_j}\right) \cdot \left(\prod_{j=1}^{q} \sigma_j^{\sigma_j}\right).$$

$(\lambda)_\nu$ $(\lambda, \nu \in \mathbb{C})$ denotes the *general* Pochhammer symbol (or the *shifted* factorial), occurring in (7), is defined, in terms of the familiar Gamma function, by

$$(\lambda)_\nu := \frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)} = \begin{cases} 1 & (\nu = 0;\ \lambda \in \mathbb{C} \setminus \{0\}) \\ \lambda(\lambda+1)\cdots(\lambda+n-1) & (\nu = n \in \mathbb{N};\ \lambda \in \mathbb{C}), \end{cases}$$

it being understood *conventionally* that $(0)_0 := 1$ and assumed *tacitly* that the above Γ-quotient exists. Moreover, the H-function involved in the right-hand side of (7) is the well-known Fox's H-function which is defined by (see, for example, [26, Chapter 2] and [12, pp. 58 *et seq.*])

$$\begin{aligned} H_{p,q}^{m,n}(z) &= H_{p,q}^{m,n}\left[z \,\middle|\, \begin{matrix} (a_p, A_p) \\ (b_q, B_q) \end{matrix}\right] \\ &= H_{p,q}^{m,n}\left[z \,\middle|\, \begin{matrix} (a_1, A_1), \cdots, (a_p, A_p) \\ (b_1, B_1), \cdots, (b_q, B_q) \end{matrix}\right] = \frac{1}{2\pi \mathrm{i}} \int_{\mathcal{L}} \Xi(\mathfrak{s}) z^{-\mathfrak{s}} \mathrm{d}\mathfrak{s}, \end{aligned} \tag{8}$$

where

$$\Xi(\mathfrak{s}) = \frac{\prod_{j=1}^{m} \Gamma\left(b_j + B_j \mathfrak{s}\right) \prod_{j=1}^{n} \Gamma\left(1 - a_j - A_j \mathfrak{s}\right)}{\prod_{j=m+1}^{q} \Gamma\left(1 - b_j - B_j \mathfrak{s}\right) \prod_{j=n+1}^{p} \Gamma\left(a_j + A_j \mathfrak{s}\right)}. \tag{9}$$

Here

$$z \in \mathbb{C} \setminus \{0\} \qquad \text{with} \qquad |\arg(z)| < \pi,$$

an empty product is interpreted as 1, m, n, p and q are integers such that $1 \leqq m \leqq q$ and $0 \leqq n \leqq p$,

$$A_j > 0 \quad (j = 1, \cdots, p) \qquad \text{and} \qquad B_j > 0 \quad (j = 1, \cdots, q),$$

$$\alpha_j \in \mathbb{C} \quad (j = 1, \cdots, p) \qquad \text{and} \qquad \beta_j \in \mathbb{C} \quad (j = 1, \cdots, q),$$

and $\mathcal{L}$ is a suitable Mellin-Barnes type contour separating the poles of the gamma functions

$$\left\{\Gamma(b_j + B_j \mathfrak{s})\right\}_{j=1}^{m}$$

from the poles of the gamma functions

$$\left\{\Gamma(1 - a_j - A_j \mathfrak{s})\right\}_{j=1}^{n}.$$

If, in the series representation (7), we make use of the following limit formula (see [20, p. 1496, Eq. (4.12)])

$$\lim_{b \to 0} \left\{ H_{0,2}^{2,0}\left[(a+n) b^{\frac{1}{\lambda}} \,\middle|\, \begin{matrix} \overline{\phantom{(s,1),(0,\frac{1}{\lambda})}} \\ (s,1), \left(0, \frac{1}{\lambda}\right) \end{matrix}\right] \right\} = \lambda \Gamma(s) \qquad (\lambda > 0), \tag{10}$$

we find for the extended Hurwitz-Lerch zeta function

$$\Phi^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}_{\lambda_1,\cdots,\lambda_p;\mu_1,\cdots,\mu_q}(z,s,a)$$

that (see [29, p. 503, Eq. (6.2)])

$$\Phi^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}_{\lambda_1,\cdots,\lambda_p;\mu_1,\cdots,\mu_q}(z,s,a) := \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p}(\lambda_j)_{n\rho_j}}{n! \cdot \prod_{j=1}^{q}(\mu_j)_{n\sigma_j}} \frac{z^n}{(n+a)^s} \tag{11}$$

$$\Big(p,q \in \mathbb{N}_0;\ \lambda_j \in \mathbb{C}\ \ (j=1,\cdots,p);\ a,\mu_j \in \mathbb{C}\setminus \mathbb{Z}_0^-\ \ (j=1,\cdots,q);$$
$$\rho_j,\sigma_k \in \mathbb{R}^+\ \ (j=1,\cdots,p;\ k=1,\cdots,q);\ \Delta > -1\ \text{ when }\ s,z \in \mathbb{C};$$
$$\Delta = -1\ \text{ and }\ s \in \mathbb{C}\ \text{ when }\ |z| < \nabla^*;$$
$$\Delta = -1\ \text{ and }\ \Re(\Xi) > \frac{1}{2}\ \text{ when }\ |z| = \nabla^*\Big),$$

which was defined by Srivastava *et al.* (see [20, p. 1496, Eq. (4.12)]). In fact, the function

$$\Phi^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}_{\lambda_1,\cdots,\lambda_p;\mu_1,\cdots,\mu_q}(z,s,a)$$

in (11), which was introduced by Srivastava *et al.* [29], is a multiparameter extension and generalization of the classical Hurwitz-Lerch zeta function $\Phi(z,s,a)$ defined by (6).

By applying Srivastava's λ-generalized Hurwitz-Lerch zeta function

$$\Phi^{(\rho_1,\cdots,\rho_p,\sigma_1,\cdots,\sigma_q)}_{\lambda_1,\cdots,\lambda_p;\mu_1,\cdots,\mu_q}(z,s,a;b,\lambda)$$

occurring on the left-hand side of (7), Srivastava and Gaboury [24] introduced the following linear operator:

$$\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(f) : \mathcal{A} \to \mathcal{A},$$

which they defined by

$$\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(f)(z) = G^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(z) * f(z), \tag{12}$$

where

$$G^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(z) = \frac{\lambda\Gamma(s)\prod_{j=1}^{q}\left(\mu_j\right)(a+1)^s}{\prod_{j=1}^{p}\left(\lambda_j\right)}[\Lambda(a+1,b,s,\lambda)]^{-1}$$
$$\cdot \left[\Phi^{(1,\cdots,1,1,\cdots,1)}_{\lambda_1,\cdots,\lambda_p;\mu_1,\cdots,\mu_q}(z,s,a;b,\lambda) - \frac{a^{-s}}{\lambda\Gamma(s)}\Lambda(a,b,s,\lambda)\right]$$
$$= z + \sum_{n=2}^{\infty} \frac{\prod_{j=1}^{p}\left(\lambda_j+1\right)_{n-1}}{\prod_{j=1}^{q}\left(\mu_j+1\right)_{n-1}} \cdot \left(\frac{a+1}{a+n}\right)^s \frac{\Lambda(a+n,b,s,\lambda)}{\Lambda(a+1,b,s,\lambda)} \frac{z^n}{n!} \tag{13}$$

with

$$\Lambda(a,b,s,\lambda) := H_{0,2}^{2,0}\left[(a+n)b^{\frac{1}{\lambda}} \,\middle|\, \begin{array}{c} \overline{\qquad\qquad} \\ (s,1),\left(0,\frac{1}{\lambda}\right) \end{array}\right]. \tag{14}$$

Now, from (12) and (13), we have

$$\mathcal{J}_{(\lambda_p),(\mu_q),b}^{s,a,\lambda} f(z) = z + \sum_{n=2}^{\infty} \frac{\prod_{j=1}^{p}\left(\lambda_j+1\right)_{n-1}}{\prod_{j=1}^{q}\left(\mu_j+1\right)_{n-1}} \cdot \left(\frac{a+1}{a+n}\right)^s \frac{\Lambda(a+n,b,s,\lambda)}{\Lambda(a+1,b,s,\lambda)}\, a_n\, \frac{z^n}{n!} \tag{15}$$

$$\Big(\lambda_j \in \mathbb{C} \ \ (j=1,\cdots,p);\ \mu_j \in \mathbb{C}\setminus\mathbb{Z}_0^- \ \ (j=1,\cdots,q);\ p \leqq q+1;\ z\in\mathbb{U};$$

$$\min\{\Re(a),\Re(s)\}>0;\ \lambda>0 \ \text{ when } \ \Re(b)>0 \ \text{ and } \ s\in\mathbb{C};\ a\in\mathbb{C}\setminus\mathbb{Z}_0^- \ \text{ when } \ b=0\Big).$$

It is easy to see from the definition (15) that

$$z\left(\mathcal{J}_{(\lambda_p),(\mu_q),b}^{s,a,\lambda} f(z)\right)' = (\lambda_1+1)\ \mathcal{J}_{(\lambda_1+1,\lambda_2,\cdots,\lambda_p),(\mu_q),b}^{s,a,\lambda} f(z) - \lambda_1\, \mathcal{J}_{(\lambda_p),(\mu_q),b}^{s,a,\lambda} f(z). \tag{16}$$

Definition 1. Let Ψ be the set of complex-valued functions $\psi(u,v,w)$ given by

$$\psi(u,v,w) : \mathbb{C}^3 \to \mathbb{C}$$

such that

(i) $\psi(u,v,w)$ is continuous in a domain $\mathbb{D}\subset\mathbb{C}^3$;

(ii) $(0,0,0)\in\mathbb{D}$ and $\left|\psi(0,0,0)\right|<1$;

(iii) The following inequality holds true:

$$\left|\psi\left(e^{i\theta}, \left[\frac{\lambda_1+t}{\lambda_1+1}\right]e^{i\theta}, \frac{1}{\lambda_1+1}\left[\lambda_1+2t+\frac{L}{\lambda_1+1}\right]e^{i\theta}\right)\right| \geqq 1$$

when $\lambda_1 \notin \mathbb{Z}_0^-$ and

$$\left(e^{i\theta}, \left[\frac{\lambda_1+t}{\lambda_1+1}\right]e^{i\theta}, \frac{1}{\lambda_1+1}\left[\lambda_1+2t+\frac{L}{\lambda_1+1}\right]e^{i\theta}\right) \in \mathbb{D}$$

with $\Re(L) \geqq t(t-1)$ for real $\theta\in\mathbb{R}$ and $t \geqq 1$.

By using the generalization of the Srivastava-Attiya operator defined by (15), we now introduce the following integral operator:

$$\mathfrak{F}_{(\lambda_p),(\mu_q),b}^{\beta,s,a,\lambda}(\gamma_1,\cdots,\gamma_k;z) : \mathcal{A}^n \to \mathcal{A}.$$

Definition 2. For $\beta,\gamma_1,\gamma_2,\cdots,\gamma_k \in \mathbb{C}$ with

$$\Re(\beta)>0 \qquad \text{and} \qquad \Re(\gamma_m)>0 \qquad (m\in\{1,\cdots,k\}),$$

we define the integral operator:

$$\mathfrak{F}_{(\lambda_p),(\mu_q),b}^{\beta,s,a,\lambda}(\gamma_1,\cdots,\gamma_k;z) : \mathcal{A}^n \to \mathcal{A}$$

by

$$\mathfrak{F}^{\beta,s,a,\lambda}_{(\lambda_p),(\mu_q),b}\ (\gamma_1,\cdots,\gamma_k;z)=\left(\beta\int_0^z t^{\beta-1}\prod_{m=1}^{k}\left[\frac{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\ f_m(t)}{t}\right]^{\frac{1}{\gamma_m}}dt\right)^{\frac{1}{\beta}}. \tag{17}$$

By suitably specializing Definition 2, we are led to the following integral operators:

$$\begin{aligned}&\mathfrak{F}^{1+k(\alpha-1),s,a,\lambda}_{(\alpha_1-1,\cdots,\alpha_p-1),(\beta_1-1,\cdots,\beta_q-1),0}\left(\frac{1}{\alpha-1},\cdots,\frac{1}{\alpha-1};z\right)\\&=F_\alpha(\alpha_1,\beta_1;z)=\left([1+k(\alpha-1)]\cdot\int_0^z(H^p_q(\alpha_1,\beta_1)f_1(t))^{\alpha-1}\cdots,\left[H^p_q(\alpha_1,\beta_1)f_k(t)\right]^{\alpha-1}\ dt\right)^{\frac{1}{1+k(\alpha-1)}},\end{aligned} \tag{18}$$

where the operator $F_\alpha(\lambda_1,\mu_1;z)$ was investigated by Selvaraj and Karthikeyan [19];

$$\begin{aligned}&\mathfrak{F}^{1+k(\alpha-1),s,a,\lambda}_{(\lambda,1),(\lambda),0}\left(\frac{1}{\alpha-1},\cdots,\frac{1}{\alpha-1};z\right)=F_{k,\alpha}(z)\\&=\left([1+k(\alpha-1)]\int_0^z[f_1(t)]^{\alpha-1}\cdots[f_k(t)]^{\alpha-1}\ dt\right)^{\frac{1}{1+k(\alpha-1)}},\end{aligned} \tag{19}$$

where the operator $F_{k,\alpha}(z)$ was investigated by Breaz *et al.* (see [1], [3] and [5]);

$$\begin{aligned}&\mathfrak{F}^{\beta,0,a,\lambda}_{(\lambda,1),(\lambda),0}\left(\frac{1}{\alpha_1},\cdots,\frac{1}{\alpha_k};z\right)=J_{\alpha_1,\cdots,\alpha_k,\beta}(z)\\&=\left[\beta\int_0^z t^{\beta-1}\left(\frac{f_1(t)}{t}\right)^{\alpha_1}\cdots\left(\frac{f_k(t)}{t}\right)^{\alpha_k}dt\right]^{\frac{1}{\beta}},\end{aligned} \tag{20}$$

where the operator $J_{\alpha_1,\cdots,\alpha_k,\beta}(z)$ was investigated by Breaz and Breaz [2] (see also Stanciu *et al.* [30]);

$$\begin{aligned}&\mathfrak{F}^{\beta,0,a,\lambda}_{(\lambda,1),(\lambda),0}\left(\frac{1}{\alpha_1},\cdots,\frac{1}{\alpha_k};z\right)=F_{\alpha_1,\cdots,\alpha_k,\beta}(z)\\&=\left[\beta\int_0^z t^{\beta-1}\left(\frac{f_1(t)}{t}\right)^{\frac{1}{\alpha_1}}\cdots\left(\frac{f_k(t)}{t}\right)^{\frac{1}{\alpha_k}}\ dt\right]^{\frac{1}{\beta}},\end{aligned} \tag{21}$$

where the operator $F_{\alpha_1,\cdots,\alpha_k,\beta}(z)$ was investigated by Seenivasagan and Breaz [18] (see also [6]);

$$\begin{aligned}&\mathfrak{F}^{1,0,a,\lambda}_{(\lambda,1),(\lambda),0}\left(\frac{1}{\alpha_1},\cdots,\frac{1}{\alpha_k};z\right)=F(z)\\&=\int_0^z\left(\frac{f_1(t)}{t}\right)^{\alpha_1}\cdots\left(\frac{f_k(t)}{t}\right)^{\alpha_k}\ dt,\end{aligned} \tag{22}$$

where the operator $F(z)$ was investigated by Breaz and Breaz [2];

$$\begin{aligned}&\mathfrak{F}^{1+k(\alpha-1),0,a,\lambda}_{(2,1,1),(1,0),0}\left(\frac{1}{\alpha-1},\cdots,\frac{1}{\alpha-1};z\right)=F_\alpha(z)=\Bigg([1+k(\alpha-1)]\\&\cdot\int_0^z t^{k(\alpha-1)}\left[f_1'(t)\right]^{\alpha-1}\cdots\left[f_k'(t)\right]^{\alpha-1}\ dt\Bigg)^{\frac{1}{1+k(\alpha-1)}},\end{aligned} \tag{23}$$

where the operator $F_\alpha(z)$ was investigated by Selvaraj and Karthikeyan [19];

$$\begin{aligned}\mathfrak{F}^{1,0,a,\lambda}_{(2,1,1),(1,0),0}\left(\frac{1}{\alpha_1},\cdots,\frac{1}{\alpha_k};z\right) &= F_{\alpha_1,\cdots,\alpha_k}(z)\\ &= \int_0^z \left[f_1'(t)\right]^{\alpha_1-1}\cdots\left[f_k'(t)\right]^{\alpha_k-1}\,dt,\end{aligned} \tag{24}$$

where the operator $F_\alpha(z)$ was investigated by Breaz *et al.* [7];

$$\mathfrak{F}^{\delta,0,a,\lambda}_{(\lambda,1),(\lambda),0}\left(\frac{1}{\alpha-1},\cdots,\frac{1}{\alpha-1};z\right) = F_\alpha(z) = \left(\alpha\int_0^z [f(t)]^{\alpha-1}\right)^{\frac{1}{\alpha}}\,dt, \tag{25}$$

where the operator $F_\alpha(z)$ was investigated by Pescar [17].

By making use of the integral operator defined in (15), we have the following definition.

Definition 3. A function $f_m \in \mathcal{A}$ $(m \in \{1,\cdots,k\})$ is said to be in the class $\mathcal{S}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}$ if it satisfy the following condition:

$$\left|\frac{z^2\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\,f_m(t)\right)'}{\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\,f_m(t)\right)^2}-1\right|<1 \qquad (z\in\mathbb{U};\ m\in\{1,\cdots,k\}). \tag{26}$$

In our investigation of the function class $\mathcal{S}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}$ given by Definition 3, we shall need the univalence criteria and other results asserted by the following lemmas.

Lemma 1. (see [14]) *Let the function f be analytic in the disk*

$$\mathbb{U}_R = \{z : z \in \mathbb{C} \qquad \text{and} \qquad |z| < R\}$$

with $|f(z)| < M$ for some fixed $M > 0$. If $f(z)$ has one zero with multiplicity order bigger that m for $z = 0$, then

$$|f(z)| \leqq \frac{M}{R^m}\,|z|^m \qquad (z \in \mathbb{U}_R). \tag{27}$$

The equality holds true in (27) *only if*

$$f(z) = e^{i\theta}\,\frac{M}{R^m}\,z^m \qquad (z \in \mathcal{U}_R),$$

where θ is real constant.

Lemma 2. (see [15] and [16]) *Let $\beta \in \mathbb{C}$ with $\Re(\beta) > 0$. If the function $f(z) \in \mathcal{A}$ is constrained by*

$$\frac{1-|z|^{2\Re(\beta)}}{\Re(\beta)}\left|\frac{zf''(z)}{f'(z)}\right| \leqq 1 \qquad (z \in \mathbb{U}),$$

then the function $F_\beta(z)$ given in terms of the following integral operator:

$$\begin{aligned}F_\beta(z) &= \left(\beta\int_0^z t^{\beta-1}f'(t)\,dt\right)^{\frac{1}{\beta}}\\ &= z + \frac{2a_2}{\beta+1}\,z^2 + \left(\frac{3a_3}{\beta+2} - \frac{2\beta(1-\beta)a_2^2}{(\beta+1)^2}\right)z^3 + \cdots\end{aligned} \tag{28}$$

is in the class $\mathcal{S}$ of normalized analytic and univalent functions in $\mathbb{U}$.

Lemma 3. (see [17]) *Let* $\beta \in \mathbb{C}$ *with*

$$\Re(\beta) > 0 \qquad \text{and} \qquad c \in \mathbb{C} \qquad |c| \leqq 1.$$

If the function $f(z) \in \mathcal{A}$ *is constrained by*

$$\left| c\,|z|^{2\beta} + (1 - |z|^{2\beta}) \frac{zf''(z)}{\beta f'(z)} \right| \leqq 1 \qquad (z \in \mathbb{U}),$$

then the function $F_\beta(z)$ *given in terms of the following integral operator*:

$$F_\beta(z) = \left(\beta \int_0^z t^{\beta-1} f'(t)\,dt \right)^{\frac{1}{\beta}} \tag{29}$$

is in the class $\mathcal{S}$ *of normalized analytic and univalent functions in* $\mathbb{U}$.

Lemma 4. (see [13]) *Let the function* $w(z)$ *given by*

$$\omega(z) = a + \omega_r z^r + \omega_{r+1} z^{r+1} + \cdots$$

be analytic in $\mathbb{U}$ *with*

$$\omega(z) \neq a \qquad \text{and} \qquad r \in \mathbb{N}.$$

If

$$z_0 = r_0 e^{i\theta} \quad (0 < r_0 < 1) \qquad \text{and} \qquad |\omega(z_0)| = \max_{|z| \leqq r_0} \{|\omega(z)|\},$$

then

$$\frac{z_0 \omega'(z_0)}{\omega(z_0)} = \tau \qquad \text{and} \qquad \Re\left(1 + \frac{z_0 \omega''(z_0)}{\omega'(z_0)}\right) \geqq \tau, \tag{30}$$

where τ *is a real number and*

$$\tau \geqq r\, \frac{|\omega(z_0) - a|^2}{|\omega(z_0)|^2 - |a|^2} \geqq r\, \frac{|\omega(z_0)| - |a|}{|\omega(z_0)| + |a|}.$$

2. Main Results and Their Corollaries

We begin by proving Theorem 1 below.

Theorem 1. *Let the functions* $f_m(z) \in \mathcal{A}$ $(m = 1, \cdots, k)$. *Suppose that* $\beta, \gamma_m \in \mathbb{C}$ $(m = 1, \cdots, k)$ *with*

$$\Re(\beta) > 0 \qquad \text{and} \qquad M_m > 0 \quad (m = 1, \cdots, k).$$

Also let

$$\sum_{m=1}^{k} \frac{2M_m + 1}{|\gamma_m|} \leqq \Re(\beta). \tag{31}$$

If, for all $m \in \{1, \cdots, k\}$,

$$f_m(z) \in \mathcal{S}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(z)$$

and

$$\left| \mathcal{J}^{s,a,\nu}_{(\lambda_p),(\mu_q),b} f_m(z) \right| \leqq M_m \qquad (z \in \mathbb{U}), \tag{32}$$

then the general integral operator defined by (17) is analytic and univalent in $\mathbb{U}$.

Proof. It is easy to verify that

$$\frac{\mathcal{J}^{s,a,\nu}_{(\lambda_p),(\mu_q),b}\, f_m(z)}{z} \neq 0.$$

Hence, for $z = 0$, we find that

$$\left(\frac{\mathcal{J}^{s,a,\nu}_{(\lambda_p),(\mu_q),b}\, f_1(z)}{z}\right)^{\frac{1}{\mu_j}} \cdots \left(\frac{\mathcal{J}^{s,a,\nu}_{(\lambda_p),(\mu_q),b}\, f_m(z)}{z}\right)^{\frac{1}{\gamma_k}} = 1.$$

Let us define the function $g(z)$ as follows:

$$g(z) = \int_0^z \prod_{m=1}^{k} \left(\frac{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(t)}{t}\right)^{\frac{1}{\gamma_m}} dt. \tag{33}$$

Then we have

$$\frac{zg''(z)}{g'(z)} = \sum_{m=1}^{k} \frac{1}{\gamma_m} \left(\frac{z\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)'}{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)} - 1\right),$$

so that

$$\left|\frac{zg''(z)}{g'(z)}\right| \leqq \sum_{m=1}^{k} \frac{1}{|\gamma_m|} \left|\frac{z\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)'}{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)} - 1\right|.$$

Therefore, we get

$$\begin{aligned}
&\frac{1-|z|^{2\Re(\beta)}}{\Re(\beta)} \left|\frac{zg''(z)}{g'(z)}\right| \\
&\leqq \frac{1-|z|^{2\Re(\beta)}}{\Re(\beta)} \left(\sum_{m=1}^{k} \frac{1}{|\gamma_m|} \left|\frac{z\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)'}{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)}\right| + 1\right) \\
&\leqq \frac{1-|z|^{2\Re(\beta)}}{\Re(\beta)} \left(\sum_{m=1}^{k} \frac{1}{|\gamma_m|} \left|\frac{z^2\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)'}{\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)^2}\right| \cdot \left|\frac{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)}{z}\right| + 1\right) \\
&\leqq \frac{1-|z|^{2\Re(\beta)}}{\Re(\beta)} \left(\sum_{m=1}^{k} \frac{1}{|\gamma_m|} \left[\left|\frac{z^2\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_j(z)\right)'}{\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_i(z)\right)^2} - 1\right| + 1\right] \cdot \left|\frac{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)}{z}\right| + 1\right) \\
&\leqq \frac{1}{\Re(\beta)} \sum_{m=1}^{k} \frac{2M_m + 1}{|\gamma_m|}.
\end{aligned}$$

By using the Schwarz lemma, we have

$$\left|\mathcal{J}^{s,a,\nu}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right| \leqq M_m\, |z| \qquad (z \in \mathcal{U}).$$

Now, from (31), we obtain

$$\frac{1-|z|^{2\Re(\beta)}}{\Re(\beta)} \left|\frac{zg''(z)}{g'(z)}\right| \leqq 1.$$

Finally, by applying Lemma 2 for the function $g(z)$, we obtain the required result asserted by Theorem 1. □

Remark 1. If, in Theorem 1, we set

$$\lambda_1 = \alpha_1 - 1, \cdots, \lambda_p = \alpha_p - 1, \qquad \mu_1 = \beta_1 - 1, \cdots, \mu_q = \beta_q - 1,$$

$$\gamma_1 = \frac{1}{\alpha - 1}, \cdots, \gamma_k = \frac{1}{\alpha - 1} \qquad \text{and} \qquad M_m = 1 \quad (1 \leqq m \leqq k),$$

we obtain a known result proven in [19].

Corollary 1. *Let the functions* $f_m(z) \in \mathcal{A}$ $(m \in \{1, \cdots, k\})$. *Also let* $\alpha \in \mathbb{C}$ *with*

$$\Re(\alpha) > 0 \qquad \text{and} \qquad |\alpha - 1| \leqq \frac{\Re(\alpha)}{3k}.$$

If

$$\left| \frac{z^2 \left(H_q^p(\alpha_1, \beta_1) f_m(t) \right)'}{\left[H_q^p(\alpha_1, \beta_1) f_m(t) \right]^2} - 1 \right| < 1$$

and

$$\left| H_q^p(\alpha_1, \beta_1) f_m(t) \right| \leqq M_m \qquad (m = 1, \cdots, k;\ z \in \mathbb{U}),$$

then the general integral operator defined by (18) *is analytic and univalent in* $\mathbb{U}$.

Remark 2. Putting

$$p = 2, \ q = 1, \ \lambda_1 = \lambda, \ \lambda_2 = 1, \ \mu_1 = \lambda, \ \gamma_j = \frac{1}{\alpha - 1} \quad (j = 1, \cdots, k)$$

and

$$M_m = 1 \qquad (1 \leqq m \leqq k)$$

in Theorem 1, we obtain another known result given in [4].

Corollary 2. *Let the functions* $f_m(z) \in \mathcal{A}$ $(m \in \{1, \cdots, k\})$. *Also let* $\alpha \in \mathbb{C}$ *with*

$$\Re(\alpha) > 0 \qquad \text{and} \qquad |\alpha - 1| \leqq \frac{\Re(\alpha)}{3k}.$$

If

$$\left| \frac{z^2 f_m'(t)}{f_m^2(t)} - 1 \right| < 1 \qquad \text{and} \qquad \left| f_m(t) \right| \leqq 1 \qquad (m = 1, \cdots, k;\ z \in \mathbb{U}),$$

then the general integral operator defined by (19) *is analytic and univalent in* $\mathbb{U}$.

We now prove another result asserted by Theorem 2 below.

Theorem 2. *Let the functions* $f_m(z) \in \mathcal{A}$ $(m = 1, \cdots, k)$. *Suppose that*

$$c, \beta \in \mathbb{C} \qquad \text{and} \qquad M_m > 0 \quad (m = 1, \cdots, k).$$

Also let

$$\gamma_m \in \left[1, \max_{1 \leqq m \leqq k} \left\{ \frac{(2M_m+1)k}{(2M_m+1)k-1} \right\}\right] \qquad (m = 1, \cdots, k)$$

and

$$|c| \leqq 1 - \frac{1}{\Re(\beta)} \max_{1 \leqq m \leqq k} \left\{ \frac{(2M_m+1)\,k}{|\gamma_m|} \right\}. \tag{34}$$

If, for all $m = 1, \cdots, k$,

$$f_m(z) \in \mathcal{S}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(z) \qquad \text{and} \qquad \left|\mathcal{J}^{s,a,\nu}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right| \leqq M_m \qquad (z \in \mathbb{U}), \tag{35}$$

then the general integral operator defined by (17) *is analytic and univalent in* $\mathbb{U}$.

Proof. From Theorem 1, we have

$$\frac{zg''(z)}{g'(z)} = \sum_{m=1}^{k} \frac{1}{\gamma_m} \left(\frac{z\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)'}{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)} - 1 \right),$$

so that

$$\begin{aligned}
&\left| c\,|z|^{2\beta} + (1-|z|^{2\beta}) \frac{zg''(z)}{\beta g'(z)} \right| \\
&= \left| c\,|z|^{2\beta} + \left(\frac{1-|z|^{2\beta}}{\beta}\right) \left[\sum_{m=1}^{k} \frac{1}{\gamma_m} \left(\frac{z\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)'}{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)} - 1 \right) \right] \right| \\
&\leqq |c| + \frac{1}{\left|\beta\right|} \left(\sum_{m=1}^{k} \frac{1}{\left|\gamma_m\right|} \left| \frac{z\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)'}{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)} - 1 \right| \right) \\
&\leqq |c| + \frac{1}{\Re(\beta)} \left(\sum_{m=1}^{k} \frac{1}{\left|\gamma_m\right|} \left| \frac{z^2\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)'}{\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)\right)^2} \right| \cdot \left| \frac{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)}{z} \right| + 1 \right) \\
&\leqq |c| + \frac{1}{\Re(\beta)} \left(\sum_{m=1}^{k} \frac{1}{\left|\gamma_m\right|} \left[\left| \frac{z^2\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_j(z)\right)'}{\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_j(z)\right)^2} - 1 \right| + 1 \right] \cdot \left| \frac{\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\, f_m(z)}{z} \right| + 1 \right) \\
&\leqq |c| + \frac{1}{\Re(\beta)} \sum_{m=1}^{k} \frac{2M_m+1}{|\gamma_m|} \\
&\leqq |c| + \frac{1}{\Re(\beta)} \max_{1 \leqq m \leqq k} \frac{(2M_m+1)\,k}{|\gamma_m|}.
\end{aligned}$$

Now, by making use of (34), we obtain

$$\left| c\,|z|^{2\beta} + \left(1-|z|^{2\beta}\right) \frac{zg''(z)}{\beta g'(z)} \right| \leqq 1.$$

Finally, if we apply Lemma 3 for the function $g(z)$, we obtain the result asserted by Theorem 2. □

Remark 3. If we set

$$p = 2, \quad q = 1, \quad \lambda_1 = \lambda, \quad \lambda_2 = 1, \quad \mu_1 = \lambda \quad \text{and} \quad \gamma_j = \frac{1}{\alpha_j} \quad (j = 1, \cdots, k)$$

in Theorem 2, we obtain a known result (see [31]).

Corollary 3. *Let the functions* $f_m(z) \in \mathcal{A} \quad (m = 1, \cdots, k)$. *Suppose that*

$$c, \beta \in \mathbb{C} \qquad \textit{and} \qquad M_m \geqq 1 \quad (m = 1, \cdots, k).$$

Also let

$$\alpha_m \in \left[1, \max_{1 \leqq m \leqq k} \left\{ \frac{(2M_m + 1)k}{(2M_m + 1)k - 1} \right\} \right] \qquad (m = 1, \cdots, k)$$

and

$$|c| \leqq 1 - \frac{k}{\Re(\beta)} \max_{1 \leqq m \leqq k} (2M_m + 1) |\alpha_m|.$$

If

$$\left| f_m(z) \right| \leqq M_m \qquad \textit{and} \qquad \left| \frac{z f_m'(z)}{f_m^2(z)} - 1 \right| \leqq 1 \qquad (z \in \mathbb{U}; \ m = 1, \cdots, k),$$

then the general integral operator defined by (20) *is analytic and univalent in* $\mathbb{U}$.

Finally, we state and prove Theorem 3 below.

Theorem 3. *Let* $\lambda_1 \notin \mathbb{Z}_0^-$. *Suppose that* $\psi(u, v, w) \in \Psi$ *and that*

$$\Big(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b} f(z), \ \mathcal{J}^{s,a,\lambda}_{(\lambda_1+1,\lambda_2,\cdots,\lambda_p),(\mu_q),b} f(z), \ \mathcal{J}^{s,a,\lambda}_{(\lambda_1+2,\lambda_2,\cdots,\lambda_p),(\mu_q),b} f(z) \Big) \in \mathbb{D} \subset \mathbb{C}^3. \tag{36}$$

If

$$\left| \psi\Big(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b} f(z), \ \mathcal{J}^{s,a,\lambda}_{(\lambda_1+1,\lambda_2,\cdots,\lambda_p),(\mu_q),b} f(z), \ \mathcal{J}^{s,a,\lambda}_{(\lambda_1+2,\lambda_2,\cdots,\lambda_p),(\mu_q),b} f(z) \Big) \right| < 1 \qquad (z \in \mathbb{U}), \tag{37}$$

then

$$\left| \Big(\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b} f(z) \Big) \right| < 1 \qquad (z \in \mathbb{U}).$$

Proof. Let

$$\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b} f(z) = \omega(z) \qquad (z \in \mathbb{U}). \tag{38}$$

Thus, clearly, it follows that $\omega(z)$ is analytic in $\mathbb{U}$,

$$\omega(0) = 1 \qquad \text{and} \qquad \omega(z) \neq 1 \qquad (z \in \mathbb{U}).$$

Upon differentiating both sides (38) with respect to z, if we make use of the identity (16), we readily obtain

$$(\lambda_1+1)\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_1+1,\lambda_2,\cdots,\lambda_p),(\mu_q),b}\,f(z)\right)=z\omega'(z)+\lambda_1\omega(z). \tag{39}$$

Moreover, by differentiating (39) with respect to z and using the following identity:

$$\begin{aligned}z\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_1+1,\lambda_2,\cdots,\lambda_p),(\mu_q),b}\,f(z)\right)'&=(\lambda_1+2)\ \mathcal{J}^{s,a,\lambda}_{(\lambda_1+2,\lambda_2,\cdots,\lambda_p),(\mu_q),b}\,f(z)\\&\quad-(\lambda_1+1)\ \mathcal{J}^{s,a,\lambda}_{(\lambda_1+1,\lambda_2,\cdots,\lambda_p),(\mu_q),b}\,f(z),\end{aligned}$$

which is a consequence of the identity (16), we obtain

$$\begin{aligned}&(\lambda_1+2)\left(\mathcal{J}^{s,a,\lambda}_{(\lambda_1+2,\lambda_2,\cdots,\lambda_p),(\mu_q),b}\,f(z)\right)\\&\qquad=\lambda_1\omega(z)+2z\omega'(z)+\frac{1}{\lambda_1+1}z^2\omega''(z)\qquad(z\in\mathbb{U}).\end{aligned} \tag{40}$$

We now claim that

$$|\omega(z)|<1\qquad(z\in\mathbb{U}).$$

Otherwise, there exists a point $z_0\in\mathbb{U}$ such that

$$\max_{|z|\leqq|z_0|}|\omega(z)|=|\omega(z_0)|=1. \tag{41}$$

Thus, by letting $\omega(z_0)=e^{i\theta}$ and using Lemma 4 with $a=1$ and $r=1$, we see that

$$\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\,f(z)=e^{i\theta},$$

$$\mathcal{J}^{s,a,\lambda}_{(\lambda_1+1,\lambda_2,\cdots,\lambda_p),(\mu_q),b}\,f(z)=\frac{1}{\lambda_1+1}\,(\lambda_1+\tau)\,e^{i\theta}$$

and

$$\mathcal{J}^{s,a,\lambda}_{(\lambda_1+2,\lambda_2,\cdots,\lambda_p),(\mu_q),b}\,f(z)=\frac{1}{\lambda_1+2}\left(\lambda_1+2\tau+\frac{L}{\lambda_1+1}\right)e^{i\theta},$$

where

$$L=\frac{z_0^2w''(z_0)}{\omega(z_0)}\qquad\text{and}\qquad\tau\geqq1.$$

Furthermore, by an application of (30) in Lemma 4, we get

$$\Re(L)\geqq\tau(\tau-1).$$

Since $\psi(u,v,w)\in\Psi$, we have

$$\left|\psi\left(e^{i\theta},\left[\frac{\lambda_1+\tau}{\lambda_1+1}\right]e^{i\theta},\frac{1}{\lambda_1+1}\left[\lambda_1+2\tau+\frac{L}{\lambda_1+1}\right]e^{i\theta}\right)\right|\geqq1, \tag{42}$$

which contradicts the condition (37) of Theorem 3. Therefore, we conclude that

$$\left|\mathcal{J}^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}\,f(z)\right|<1\qquad(z\in\mathbb{U}),$$

which evidently completes the proof of Theorem 3. □

3. Concluding Remarks and Observations

In our present investigation, we have introduced and studied systematically the univalence criteria of a new family of integral operators by using a substantially general form of the widely-investigated Srivastava-Attiya operator. In particular, we have derived new sufficient conditions of univalence for this generalized Srivastava-Attiya operator. Our main results are contained in Theorems 1, 2 and 3. By suitably specializing these main results, we have deduced several corollaries and consequences which were derived in a number related earlier works on the subject of investigation here (see also the recent works [9], [10], [11] and [23]).

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