Filomat 31:20 (2017), 6543–6549
https://doi.org/10.2298/FIL1720543E

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

φ-Contractibility of Some Classes of Banach Function Algebras

Morteza Essmaili[a], Amir H. Sanatpour[a]

[a]*Department of Mathematics, Kharazmi University, Tehran, Iran*

Abstract. In this paper, we study φ-contractibility of natural Banach function algebras on a compact Hausdorff space. As a consequence, we characterize φ-contractibility of the Lipschitz algebra $Lip(X, d^\alpha)$, for a compact metric space (X, d). We also characterize φ-contractibility of certain subalgebras of Lipschitz functions including rational Lipschitz algebras, analytic Lipschitz algebras and differentiable Lipschitz algebras.

1. Introduction and Preliminaries

As a generalization of the left amenability of Lau algebras, Kaniuth, Lau and Pym introduced and studied the notion of φ-amenability for Banach algebras in [13] and [14], where $\varphi : A \longrightarrow \mathbb{C}$ is a character, i.e., a non-zero homomorphism of A. Independently, Sangani Monfared introduced and studied the notion of character amenability for Banach algebras in [18]. We say that the Banach algebra A is *left* (*right*) *φ-amenable* if for all Banach A-bimodules E for which the left (right) module action is given by

$$a \cdot x = \varphi(a)x, \quad (x \cdot a = \varphi(a)x) \qquad (a \in A, x \in E),$$

every derivation $D : A \longrightarrow E'$ is inner. Let $\Delta(A)$ denote the set of all characters on A. We say that A is *left* (*right*) *character amenable* if A is left (right) φ-amenable for all $\varphi \in \Delta(A) \cup \{0\}$. In [11, Theorem 2.3], the authors characterized φ-amenability of Banach algebras in terms of the existence of bounded approximate φ-diagonals and φ-virtual diagonals. According to this characterization, the authors in [11] defined the notions of φ-contractibility and character contractibility of Banach algebras. A Banach algebra A is called *right φ-contractible* if there exists $\mathbf{m} \in A\hat{\otimes}A$ such that

$$a \cdot \mathbf{m} = \varphi(a)\mathbf{m} \quad \text{and} \quad \varphi(\pi(\mathbf{m})) = 1 \qquad (a \in A),$$

where $\pi : A\hat{\otimes}A \longrightarrow A$ is the diagonal operator defined by $\pi(a \otimes b) = ab$. *Left φ-contractibility* and *character contractibility* of Banach algebras can be defined in a similar way. These notions have been studied for some classes of Banach algebras. For example, Alaghmandan et al. studied character contractibility of abstract Segal algebras [2]. See, also [3] and [19] for other types of character amenability.

2010 *Mathematics Subject Classification*. 16E40, 46J15.
Keywords. φ-contractibility, φ-amenability, Lipschitz algebras, rational Lipschitz algebras, analytic Lipschitz algebras, differentiable Lipschitz algebras.
Received: 11 June 2016; Accepted: 12 April 2017
Communicated by Dragan S. Djordjević
Email addresses: m.essmaili@khu.ac.ir (Morteza Essmaili), a_sanatpour@khu.ac.ir, a.sanatpour@gmail.com (Amir H. Sanatpour)

For a topological space X, let $C(X)$ denote the algebra of all complex-valued continuous functions on X. A *function algebra* on a compact Hausdorff space X is a subalgebra of $C(X)$ which separates the points of X and contains the constant functions. A *Banach function algebra* on X is a function algebra on X which is complete under an algebra norm (see [4], [5] and [15]). If the norm of a Banach function algebra is the *uniform norm* on X, that is

$$\|f\|_X = \sup_{x\in X} \left|f(x)\right|,$$

then it is called a *uniform algebra*.

A function algebra A on a compact Hausdorff space X is said to be *natural*, if every character φ on A is an evaluation homomorphism at some point of X [5, 4.1.3], i.e. there exists a point $x_0 \in X$ such that $\varphi(f) = \delta_{x_0}(f) = f(x_0)$ for every $f \in A$. In the case where A is a Banach function algebra on a compact Hausdorff space X, A is natural if the character space of A is the *weak** homeomorphic image of X under the mapping $x \mapsto \delta_x$.

Let (X, d) be a compact metric space and $0 < \alpha \leq 1$. The *Lipschitz algebra of order* α, denoted by $Lip(X, d^\alpha)$, is the algebra of all complex-valued functions f on X for which

$$p_\alpha(f) = \sup\left\{\frac{|f(x) - f(y)|}{d(x, y)^\alpha} : x, y \in X \text{ and } x \neq y\right\} < \infty.$$

For each $0 < \alpha < 1$, the subalgebra of those functions $f \in Lip(X, d^\alpha)$ for which

$$\lim_{d(x,y)\to 0} \frac{|f(x) - f(y)|}{d^\alpha(x, y)} = 0,$$

is called *little Lipschitz algebra of order* α, denoted by $\ell ip(X, d^\alpha)$. These Lipschitz algebras were studied by Sherbert [20]. The algebras $Lip(X, d^\alpha)$ for $0 < \alpha \leq 1$ and $\ell ip(X, d^\alpha)$ for $0 < \alpha < 1$ are natural Banach function algebras on X if equipped with the norm

$$\|f\|_\alpha = \|f\|_X + p_\alpha(f),$$

where

$$\|f\|_X = \sup_{x\in X} |f(x)|.$$

In the rest of this paper, when using the notation of (big) Lipschitz algebras $Lip(X, d^\alpha)$ we assume that $0 < \alpha \leq 1$, and when using the notation of (little) Lipschitz algebras $\ell ip(X, d^\alpha)$ we assume that $0 < \alpha < 1$. Also, when X is a compact *plane set* we simply write $Lip(X, \alpha)$ and $\ell ip(X, \alpha)$ instead of $Lip(X, |\cdot|^\alpha)$ and $\ell ip(X, |\cdot|^\alpha)$, respectively.

For a compact metric space (X, d), φ-amenability of the Lipschitz algebras $Lip(X, d^\alpha)$ and $\ell ip(X, d^\alpha)$ have been studied by Kaniuth et al. in [14, Example 5.3], (see also [8, Theorem 2.4]). Recently, Dashti et al. in [7] characterized φ-amenability of $Lip(X, d^\alpha)$, where (X, d) is a locally compact metric space. Furthermore, the authors in [11] characterized character amenability of natural unital uniform algebras. This paper is organized as follows. In section 2, we first consider φ-contractibility of commutative semisimple Banach algebras. Then, we investigate φ-contractibility and character contractibility of natural Banach function algebras on a compact Hausdorff space. As a consequence, we characterize φ-contractibility of $Lip(X, d^\alpha)$, for a compact metric space. In section 3, we apply these results for characterization of φ-contractibility for large classes of Banach function algebras such as analytic Lipschitz algebras on a compact plane space. Moreover, we prove that for the differentiable Lipschitz algebras $Lip^n(X, \alpha)$, $\ell ip^n(X, \alpha)$, or $Lip^n_R(X, \alpha)$ on a perfect compact plane set X, the notions of φ-amenability and φ-contractibility are equivalent.

2. Main Results

In this section, we first consider φ-contractibility of commutative Banach algebras. Indeed, we characterize φ-contractibility of semisimple commutative Banach algebras in terms of topological structure of $(\Delta(A), wk^*)$. Then, we apply our results for certain natural Banach function algebras.

Theorem 2.1. *Let A be a commutative Banach algebra and $\varphi \in \Delta(A)$.*

(*i*) *If A is φ-contractible, then $\{\varphi\}$ is open in $(\Delta(A), wk^*)$.*

(*ii*) *If A is semisimple and $\{\varphi\}$ is open in $(\Delta(A), wk^*)$, then A is φ-contractible.*

Proof. (*i*) Let A be φ-contractible. By [19, Proposition 4.1], there exists $m \in A$ such that $\varphi(m) = 1$ and $am = \varphi(a)m$ for all $a \in A$. Consequently, $\hat{a}\hat{m} = \varphi(a)\hat{m}$ and therefore

$$\psi(a)\psi(m) = \varphi(a)\psi(m), \tag{1}$$

for all $\psi \in \Delta(A)$ and $a \in A$. Equation (1) implies that for each $\psi \in \Delta(A)$ and $a \in A$, if $\psi(m) \neq 0$ then $\psi(a) = \varphi(a)$. Therefore, by letting

$$K = \{\psi \in \Delta(A) \ : \ \psi(m) \neq 0\},$$

we have

$$\hat{a}|_K = \varphi(a), \quad \text{for all } a \in A. \tag{2}$$

Note that $\varphi \in K$, since $\varphi(m) = 1$. Also, if $\psi \in K$ then (2) implies that

$$\psi(a) = \hat{a}(\psi) = \varphi(a) \qquad (a \in A).$$

Consequently, $K = \{\varphi\}$ and therefore $\hat{m} = \chi_{\{\varphi\}}$. Since $\hat{m} \in C(\Delta(A), wk^*)$, we conclude that $\{\varphi\}$ is open in $(\Delta(A), wk^*)$,

(*ii*) Let A be semisimple and $\{\varphi\}$ be open in $(\Delta(A), wk^*)$. Then, by Šilov's idempotent theorem, there exists a unique element $m \in A$ such that

$$\varphi(m) = 1 \quad \text{and} \quad \psi(m) = 0, \qquad (\psi \in \Delta(A) \setminus \{\varphi\}).$$

Therefore, for each $a \in A$ and $\psi \in \Delta(A) \setminus \{\varphi\}$ we have $\psi(am) = 0 = \psi(\varphi(a)m)$. Also, $\varphi(am) = \varphi(\varphi(a)m)$ for each $a \in A$. Hence, by semisimplicity of A, we conclude that $am = \varphi(a)m$ for each $a \in A$ and this completes the proof. □

Let A be a commutative semisimple Banach algebra and $\varphi \in \Delta(A)$. By Theorem 2.1, φ-contractibility of A is equivalent to $\{\varphi\}$ being open (isolated) in $(\Delta(A), wk^*)$. On the other hand, by [14, Remark 5.1], if A is φ-amenable then $\{\varphi\}$ is open in $(\Delta(A), wk)$. This arises the natural question that if φ-amenability of A is equivalent to $\{\varphi\}$ being open (isolated) in $(\Delta(A), wk)$?

The following example gives a negative answer to the above question.

Example 2.2. *Let $S = \mathbb{N} \cup \{0\}$ and define the semigroup operation on S by*

$$m * n = \begin{cases} m & \text{if} \quad m = n \\ 0 & \text{if} \quad m \neq n \end{cases} \qquad (m, n \in \mathbb{N} \cup \{0\}).$$

Considering the semigroup algebra $A = \ell^1(S)$ with convolution product, it is easy to see that $\Delta(A) = \{\varphi_t : t \in \mathbb{N}\} \cup \{\varphi_S\}$, where for each $t \in \mathbb{N}$, $\varphi_S(\delta_m) = 1$ and

$$\varphi_t(\delta_m) = \begin{cases} 1 & \text{if} \quad m = t \\ 0 & \text{if} \quad m \neq t \end{cases} \qquad (m \in \mathbb{N} \cup \{0\}).$$

Moreover, $A^\sharp$, the unitization of A, is a semisimple commutative Banach algebra and $\Delta(A^\sharp) = \Delta(A) \cup \{\varphi_\infty\}$, where

$$\varphi_\infty(\delta_m) = 0 \quad \text{and} \quad \varphi_\infty(e_{A^\sharp}) = 1, \qquad (m \in \mathbb{N} \cup \{0\}).$$

Since $A = \ker \varphi_\infty$ has no bounded approximate identity, by [13, Priposition 2.2] it follows that the semigroup algebra $A^\sharp$ is not φ_∞-amenable. It suffices to show that φ_∞ is isolated in $(\Delta(A^\sharp), wk)$. To do this, assume towards a contradiction that (φ_α) is a net in $\Delta(A)$ such that $wk - \lim \varphi_\alpha = \varphi_\infty$. Therefore, it follows that $\varphi_\infty(e_{A^\sharp}) = \lim \phi_\alpha(e_{A^\sharp}) = 0$ which is a contradiction.

As a consequence of Theorem 2.1, we characterize φ-contractibility of a natural Banach function algebra on a compact Hausdorff space.

Theorem 2.3. *Let (X, τ) be a compact Hausdorff space and A be a natural Banach function algebra on X. Then, A is φ_x-contractible if and only if $\{x\}$ is an isolated point of (X, τ).*

Proof. Since A is a natural Banach function algebra on (X, τ), the map $x \leftrightarrow \varphi_x$ is a homeomorphism between spaces (X, τ) and $(\Delta(A), wk^*)$. Hence, the result follows immediately from Theorem 2.1. $\square$

Corollary 2.4. *Let A be a natural Banach function algebra on compact Hausdorff space X. Then, A is character contractible if and only if X is finite.*

For a compact plane set X, $R(X)$ denotes the uniform closure of $R_0(X)$ where $R_0(X)$ is the set of all rational functions on X with poles off X. Also, for a compact plane set X with nonempty interior, $A(X)$ denotes the uniform algebra of all functions in $C(X)$ which are analytic in the interior of X.

In the rest of this paper, when using the notations $R(X)$ or $A(X)$, the underlying set X is assumed to satisfy the above mentioned descriptions. Also, the notation $C(X)$ is used when X is a compact and Hausdorff space.

Applying Theorem 2.3 and Corollary 2.4 for the natural uniform algebras $C(X)$, $R(X)$ and $A(X)$, we get the following result.

Theorem 2.5. *Let A be any of the uniform algebras $C(X)$, $R(X)$ or $A(X)$. Then,*

(*i*) *A is φ_x-contractible if and only if $\{x\}$ is an isolated point of X.*

(*ii*) *A is character contractible if and only if X is finite.*

In the next theorem, we investigate φ-contractibility of certain natural Banach function algebras of Lipschitz functions. Before stating the next theorem, we recall that in general for the norms $\|\cdot\|_1$ and $\|\cdot\|_2$, the notation $\|\cdot\|_1 \lesssim \|\cdot\|_2$ means that $\|\cdot\|_1 \leq c\|\cdot\|_2$ for some positive constant c.

Theorem 2.6. *Let (X, d) be a compact metric space and $(A, \|\cdot\|_A)$ be a natural Banach function algebra contained in $Lip(X, d^\alpha)$ with $\|\cdot\|_{Lip(X,d^\alpha)} \lesssim \|\cdot\|_A$. Then, the following are equivalent:*

(*i*) *A is φ_x-amenable*

(*ii*) *A is φ_x-contractible*

(*iii*) *$\{x\}$ is an isolated point of (X, d)*

Proof. First we show that for the algebra $(A, \|\cdot\|_A)$, weak topology and weak* topology coincide on $X = \Delta(A)$ (see also [17, Theorem 2.2]). Note that since the algebra A is natural, we have $\tau_d = \tau_{wk^*}$. On the other hand, since

$$\tau_{wk^*} \subseteq \tau_{wk} \subseteq \tau_{\|\cdot\|},$$

it is enough to show that norm topology on $\Delta(A)$ is weaker than weak* topology on $X = \Delta(A)$. To see this, note that for each $x, y \in X$ and $f \in A$ we have

$$\begin{aligned} |\varphi_x(f) - \varphi_y(f)| &= |f(x) - f(y)| \\ &\leq p_\alpha(f) d^\alpha(x, y) \\ &\leq \|f\|_{Lip(X,d^\alpha)} d^\alpha(x, y) \\ &\lesssim \|f\|_A d^\alpha(x, y). \end{aligned}$$

Consequently, for each $x, y \in X$ we have

$$\|\varphi_x - \varphi_y\| \lesssim d^\alpha(x, y), \tag{3}$$

which implies the desired result. Now, let A be φ_x-amenable for some $x \in X$. Then, by [14, Remark 5.1], $\{\varphi_x\}$ is open in $(\Delta(A), wk)$ and hence in $(\Delta(A), wk^*)$. Therefore, Theorem 2.1 implies that A is φ_x-contractible and hence (*i*) and (*ii*) are equivalent. On the other hand, since the algebra A is natural (*ii*) and (*iii*) are equivalent, by Theorem 2.1. $\square$

Corollary 2.7. *Let (X, d) be a compact metric space and $(A, \|\cdot\|_A)$ be a natural Banach function algebra contained in $Lip(X, d^\alpha)$ with $\|\cdot\|_{Lip(X,d^\alpha)} \lesssim \|\cdot\|_A$. Then, the following are equivalent:*

(*i*) *A is character amenable*

(*ii*) *A is character contractible*

(*iii*) *X is finite*

Remark 2.8.

(*i*) *Clearly, one can apply Theorem 2.6 and Corollary 2.7 for the algebras $Lip(X, d^\alpha)$ and $\ell ip(X, d^\alpha)$ when (X, d) is a compact metric space. In the next section, we give more examples of well known Banach function algebras A satisfying hypothesis of Theorem 2.6 and Corollary 2.7.*

(*ii*) *In Theorem 2.6 and Corollary 2.7, when (X, d) is not a compact metric space, the Banach function algebra A fails to be natural. But, in this case the character space of the algebra A contains the evaluation homomorphisms φ_x for all $x \in X$. Therefore, (3) implies that if φ_x is an isolated point in $(\Delta(A), wk^*)$, then $\{x\}$ is an isolated point in the metric space (X, d). This, along with Theorem 2.1(i), implies that if A is φ_x-contractible, then $\{x\}$ is an isolated point in (X, d). Consequently, if A is character contractible, then (X, d) has no limit point.*

3. Some Examples

It is known that for a compact plane set X and $0 < \alpha < 1$

$$R_0(X) \subseteq Lip(X, 1) \subseteq \ell ip(X, \alpha) \subseteq Lip(X, \alpha).$$

For each compact plane set X and $0 < \alpha \leq 1$, the *rational Lipschitz algebra of order α*, is defined by

$$Lip_R(X, \alpha) = \overline{R_0(X)}^{\|\cdot\|_\alpha}.$$

Note that for each $0 < \alpha < 1$

$$Lip_R(X, \alpha) = \ell ip_R(X, \alpha) = \overline{R_0(X)}^{\|\cdot\|_\alpha}.$$

It is known that $Lip_R(X, \alpha)$ is a natural Banach function algebra, for each $0 < \alpha \leq 1$ [10].

For a compact plane set X with nonempty interior and $0 < \alpha \leq 1$ the *analytic Lipschitz algebra of order α*, is defined by

$$Lip_A(X, \alpha) = A(X) \cap Lip(X, \alpha).$$

Similarly, for each $0 < \alpha < 1$ the *little analytic Lipschitz algebra of order α*, is defined by

$$\ell ip_A(X, \alpha) = A(X) \cap \ell ip(X, \alpha).$$

It is known that analytic Lipschitz algebras $Lip_A(X, \alpha)$ and $\ell ip_A(X, \alpha)$ are natural Banach function algebras [12], and moreover

$$Lip_R(X, \alpha) \subseteq \ell ip_A(X, \alpha) \subseteq Lip_A(X, \alpha) \subseteq Lip(X, \alpha).$$

In the next theorem, for the Lipschitz algebras $Lip_R(X, \alpha)$, $Lip_A(X, \alpha)$ and $\ell ip_A(X, \alpha)$, the underlying set X is assumed to be a compact plane set which has nonempty interior in the case of $Lip_A(X, \alpha)$ and $\ell ip_A(X, \alpha)$.

Theorem 3.1. *Let A be any of the Lipschitz algebras $Lip_R(X, \alpha)$, $Lip_A(X, \alpha)$, or $\ell ip_A(X, \alpha)$. Then, the following are equivalent:*

(*i*) *A is φ_x-amenable,*

(*ii*) *A is φ_x-contractible,*

(*iii*) *$\{x\}$ is an isolated point of X.*

Proof. Since norm of the algebra A is equal to the Lipschitz norm, the proof follows immediately from Theorem 2.6. □

A complex-valued function f on a perfect plane set X is called *differentiable* on X if at each point $z_0 \in X$ the following limit exists

$$f'(z_0) = \lim_{\substack{z \to z_0 \\ z \in X}} \frac{f(z) - f(z_0)}{z - z_0}.$$

For a perfect compact plane set X and $n \in \mathbb{N}$, the algebra of *n-times continuously differentiable functions* on X is denoted by $D^n(X)$. These algebras were originally studied by Dales and Davie in [6]. The algebra $D^n(X)$ with the norm

$$\|f\|_n = \sum_{k=0}^{n} \frac{\|f^{(k)}\|_X}{k!},$$

is a normed function algebra on X.

Similarly, for a perfect compact plane set X, $0 < \alpha \leq 1$ and $n \in \mathbb{N}$, the algebra of all complex-valued functions f on X whose derivatives up to order n exist and $f^{(k)} \in Lip(X, \alpha)$ for each k $(0 \leq k \leq n)$, is denoted by $Lip^n(X, \alpha)$. Also, for each $0 < \alpha < 1$, the algebra $\ell ip^n(X, \alpha)$ is defined, which is a closed subalgebra of $Lip^n(X, \alpha)$. These *differentiable Lipschitz algebras* were first studied in [9]. The algebras $Lip^n(X, \alpha)$ and $\ell ip^n(X, \alpha)$ equipped with the norm

$$\|f\|_{n,\alpha} = \sum_{k=0}^{n} \frac{\|f^{(k)}\|_\alpha}{k!} = \sum_{k=0}^{n} \frac{\|f^{(k)}\|_X + p_\alpha(f^{(k)})}{k!},$$

are normed function algebras on X which are not necessarily complete. It is known that completeness of $D^1(X)$ implies completeness of all the algebras $D^n(X)$, $Lip^n(X, \alpha)$ and $\ell ip^n(X, \alpha)$ [16]. In order to give sufficient conditions for the completeness and naturality of these algebras, we next recall the definition of *regularity* of a compact plane set X.

Definition 3.2. *Let X be a compact plane set which is connected by rectifiable arcs. Let $\delta(z, w)$ be the geodesic metric on X, the infimum of lengths of arcs joining z and w.*

(*i*) *X is pointwise regular if for each $z \in X$ there exists a constant $c_z > 0$ such that for every $w \in X$, $\delta(z, w) \leq c_z|z - w|$.*

(*ii*) *X is uniformly regular if there exists a constant $c > 0$ such that for every $z, w \in X$, $\delta(z, w) \leq c|z - w|$.*

Note that every convex compact plane set is obviously uniformly regular. There are also non-convex uniformly regular sets like the Swiss cheese set. Dales and Davie in [6] proved that if X is a finite union of uniformly regular sets, then $D^1(X)$ is complete. The proof given in [6] is also valid when X is a finite union of pointwise regular sets [9]. It is known that if X is a uniformly regular plane set, then $D^1(X)$ is a proper (closed) subalgebra of $Lip(X, 1)$ and norms of $D^1(X)$ and $Lip(X, 1)$ are equivalent on $D^1(X)$. For any such X we therefore have

$$D^{n+1}(X) \subseteq Lip^n(X, 1) \subseteq \ell ip^n(X, \alpha) \subseteq Lip^n(X, \alpha) \subseteq D^n(X).$$

Also, norms of $D^{n+1}(X)$ and $Lip^n(X, 1)$ are equivalent on $D^{n+1}(X)$ [16].

As proved in [9], the algebras $Lip^n(X, \alpha)$ and $\ell ip^n(X, \alpha)$ are natural, when X is uniformly regular. However, by applying the same method used by Jarosz in [12], one can show that the algebras $Lip^n(X, \alpha)$ and $\ell ip^n(X, \alpha)$ are natural for every perfect compact plane set X.

In the next theorem, the underlying set X is assumed to be a perfect compact plane set such that $Lip^n(X, \alpha)$ is a Banach algebra.

Theorem 3.3. *Let A be any of the differentiable Lipschitz algebras $Lip^n(X, \alpha)$, $\ell ip^n(X, \alpha)$, or $Lip^n_R(X, \alpha)$. Then, the following statements are equivalent:*

(*i*) *A is φ_x-amenable,*

(*ii*) *A is φ_x-contractible,*

(*iii*) $\{x\}$ *is an isolated point of X,*

and therefore, since X is assumed to be perfect, none of these statements hold true.

Proof. Obviously, $\|\cdot\|_{Lip(X,\alpha)} \leq \|\cdot\|_A$. Therefore, the proof follows immediately from Theorem 2.6. □

Remark 3.4. *Let A be any of the algebras in Theorem 3.1 or Theorem 3.3. Then, by Corollary 2.7, character amenability of A, character contractibility of A, and finiteness of X are equivalent.*

Remark 3.5. *Let X be a uniformly regular perfect compact plane set. Similar to the case of $Lip^n_R(X,\alpha)$, the natural subalgebra $D^n_R(X)$ of $D^n(X)$ is defined (see, [1] and [6]). The algebras $D^n_R(X)$ and $D^n(X)$ satisfy conditions of Theorem 2.6 and Corollary 2.7. Therefore, the results of Theorem 3.3 and Remark 3.4 are also valid for these algebras.*

References

[1] M. Abtahi and T. G. Honary, Properties of certain subalgebras of Dales-Davie algebras, Glasg. Math. J. 49 (2007) 225–233.
[2] M. Alaghmandan, R. Nasr-Isfahani and M. Nemati, Character amenability and contractibility of abstract Segal algebras, Bull. Aust. Math. Soc. 82 (2010) 274–281.
[3] A. Bodaghi, H. Ebrahimi, M. Lashkarizadeh Bami, Generalized notions of module character amenability, Filomat, to appear.
[4] A. Browder, Introduction to Function Algebras, W. A. Benjamin, Inc., New York, 1969.
[5] H. G. Dales, Banach Algebras and Automatic Continuity, London Mathematical Society Monographs, New Series, 24, The Clarendon Press, Oxford, 2000.
[6] H. G. Dales and A. M. Davie, Quasianalytic Banach function algebras, J. Funct. Anal. 13 (1973) 28–50.
[7] M. Dashti, R. Nasr-Isfahani and S. Soltani Renani, Character Amenability of Lipschitz Algebras, Canad. Math. Bull. 57 (2014) 37–41.
[8] M. Eshaghi Gordji, A. Jabbari, and G. Hui Kim, Sme characterizations of character amenable Banach algebras, Bull. Korean Math. Soc. 52 (2015) 761–769.
[9] T. G. Honary and H. Mahyar, Approximation in Lipschitz algebras of infinitely differentiable functions, Bull. Korean Math. Soc. 36 (1999) 629–636.
[10] T. G. Honary and H. Mahyar, Approximation in Lipschitz algebras, Quaest. Math. 23 (2000) 13–19.
[11] Z. Hu, M. S. Monfared and T. Traynor, On character amenable Banach algebras, Studia Math. 193 (2009) 53–78.
[12] K. Jarosz, $\mathrm{Lip}_{Hol}(X,\alpha)$, Proc. Amer. Math. Soc. 125 (1997) 3129–3130.
[13] E. Kaniuth, A. T. Lau and J. S. Pym, On φ-amenability of Banach algebras, Math. Proc. Cambridge Philos. Soc. 144 (2008) 85–96.
[14] E. Kaniuth, A. T. Lau and J. Pym, On character amenability of Banach algebras, J. Math. Anal. Appl. 344 (2008) 942–955.
[15] G. M. Leibowitz, Lectures on Complex Function Algebras, Scott, Foresman and Co., Glenview, Illinois, 1970.
[16] H. Mahyar, Approximation in Lipschitz algebras and their maximal ideal spaces, PhD Thesis, Tarbiat Moallem University, Tehran, Iran, 1994.
[17] H. Mahyar and A. H. Sanatpour, Quasicompact endomorphisms of Lipschitz algebras of analytic functions, Publ. Math. Debrecen 76 (2010) 135–145.
[18] M. S. Monfared, Character amenability of Banach algebras, Math. Proc. Cambridge Philos. Soc. 144 (2008) 697–706.
[19] H. Pourmahmood Aghababa, L. Yi Shi and Y. Jing Wu, Generalized notions of character amenability, Acta. Math. Sinica 29 (2013) 1329–1350.
[20] D. R. Sherbert, Banach algebras of Lipschitz functions, Pacific J. Math. 13 (1963) 1387–1399.