Modified Laguerre Matrix Polynomials

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Abstract. In this paper, modified Laguerre matrix polynomials which appear as finite series solutions of second-order matrix differential equation are introduced. Some formulas related to an explicit expression, a three-term matrix recurrence relation and a Rodrigues formula are obtained. Several families of bilinear and bilateral generating matrix functions for modified Laguerre matrix polynomials are derived. Finally a new generalization of the Laguerre-type matrix polynomials is introduced.

1. Introduction

Matrix generalization of special functions has become important in the last two decades. The reason of importance have many motivations. For instance, using special matrix functions provides solutions for some physical problems. Also, special matrix functions are in connection with different matrix functions. Jódar et al introduced Laguerre matrix polynomials in [9]. Some important properties of Laguerre matrix polynomials such as asymptotic expressions [10, 11, 13–15], relations between different matrix functions and generating matrix functions [1–4, 7, 15, 17, 18] are studied. In this paper, we deal with a new generalization of Laguerre matrix polynomials which we call modified Laguerre matrix polynomials. The organization of this paper is as follows. In section 2 starting from an appropriate matrix generating function, an explicit expression, a three-term recurrence relation, Rodrigues formula and second-order matrix differential equation for modified Laguerre matrix polynomials are given. Section 3 deals with bilinear and bilateral matrix generating function for modified Laguerre matrix polynomials. Finally, in Section 4, examining the explicit expression of both Laguerre matrix polynomials and modified Laguerre matrix polynomials, we give a generalization of Laguerre-type matrix polynomials.

Throughout this paper, for a matrix A in $\mathbb{C}^{r \times r}$, its spectrum $\sigma(A)$ denotes the set of all eigenvalues of A. If $f(z)$ and $g(z)$ are holomorphic functions of the complex variable z, which are defined in an open set Ω of the complex plane and A is a matrix in $\mathbb{C}^{r \times r}$ such that $\sigma(A) \subset \Omega$, then from the properties of matrix functional calculus in [6, p. 558], it follows that: $f(A)g(A) = g(A)f(A)$. Hence, if $B \in \mathbb{C}^{r \times r}$ is a matrix for which $\sigma(B) \subset \Omega$ and $AB = BA$, then $f(A)g(B) = g(B)f(A)$.

The matrix analogues of Pochhammer symbol or shifted factorial is defined by

$$(A)_n = A(A + I)(A + 2I)\ldots(A + (n-1)I), \quad n \geq 1, (A)_0 = I, \tag{1}$$

where $A \in \mathbb{C}^{r \times r}$. The hypergeometric matrix function $F(A; B; C; z)$ is defined by

$$F(A, B; C; z) = \sum_{n \geq 0} \frac{(A)_n (B)_n [(C)_n]^{-1}}{n!} z^n, \tag{2}$$

for matrices A, B, C in $\mathbb{C}^{r \times r}$ such that $C + nI$ is invertible for all integers $n \geq 0$ and for $|z| < 1$ (see [8]). According to (1) if $A = -iI$ where i is a natural number then $(A)_{i+j} = 0$ for $j \geq 1$. Thus $F(A, B; C; z)$ becomes a matrix polynomial of degree i. Furthermore, for a matrix A in $\mathbb{C}^{r \times r}$ the authors exploited the following relation due to [8]:

$$(1 - y)^{-A} = \sum_{n=0}^{\infty} \frac{(A)_n}{n!} y^n, \quad |y| < 1. \tag{3}$$

Also, for a matrix $A(k, n)$ in $\mathbb{C}^{r \times r}$ for $n \geq 0$ and $k \geq 0$, the following relation is given by Defez and Jódar in [5]

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} A(k, n) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} A(k, n - k). \tag{4}$$

We conclude this section by recalling the Laguerre matrix polynomials. Let A be a matrix in $\mathbb{C}^{r \times r}$ such that $-k \notin \sigma(A)$ for every integer $k > 0$ and λ be a complex number whose real part is positive. Then the Laguerre matrix polynomials $L_n^{(A,\lambda)}(x)$ are defined by [9]:

$$L_n^{(A,\lambda)}(x) = \sum_{k=0}^{n} \frac{(-1)^k (A + I)_n [(A + I)_k]^{-1} \lambda^k x^k}{k! (n - k)!}. \tag{5}$$

The generating function of Laguerre matrix polynomials is given in [9]

$$(1 - t)^{-(A+I)} \exp\left(\frac{-\lambda x t}{1 - t}\right) = \sum_{n=0}^{\infty} L_n^{(A,\lambda)}(x) t^n, \quad t \in \mathbb{C}, \quad |t| < 1, \quad x \in \mathbb{C},$$

and Rodrigues formula is

$$L_n^{(A,\lambda)}(x) = \frac{x^{-A} \exp(\lambda x)}{n!} D^n \left[x^{A+nI} \exp(-\lambda x)\right], \quad n \geq 0. \tag{6}$$

Also, Laguerre matrix polynomials satisfy the three-term recurrence relation

$$(n + 1) L_{n+1}^{(A,\lambda)}(x) + [\lambda x I - (A + (2n + 1) I)] L_n^{(A,\lambda)}(x) + (A + nI) L_{n-1}^{(A,\lambda)}(x) = 0, \tag{7}$$

and second order matrix differential equation

$$\left[x D^2 + ((A + I) - \lambda x I) D + \lambda n I\right] L_n^{(A,\lambda)}(x) = 0. \tag{8}$$

2. Modified Laguerre Matrix Polynomials: Definition and Properties

Let us assume that B is an arbitrary matrix in $\mathbb{C}^{r \times r}$, λ is a complex number whose real part is positive and let us consider the matrix-valued function

$$G(x, t) = (1 - t)^{-B} \exp(\lambda x t), \tag{9}$$

defined for the complex values of x and t with $|t| < 1$. $G(x,t)$, regarded as a function of the complex variable t, is holomorphic in $|t| < 1$, and therefore has the Taylor expansion about $t = 0$, of the form

$$G(x,t) = \sum_{n=0n}^{\infty} f_n^{(B,\lambda)}(x) t^n. \tag{10}$$

From (3) and (4), we acquire

$$\begin{aligned} G(x,t) &= \sum_{n=0}^{\infty} \frac{(B)_n}{n!} t^n \sum_{n=0}^{\infty} \frac{\lambda^n t^n}{n!} \\ &= \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{(B)_{n-k} \lambda^k x^k}{k!(n-k)!} t^n. \end{aligned} \tag{11}$$

From (10) and (11) the matrix coefficients $f_n^{(B,\lambda)}(x)$ take the form:

$$f_n^{(B,\lambda)}(x) = \sum_{k=0}^{n} \frac{(B)_{n-k} \lambda^k x^k}{k!(n-k)!}. \tag{12}$$

It is clear from (12) that $f_n^{(B,\lambda)}(x)$ is a matrix polynomial of degree n with leading coefficients $\frac{I}{n!}$ and the first few modified Laguerre matrix polynomials are listed below:

$$f_0^{(B,\lambda)}(x) = I, \quad f_1^{(B,\lambda)}(x) = B + \lambda x I, \quad f_2^{(B,\lambda)}(x) = (\lambda x)^2 I + B\lambda x + \frac{B(B+I)}{2}.$$

If we take $B = -(A + nI)$ such that $-k \notin \sigma(A)$ for every integer $k > 0$ we get

$$(-A - nI)_{n-k} = (-1)^{n-k}(A+I)_n [(A+I)_k]^{-1}. \tag{13}$$

Using (13), we get $f_n^{(-A-nI,\lambda)}(x) = (-1)^n L_n^{(A,\lambda)}(x)$. Besides it is easy to show that modified Laguerre scalar polynomials $f_n^{\beta}(x)$ are a particular case of $f_n^{(B,\lambda)}(x)$:

$$f_n^{\beta}(x) = f_n^{(B,1)}(x), \quad B = [\beta]_{1\times1}$$

(cf. [12, 16]). Therefore we call modified Laguerre matrix polynomials for $f_n^{(B,\lambda)}(x)$.

From (9), it is obvious that $G(x,t)$ is an analytic $\mathbb{C}^{r\times r}$−valued function of the variable t in $|t| < 1$. Thereby differentiating $G(x,t)$ with respect to t we get

$$\frac{\partial G(x,t)}{\partial t} = (1-t)^{-1} [B + \lambda x(1-t)I] (1-t)^{-B} \exp(\lambda x t), \tag{14}$$

$$(1-t)\frac{\partial G(x,t)}{\partial t} - [B + \lambda x(1-t)I] G(x,t) = 0.$$

Hence,

$$\sum_{n=1}^{\infty} n f_n^{(B,\lambda)}(x) t^{n-1} - \sum_{n=1}^{\infty} n f_n^{(B,\lambda)}(x) t^n - \sum_{n=0}^{\infty} B f_n^{(B,\lambda)}(x) t^n$$

$$- \sum_{n=0}^{\infty} \lambda x f_n^{(B,\lambda)}(x) t^n + \sum_{n=0}^{\infty} \lambda x f_n^{(B,\lambda)}(x) t^{n+1} = 0.$$

Making appropriate changes of indices and comparing the coefficients of each power t^n, we obtain a three-term matrix recurrence relation as:

$$(n+1) f_{n+1}^{(B,\lambda)}(x) - [\lambda x I + (B + nI)] f_n^{(B,\lambda)}(x) + \lambda x f_{n-1}^{(B,\lambda)}(x) = 0, \quad n \geq 1, \tag{15}$$

which will be useful to calculate the *nth* polynomial in terms of the polynomials of order $n-1$ and $n-2$.

Now, we get the matrix differential equation of modified Laguerre matrix polynomials. It is clear from (9) that $G(x,t)$ is an entire $\mathbb{C}^{r \times r}$−valued function of the variable x. Using the operator D to denote $D = \frac{d}{dx}$ we have

$$DG(x,t) - \lambda t G(x,t) = 0. \tag{16}$$

From Lemma 14 of [6, p. 571], the derivative of $DG(x,t)$ can be computed in the series expansion (10). From (16), we acquire

$$\sum_{n=0}^{\infty} Df_n^{(B,\lambda)}(x) t^n - \sum_{n=0}^{\infty} \lambda f_n^{(B,\lambda)}(x) t^{n+1} = 0.$$

Equating the coefficients of t^n to zero matrix we have

$$Df_n^{(B,\lambda)}(x) = \lambda f_{n-1}^{(B,\lambda)}(x). \tag{17}$$

Differentiating with respect to x in the three-term recurrences relation (15) and using (17) we get

$$(n+1)Df_{n+1}^{(B,\lambda)}(x) - [\lambda x I + (B+nI)]Df_n^{(B,\lambda)}(x) \tag{18}$$
$$+\lambda x Df_{n-1}^{(B,\lambda)}(x) + Df_n^{(B,\lambda)}(x) - \lambda f_n^{(B,\lambda)}(x) = 0.$$

Replacing n by $n-1$ in (17) and adding the result to (18) give

$$(n+1)\lambda f_n^{(B,\lambda)}(x) - [\lambda x I + (B+nI)]Df_n^{(B,\lambda)}(x) \tag{19}$$
$$+\lambda x Df_{n-1}^{(B,\lambda)}(x) + Df_n^{(B,\lambda)}(x) - \lambda f_n^{(B,\lambda)}(x) = 0.$$

Also it is obvious from (17) that $\lambda Df_{n-1}^{(B,\lambda)}(x) = D^2 f_n^{(B,\lambda)}(x)$. Thus we obtain the second order matrix differential equation for modified Laguerre matrix polynomials as

$$\left[xD^2 - (\lambda x I + (B+(n-1)I))D + \lambda n I\right] f_n^{(B,\lambda)}(x) = 0. \tag{20}$$

Now, we prove Rodrigues formula for the modified Laguerre matrix polynomials. Let B be a matrix in $\mathbb{C}^{r \times r}$. It is apparent that

$$D^{n-k}x^{-B} = (-1)^{n-k}(B)_{n-k}x^{-B-(n-k)I}.$$

Since $D^k \exp(-\lambda x) = (-1)^k \lambda^k$, from Leibnitz formula and the properties of the matrix functional calculus we have

$$D^n \left[x^{-B}\exp(-\lambda x)\right] = (-1)^n n! x^{-B-nI}\exp(-\lambda x)\sum_{k=0}^{n} \frac{(B)_{n-k}\lambda^k x^k}{k!(n-k)!}. \tag{21}$$

From (12) and (21) we get

$$f_n^{(B,\lambda)}(x) = \frac{(-1)^n}{n!}x^{B+nI}\exp(\lambda x)D^n\left[x^{-B}\exp(-\lambda x)\right] \tag{22}$$

Summary of these results is given in the following theorem.

Theorem 2.1. *Let B be an arbitrary matrix in $\mathbb{C}^{r \times r}$, and let λ be complex number such that $Re(\lambda) > 0$. Then the modified Laguerre matrix polynomials satisfy the following properties:*

1. For $n \geq 1$

$$(n+1)f_{n+1}^{(B,\lambda)}(x) - [\lambda x I + (B+nI)]f_n^{(B,\lambda)}(x) + \lambda x f_{n-1}^{(B,\lambda)}(x) = 0.$$

2. The *n th* modified Laguerre matrix polynomial $f_n^{(B,\lambda)}(x)$ is a solution of second order matrix differential equation (20).

3. The *n th* modified Laguerre matrix polynomial $f_n^{(B,\lambda)}(x)$ is given by the Rodrigues formula (22).

3. More Generating Matrix Functions for Modified Laguerre Matrix Polynomials

Generating matrix functions are fairly important in the theory of matrix polynomials because several important properties of the family of matrix polynomials can be obtained from them. Therefore, in this section we give more generating matrix functions for modified Laguerre matrix polynomials.

Theorem 3.1. *Let B and C be commutative matrices in $\mathbb{C}^{r \times r}$ and λ be complex number such that $Re(\lambda) > 0$. Then modified Laguerre matrix polynomials have the following generating matrix function:*

$$\sum_{n=0}^{\infty} (C)_n f_n^{(B,\lambda)}(x) t^n = (1 - \lambda xt)^{-C} F\left(C, B; -; \frac{t}{1 - \lambda xt}\right). \tag{23}$$

Proof. Using the explicit representation of modified Laguerre matrix polynomials given in (12), we get

$$\sum_{n=0}^{\infty} (C)_n f_n^{(B,\lambda)}(x) t^n = \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{(C)_n (B)_{n-k} \lambda^k x^k}{k! (n-k)!} t^n$$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{(C)_{n+k} (B)_n \lambda^k x^k}{k! n!} t^{n+k}.$$

Using the fact that $(C)_{n+k} = (C)_n (C + nI)_k$ the factors on the right-hand side can be rearranged as

$$\sum_{n=0}^{\infty} (C)_n f_n^{(B,\lambda)}(x) t^n = \sum_{n=0}^{\infty} \frac{(C)_n (B)_n}{n!} t^n \sum_{k=0}^{\infty} \frac{(C + nI)_k (\lambda xt)^k}{k!}.$$

By using (3) and (2), we get (23). □

Theorem 3.2. *Let B be an arbitrary matrix in $\mathbb{C}^{r \times r}$, k be nonnegative integer and λ be complex number such that $Re(\lambda) > 0$. Then we have the following generating matrix functions:*

$$\sum_{n=0}^{\infty} \frac{(n+k)!}{n! k!} f_{n+k}^{(B,\lambda)}(x) t^n = \exp(\lambda xt) (1 - t)^{-(B+kI)} f_k^{(B,\lambda)}(x(1-t)). \tag{24}$$

Proof. Replacing t by $t + v$ in (9), we have

$$(1 - (t+v))^{-B} \exp(\lambda x(t+v)) = \sum_{n=0}^{\infty} f_n^{(B,\lambda)}(x)(t+v)^n. \tag{25}$$

Firstly, expanding the binomial $(t+v)^n$ and then simplifying it we have

$$\sum_{n=0}^{\infty} f_n^{(B,\lambda)}(x)(t+v)^n = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{(n+k)! f_{n+k}^{(B,\lambda)}(x) t^n}{n! k!} v^k.$$

Now we try to expand left-hand side of (25) in a different way. It is easy to write that

$$(1 - t - v)^{-B} \exp(\lambda x(t+v))$$

$$= \exp(\lambda xt)(1-t)^{-B}\left[\exp(\lambda xv)\left(1 - \frac{v}{1-t}\right)\right]$$

$$= \exp(\lambda xt)(1-t)^{-B} \sum_{k=0}^{\infty} f_k^{(B,\lambda)}(x(1-t))\left(\frac{v}{1-t}\right)^k.$$

Comparing the coefficients of v^k in the two expansions, we get (24). □

Theorem 3.3. *Let B be an arbitrary matrix in $\mathbb{C}^{r \times r}$ and λ be a complex number such that $Re(\lambda) > 0$. Then we obtain the bilinear generating matrix function for modified Laguerre matrix polynomials as follows:*

$$\sum_{n=0}^{\infty} n! f_n^{(B,\lambda)}(x) f_n^{(B,\lambda)}(y) t^n \tag{26}$$

$$= e^{\lambda^2 xyt} (1 - \lambda xt)^{-B} (1 - \lambda yt)^{-B} F\left(B, B; -; \frac{t}{(1 - \lambda xt)(1 - \lambda yt)}\right).$$

Proof. Using (4) and (12), we have

$$\sum_{n=0}^{\infty} n! f_n^{(B,\lambda)}(x) f_n^{(B,\lambda)}(y) t^n = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{(n+k)! f_{n+k}^{(B,\lambda)}(x)(\lambda yt)^n}{n!k!} (B)_k t^k.$$

Substituting (24) in the above series we get

$$\sum_{n=0}^{\infty} n! f_n^{(B,\lambda)}(x) f_n^{(B,\lambda)}(y) t^n$$

$$= \sum_{k=0}^{\infty} e^{\lambda^2 xyt} (1 - \lambda yt)^{-(B+kI)} f_k^{(B,\lambda)}(x(1 - \lambda yt))(B)_k t^k$$

$$= e^{\lambda^2 xyt} (1 - \lambda yt)^{-B} \sum_{k=0}^{\infty} (B)_k f_k^{(B,\lambda)}(x(1 - \lambda yt)) \left(\frac{t}{1 - \lambda yt}\right)^k.$$

From (23) with $C = B$, we get (26). $\square$

Theorem 3.4. *Let B and C be commutative matrices in $\mathbb{C}^{r \times r}$ and λ be a complex number such that $Re(\lambda) > 0$. Then we obtain the bilateral generating matrix function for modified Laguerre matrix polynomials as follows:*

$$\sum_{n=0}^{\infty} F(C, -nI; -; y) f_n^{(B,\lambda)}(x) t^n \tag{27}$$

$$= e^{\lambda xt} (1 - t)^{-B} (1 + \lambda xyt)^{-C} F\left(C, B; -; \frac{-yt}{(1 - t)(1 + \lambda xyt)}\right).$$

Proof. If we replace x with $x(1 - t)$ and t with $\frac{yt}{1-t}$ in (23), we get

$$\sum_{k=0}^{\infty} (C)_k f_k^{(B,\lambda)}(x(1 - t)) \left(\frac{yt}{1 - t}\right)^k \tag{28}$$

$$= (1 - \lambda xyt)^{-C} F\left(C, B; -; \frac{yt}{(1 - t)(1 - \lambda xyt)}\right)$$

If we multiply the both sides of (28) by $e^{\lambda xt} (1 - t)^{-B}$ we have

$$e^{\lambda xt} (1 - t)^{-B} (1 - \lambda xyt)^{-C} F\left(C, B; -; \frac{yt}{(1 - t)(1 - \lambda xyt)}\right) \tag{29}$$

$$= \sum_{k=0}^{\infty} (C)_k y^k t^k \left[e^{\lambda xt} (1 - t)^{-B-kI} f_k^{(B,\lambda)}(x(1 - t))\right].$$

By means of (24), equation (29) becomes

$$e^{\lambda xt} (1-t)^{-B} (1-\lambda xyt)^{-C} F\left(C, B; -; \frac{yt}{(1-t)(1-\lambda xyt)}\right)$$

$$= \sum_{k=0}^{\infty} (C)_k y^k t^k \sum_{n=0}^{\infty} \frac{(n+k)!}{n!k!} f_{n+k}^{(B,\lambda)} (x) t^n.$$

After simplifying it and replacing y with $-y$, we have (27). $\square$

Although the generating relation (27) is in a bilateral form, we may change it into a bilinear form by the following corollary.

Corollary 3.5. *Let B and C be commutative matrices in $\mathbb{C}^{r \times r}$ and λ be a complex number such that $Re\,(\lambda) > 0$. Then modified Laguerre matrix polynomials satisfy the following bilinear generating relation:*

$$\sum_{n=0}^{\infty} n! f_n^{(B,\lambda)} (x) f_n^{(C,\lambda)} (y) t^n \tag{30}$$

$$= e^{\lambda xt} (1-yt)^{-B} (1-\lambda xt)^{-C} F\left(C, B; -; \frac{t}{(1-yt)(1-\lambda xt)}\right).$$

Proof. One can easily get

$$f_n^{(C,\lambda)} (y) = \frac{(\lambda y)^n}{n!} F\left(C, -nI; -; \frac{-1}{\lambda y}\right). \tag{31}$$

Replacing y with $\frac{-1}{y}$ and t with yt in (27) and using (31), we get (30). $\square$

Remark 3.6. *The generating relation (26) is the special case of (30) with $C = B$.*

4. Generalized Laguerre Matrix Polynomials

As mentioned before Laguerre matrix polynomials and modified Laguerre matrix polynomials have the explicit expressions respectively as

$$L_n^{(A,\lambda)} (x) = \sum_{k=0}^{n} (-1)^k (A+I)_n [(A+I)_k]^{-1} \frac{\lambda^k x^k}{k!(n-k)!},$$

and

$$f_n^{(A,\lambda)} (x) = (-1)^n L_n^{(-A-nI,\lambda)} = \sum_{k=0}^{n} (A)_{n-k} \frac{\lambda^k x^k}{k!(n-k)!},$$

Examining these two polynomials we are able to construct a generalization $p_n^{(A,\lambda)} (m; x)$ in the form:

$$p_n^{(A,\lambda)} (m; x) = \sum_{k=0}^{n} (-1)^{mk} (A + (mk+1) I)_{n-k} \frac{\lambda^k x^k}{k!(n-k)!},$$

where m is a nonnegative integer. Then

$$p_n^{(A,\lambda)} (1; x) = L_n^{(A,\lambda)} (x),$$

where $-k \notin \sigma(A)$ for every integer $k > 0$ and

$$p_n^{(A-I,\lambda)}(0;x) = (-1)^n L_n^{(-A-nI,\lambda)} = f_n^{(A,\lambda)}(x).$$

Using the direct summation techniques we obtain a generating function for $p_n^{(A,\lambda)}(m;x)$ as

$$\sum_{n=0}^{\infty} p_n^{(A,\lambda)}(m;x)\, t^n = (1-t)^{-(A+I)} \exp\left(\frac{\lambda x t}{(1-t)^m}\right).$$

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