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Some Subordination and Superordination Results Associated with Generalized Srivastava-Attiya Operator

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Abstract. By using the generalized Srivastava-Attiya operator we give some results of differential subordination and superordination of analytic functions. Some applications and examples are also obtained.

1. Introduction

Let $A(p)$ denote the class of functions $f(z)$ of the form

$$f(z) = z^p + \sum_{k=1}^{\infty} a_{k+p}\, z^{k+p} \ , \tag{1.1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z \in \mathbb{C} : \ |z| < 1\}$. Also, let $A = A(1)$.

Moreover, we denote by $\mathcal{H}[a, n]$, the class of analytic functions in $\mathbb{U}$ in the form

$$f(z) = a + \sum_{k=n}^{\infty} a_k z^k \quad (a \in \mathbb{C};\ n \in \mathbb{N} = \{1, 2, \cdots\}).$$

Furthermore, Let $\mathbb{Q}$ be the set of analytic functions $q(z)$ and univalent on $\bar{\mathbb{U}} \backslash E(q)$, where

$$E(q) = \left\{ \zeta \in \partial\mathbb{U} : \lim_{z \to \zeta} q(z) = \infty \right\},$$

is such that $\min q^{'}(\zeta) \neq 0$ for $\zeta \in \partial\mathbb{U} \backslash E(q)$.

The general Hurwitz-Lerch Zeta function $\Phi(z, s, b)$ defined by (cf., eg., [24, P. 121 et seq.])

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$$\Phi(z,s,b) = \sum_{k=0}^{\infty} \frac{z^k}{(k+b)^s}, \tag{1.2}$$

$(b \in \mathbb{C} \setminus \mathbb{Z}_0^-,\ \mathbb{Z}_0^- = \mathbb{Z}^- \cup \{0\} = \{0,-1,-2,\dots\},\ s \in \mathbb{C}$ *when* $z \in \mathbb{U}$, $\mathrm{Re}(s) > 1$ *when* $|z| = 1)$

Several properties of $\Phi(z,s,b)$ can be found in many papers, for example Attiya and Hakami [2], Attiya *et el.[3],* Choi et al. [6], Ferreira and López [11], Gupta *et al.* [12] and Luo and Srivastava [18]. See, also Kutbi and Attiya ([14], [15]), Srivastava and Attiya [23], Srivastava and Gaboury [25], Srivastava *et al. [26], Srivastava et al. [27]*, and Owa and Attiya [21].

We define the function $G_{s,b,t}$ by

$$G_{s,b,t} = 1 + (t+b)^s z\Phi(z,s,1+t+b) \tag{1.3}$$

$$\left(z \in \mathbb{U};\ b \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ \ s \in \mathbb{C};\ t \in \mathbb{R}\right),$$

Attiya and Alhakami [2], defined the operator $\mathcal{J}_{s,b}^t(f)$ by

$$\mathcal{J}_{s,b}^t(f):\ A(p) \longrightarrow A(p), \tag{1.4}$$

and

$$\mathcal{J}_{s,b}^t(f)(z) = z^p G_{s,b,t} * f(z) \tag{1.5}$$

$$\left(z \in \mathbb{U};\ f \in A(p); b \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ \ s \in \mathbb{C};\ t \in \mathbb{R}\right),$$

where $*$ denotes the convolution or Hadamard product.

Attiya and Alhakami [2] showed that

$$\mathcal{J}_{s,b}^t(f)(z) = z^p + \sum_{k=1}^{\infty} \left(\frac{t+b}{k+t+b}\right)^s a_{k+p}\ z^{k+p} \tag{1.6}$$

$$\left(z \in \mathbb{U};\ f \in A(p); b \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ \ s \in \mathbb{C};\ t \in \mathbb{R}\right)$$

The operator $\mathcal{J}_{s,b}^t(f)$ generalizes many well known operators in *Geometric Function Theory* eg. Alexander operator $A(f)$ [1] , Libera operator $L(f)$ [16], Bernardi operator $L_n(f)$ [4], Jung-Kim-Srivastava integral operator $I^\sigma(f)$ [13], Salagean operator $D^n(f)$ [22], the operator $I_\lambda^n(f)$ was studied in ([9], [7]), the operator $I_n(f)$ was studied in [28], the operator $J_{s,b}^p(f)$ was studied in [17] and others.

Definition 1.1. *Let $f(z)$ and $F(z)$ be analytic functions. The function $f(z)$ is said to be subordinate to $F(z)$, written $f(z) \prec F(z)$, if there exists a function $w(z)$ analytic in $\mathbb{U}$, with $w(0) = 0$ and $|w(z)| \le 1$, and such that $f(z) = F(w(z))$. If $F(z)$ is univalent, then $f(z) \prec F(z)$ if and only if $f(0) = F(0)$ and $f(\mathbb{U}) \subset F(\mathbb{U})$.*

In our investigations we need the following results:

Theorem 1.1. *[5] Let $q(z)$ be an univalent function in $\mathbb{U}$ and $\gamma \in \mathbb{C}^*$ such that*

$$\mathrm{Re}\left(\frac{zq''(z)}{q'(z)} + 1\right) \ge \max\left\{0, -\frac{1}{\gamma}\right\}.$$

If $p(z)$ is an analytic function in $\mathbb{U}$ with $p(0) = q(0)$ and

$$p(z) + \gamma z p'(z) \prec q(z) + \gamma z q'(z), \tag{1.7}$$

then $p(z) \prec q(z)$ and $q(z)$ is the best dominant of (1.7).

Corollary 1.1. *[5] Let $q(z)$ be a convex function in $\mathbb{U}$ with $q(0) = a$ and $\gamma \in \mathbb{C}^*$ such that* $\mathrm{Re}(\gamma) > 0$.*If $p(z) \in \mathcal{H}[a,1] \cap \mathbb{Q}$ and $p(z) + \gamma zp'(z)$ is univalent function in $\mathbb{U}$,and*

$$q(z) + \gamma zq'(z) \prec p(z) + \gamma zp'(z), \tag{1.8}$$

then $q(z) \prec p(z)$ and $q(z)$ is the best subordinant of (1.8).

Lemma 1.1. *[2] Let $f(z)$ be in the class $A(p)$, then*

$$z\left(\mathcal{J}^t_{s+1,b}\, f(z)\right)' = (t+b)\mathcal{J}^t_{s,b} f(z) - (t+b-p)\mathcal{J}^t_{s+1,b} f(z), \tag{1.9}$$

$$\left(z \in \mathbb{U};\ b \in \mathbb{C}\backslash\mathbb{Z}_0^-;\ s \in \mathbb{C};\ t \in \mathbb{R}\right)$$

In this paper, we give some results of differential subordination and superordination of analytic functions associated with the operator $\mathcal{J}^t_{s,b}(f)$.Also, we give some applications and examples of our results.

2. Main Results

Theorem 2.1. *Let $q(z)$ be an univalent function in $\mathbb{U}$, with $q(0) = 1$ and*

$$\mathrm{Re}\left(\frac{zq''(z)}{q'(z)} + 1\right) > \max\left\{0, -\mathrm{Re}\left(\frac{1}{\gamma}\right)\right\} \quad (\gamma \in \mathbb{C}^*), \tag{2.1}$$

If $f(z) \in A(p)$ and

$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)} + \gamma(t+b)\left(1 - \frac{\mathcal{J}^t_{s-1,b}(f)(z)\mathcal{J}^t_{s+1,b}(f)(z)}{\left(\mathcal{J}^t_{s,b}(f)(z)\right)^2}\right) \prec q(z) + \gamma zq'(z), \tag{2.2}$$

then

$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)} \prec q(z) \tag{2.3}$$

and $q(z)$ is the best dominant of (2.3).

Proof. If we define the function

$$p(z) = \frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}$$

by differentiating logarithmically with respect to z, and using (**??**), we have

$$\frac{zp'(z)}{p(z)} = (t+b)\left(\frac{\mathcal{J}^t_{s,b}(f)(z)}{\mathcal{J}^t_{s+1,b}(f)(z)} - \frac{\mathcal{J}^t_{s-1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}\right),$$

which gives

$$\frac{zp'(z)}{p(z)} = (t+b)\left(\frac{1}{p(z)} - \frac{\mathcal{J}^t_{s-1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}\right),$$

therefore,

$$p(z)+\gamma z p'(z)=$$
$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}+\gamma(t+b)\left(1-\frac{\mathcal{J}^t_{s-1,b}(f)(z)\mathcal{J}^t_{s+1,b}(f)(z)}{\left(\mathcal{J}^t_{s,b}(f)(z)\right)^2}\right),$$

applying Theorem 1.1, we deduce the result of the theorem. □

The following example is an application of Theorem 1.1, when we put $q(z)=\frac{1+(1-2\alpha)z}{1-z}$, $\alpha\in[0,1)$.

Example 2.1. *Let $\alpha\in[0,1)$ and $\gamma\in\mathbb{C}^*$ with $\mathrm{Re}(\gamma)\geq 0$, for $f(z)\in A(p)$ satisfies*

$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}+\gamma(t+b)\left(1-\frac{\mathcal{J}^t_{s-1,b}(f)(z)\mathcal{J}^t_{s+1,b}(f)(z)}{\left(\mathcal{J}^t_{s,b}(f)(z)\right)^2}\right) \prec \frac{1+(1-2\alpha)z}{1-z}+\frac{2(1-\alpha)\gamma z}{(1-z)^2}, \tag{2.4}$$

then

$$\mathrm{Re}\left(\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}\right)>\alpha$$

and α is the best possible.

Theorem 2.2. *Let $q(z)$ be a convex function in $\mathbb{U}$ with $q(0)=1$ and $\gamma\in\mathbb{C}^*$ such that $\mathrm{Re}(\gamma)>0$.If $f(z)\in A(p)$ satisfies*

$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}\in\mathbb{Q},$$

also, let

$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}+\gamma(t+b)\left(1-\frac{\mathcal{J}^t_{s-1,b}(f)(z)\mathcal{J}^t_{s+1,b}(f)(z)}{\left(\mathcal{J}^t_{s,b}(f)(z)\right)^2}\right) \tag{2.5}$$

is univalent in $\mathbb{U}$ and

$$q(z)+\gamma z q'(z)\prec\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}+\gamma(t+b)\left(1-\frac{\mathcal{J}^t_{s-1,b}(f)(z)\mathcal{J}^t_{s+1,b}(f)(z)}{\left(\mathcal{J}^t_{s,b}(f)(z)\right)^2}\right) \tag{2.6}$$

then

$$q(z)\prec\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)} \tag{2.7}$$

and $q(z)$ is the best subordinant of (2.7).

Proof. Define the function

$$p(z) = \frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)}$$

therefore, $p(z)$ and $q(z)$ satisfy the following subordination relation

$$q(z) + \gamma z q'(z) \prec p(z) + \gamma z p'(z),$$

applying Corollary 1.1, we have the result of the corollary. □

Combining Theorem 2.1 and Theorem 2.2, we obtain the following *sandwich result:*

Theorem 2.3. *Let* $q_1(z)$ *and* $q_2(z)$ *be convex function in* $\mathbb{U}$, *with* $q_1(0) = q_2(0) = 1$*and* $\gamma \in \mathbb{C}$ *such that* $\mathrm{Re}(\gamma) > 0$.*Also, let* $f(z) \in A(p)$ *and*

$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)} + \gamma(t+b)\left(1 - \frac{\mathcal{J}^t_{s-1,b}(f)(z)\mathcal{J}^t_{s+1,b}(f)(z)}{\left(\mathcal{J}^t_{s,b}(f)(z)\right)^2}\right)$$

is univalent in $\mathbb{U}$ *and*

$$\begin{gathered} q_1(z) + \gamma z q_1'(z) \prec \\ \frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)} + \gamma(t+b)\left(1 - \frac{\mathcal{J}^t_{s-1,b}(f)(z)\mathcal{J}^t_{s+1,b}(f)(z)}{\left(\mathcal{J}^t_{s,b}(f)(z)\right)^2}\right) \prec \\ q_2(z) + \gamma z q_2'(z) \end{gathered}$$

then

$$q_1(z) \prec \frac{\mathcal{J}^t_{s+1,b}(f)(z)}{\mathcal{J}^t_{s,b}(f)(z)} \prec q_2(z) \tag{2.8}$$

and $q_1(z)$ *and* $q_2(z)$ *are the best subordinant and the best dominant of* (2.8).

Alternating $p(z)$ in Theorem 2.1 and Theorem 2.2 by $p(z) = \frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p}$, we obtain Theorem 2.4, Example 2.2, Theorem 2.5 and Theorem 2.6 as follows:

Theorem 2.4. *Let* $q(z)$ *be an univalent function in* $\mathbb{U}$, *with* $q(0) = 1$ *and*

$$\mathrm{Re}\left(\frac{zq''(z)}{q'(z)} + 1\right) > \max\left\{0, -\mathrm{Re}\left(\frac{1}{\gamma}\right)\right\} \quad (\gamma \in \mathbb{C}^*), \tag{2.9}$$

If $f(z) \in A(p)$ *and*

$$\begin{aligned} &\gamma(t+b)\frac{\mathcal{J}^t_{s,b}(f)(z)}{z^p} + (1-\gamma(t+b))\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \\ &\prec q(z) + \gamma z q'(z), \end{aligned} \tag{2.10}$$

then

$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \prec q(z) \tag{2.11}$$

and $q(z)$ *is the best dominant of* (2.11).

Example 2.2. *Let* $\alpha \in [0,1)$ *and* $\gamma \in \mathbb{C}^*$ *with* $\mathrm{Re}(\gamma) \geq 0$ *, for* $f(z) \in A(p)$ *satisfies*

$$\begin{aligned} &\gamma(t+b)\frac{\mathcal{J}^t_{s,b}(f)(z)}{z^p} + (1-\gamma(t+b))\,\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \\ &\prec \frac{1+(1-2\alpha)z}{1-z} + \frac{2(1-\alpha)\gamma z}{(1-z)^2}, \end{aligned} \tag{2.12}$$

then

$$\mathrm{Re}\left(\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p}\right) > \alpha$$

and α *is the best possible.*

Theorem 2.5. *Let* $q(z)$ *be a convex function in* $\mathbb{U}$ *with* $q(0) = 1$ *and* $\gamma \in \mathbb{C}^*$ *such that* $\mathrm{Re}(\gamma) > 0$.*If* $f(z) \in A(p)$ *satisfies*

$$\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \in \mathbb{Q},$$

also, let

$$\gamma(t+b)\frac{\mathcal{J}^t_{s,b}(f)(z)}{z^p} + (1-\gamma(t+b))\,\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \tag{2.13}$$

is univalent in $\mathbb{U}$ *and*

$$q(z) + \gamma z q'(z) \prec \gamma(t+b)\frac{\mathcal{J}^t_{s,b}(f)(z)}{z^p} + (1-\gamma(t+b))\,\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \tag{2.14}$$

then

$$q(z) \prec \frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \tag{2.15}$$

and $q(z)$ *is the best subordinant of* (2.15).

Theorem 2.6. *Let* $q_1(z)$ *and* $q_2(z)$ *be convex function in* $\mathbb{U}$, *with* $q_1(0) = q_2(0) = 1$*and* $\gamma \in \mathbb{C}$ *such that* $\mathrm{Re}(\gamma) > 0$.*Also, let* $f(z) \in A(p)$ *and*

$$\gamma(t+b)\frac{\mathcal{J}^t_{s,b}(f)(z)}{z^p} + (1-\gamma(t+b))\,\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p}$$

is univalent in $\mathbb{U}$ *and*

$$\begin{gathered} q_1(z) + \gamma z q_1'(z) \prec \\ \gamma(t+b)\frac{\mathcal{J}^t_{s,b}(f)(z)}{z^p} + (1-\gamma(t+b))\,\frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \prec \\ q_2(z) + \gamma z q_2'(z) \end{gathered}$$

then

$$q_1(z) \prec \frac{\mathcal{J}^t_{s+1,b}(f)(z)}{z^p} \prec q_2(z) \tag{2.16}$$

and $q_1(z)$ *and* $q_2(z)$ *are the best subordinant and the best dominant of* (2.16).

Theorem 2.7. *Let $q(z)$ be a convex function in $\mathbb{U}$ with $q(0) = 1$ and $\gamma \in \mathbb{C}^*$ such that* $\mathrm{Re}(\gamma) > 0$.*If $f(z) \in A(p)$ satisfies*

$$\frac{\mathcal{J}^t_{s,b}(f)(z)}{z^p} \prec q(z) + \gamma z q'(z) ,$$

then

$$\frac{1}{z^p}\mathcal{J}^t_{1,\frac{1}{\gamma}-t}(\mathcal{J}^t_{s,b}(f))(z) \prec q(z) \tag{2.17}$$

and $q(z)$ is the best dominant of (2.17).

Proof. Let we define the function

$$p(z) = \frac{1}{z^p}\mathcal{J}^t_{1,\frac{1}{\gamma}-t}(\mathcal{F})(z) \tag{2.18}$$

where $\mathcal{F}(z) = \mathcal{J}^t_{s,b}(f)(z)$, then by using 1.1 we have,

$$z\left(\mathcal{J}^t_{1,\frac{1}{\gamma}-t}(\mathcal{F})(z)\right)' = \frac{1}{\gamma}\left(\mathcal{J}^t_{0,\frac{1}{\gamma}-t}(\mathcal{F})(z)\right) - \left(\frac{1}{\gamma} - p\right)\mathcal{J}^t_{1,\frac{1}{\gamma}-t}(\mathcal{F})(z)$$

by using (2.18), we have

$$p(z) + \gamma z p'(z) = \frac{\mathcal{F}(z)}{z^p},$$

since $q(z)$ is convex function, therefore

$$\mathrm{Re}\left(\frac{zq''(z)}{q'(z)} + 1\right) > 0 > \max\left\{0, -\mathrm{Re}\left(\frac{1}{\gamma}\right)\right\} \quad (\mathrm{Re}(\gamma) > 0), \tag{2.19}$$

using Theorem 1.1, we have the theorem. □

Remark 2.1. *The operator $\mathcal{J}^t_{s,b}(f)(z)$ generalizes the generalized Bernardi operator as follows:*

$$\mathcal{J}^t_{1,p+\beta-t}(f)(z) = L_\beta(f)(z) = \frac{p+\beta}{z^\beta}\int_0^z f(u)\, u^{\beta-1}du \tag{2.20}$$

$$(z \in \mathbb{U};\ (z) \in A(p); \mathrm{Re}(\gamma) > 0)$$

By using the above remark and Theorem 2.7 , we get the following corollary.

Corollary 2.1. *Let $q(z)$ be a convex function in $\mathbb{U}$ with $q(0) = 1$ and $\gamma \in \mathbb{C}^*$ such that* $\mathrm{Re}(\gamma) > 0$. *If $f(z) \in A(p)$ satisfies*

$$\frac{\mathcal{J}^t_{s,b}(f)(z)}{z^p} \prec q(z) + \gamma z q'(z) ,$$

then

$$\frac{1}{z^p}L_{\frac{1}{\gamma}-p}(\mathcal{J}^t_{s,b}(f))(z) \prec q(z) \tag{2.21}$$

where $L_\beta(f)$ is generalized Bernardi operator defined by (2.20) *and $q(z)$ is the best dominant of* (2.21).

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