Filomat 30:7 (2016), 2067–2074
DOI 10.2298/FIL1607067A

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Majorization for Some Classes of Analytic Functions Associated with the Srivastava-Attiya Operator

Adel A. Attiya[a], M. F. Yassen[b]

[a]Department of Mathematics, Faculty of Science, University of Mansoura, Mansoura, Egypt
Current address: Department of Mathematics, Faculty of Science, University of Hail, Hail, Saudi Arabia.
[b]Department of Mathematics, Faculty of Science, Damietta University, New Damietta, 34517, Egypt
Current address: Department of Mathematics, Faculty of Sciences and Humanities Aflaj,
Prince Sattam bin Abdulaziz University, Kingdom of Saudi Arabia

Abstract. In the present paper, we investigate the majorization properties for some classes of analytic functions associated with Srivastava-Attiya operator. Moreover, some applications of the main result are btained which give a number of interesting results.

1. Introduction

Let $\mathcal{A}$ denote the class of functions of the from $f(z)$ normalized by

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U}$.

Definition 1.1. *Let $f(z)$ and $g(z)$ be two analytic functions in the open unit disk $\mathbb{U}=\{z : |z| < 1\}$. We say that $f(z)$ is majorized by $g(z)$ in $\mathbb{U}$ (see [13], [17]), and we write $f(z) \ll g(z)$, $\quad z \in\mathbb{U}$, if there exists a function $\varphi(z)$, analytic in $\mathbb{U}$ such that*

$$\left|\varphi(z)\right| \leq 1 \quad \text{and} \quad f(z) = \varphi(z)g(z) \quad (z \in \mathbb{U}). \tag{1.2}$$

It may be noted that the notion of majorization is closely related to the concept of quasi-subordination between analytic functions (see [17]).

Definition 1.2. *Let $f(z)$ and $F(z)$ be analytic functions. The function $f(z)$ is said to be subordinate to $F(z)$, written $f(z) \prec F(z)$, if there exists a function $w(z)$ analytic in $\mathbb{U}$ with $w(0) = 0$ and $|w(z)| < 1$, and such that $f(z) = F(w(z))$. in particular, if $F(z)$ is univalent, then $f(z) \prec F(z)$ if and only if $f(0) = F(0)$ and $f(\mathbb{U}) \subset F(\mathbb{U})$.*

2010 *Mathematics Subject Classification*. Primary 30C45
Keywords. Analytic functions, Hurwitz-Lerch Zeta function, Majorization property, Subordination,Srivastava-Attiya operator.
Received: 26 December 2014; Accepted: 28 May 2015
Communicated by Hari M. Srivastava
Email addresses: first.author@email.address (Adel A. Attiya), first.author@email.address (M. F. Yassen)

We begin by recalling that a general Hurwitz-Lerch Zeta function $\Phi(z, s, b)$ defined by (*cf., e.g.,* [19, P. 121 et seq.])

$$\Phi(z, s, b) = \sum_{k=0}^{\infty} \frac{z^k}{(k+b)^s}, \tag{1.3}$$

$(b \in \mathbb{C} \setminus \mathbb{Z}_0^-,\ \mathbb{Z}_0^- = \mathbb{Z}^- \cup \{0\} = \{0, -1, -2, \dots\},\ s \in \mathbb{C} \text{ when } z \in \mathbb{U},\ \mathrm{Re}(s) > 1 \text{ when } |z| = 1)$.

Several properties of $\Phi(z, s, b)$ can be found in many papers, for example Attiya and Hakami [2], Choi *et al.* [5], Cho *et al.* [4], Ferreira and López [6], Gupta *et. al.* [7] and Luo and Srivastava [12]. See, also Kutbi and Attiya([9], [10]), Srivastava and Attiya [18], Srivastava et al. [24] and Owa and Attiya [16].

Srivastava and Attiya [18] introduced the operator $J_{s,b}(f)$ which makes a connection between *Geometric Function Theory* and *Analytic Number Theory*, defined by

$$J_{s,b}(f)(z) = G_{s,b}(z) * f(z) \tag{1.4}$$

$$\left(z \in \mathbb{U};\ f \in A;\ b \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ s \in \mathbb{C}\right)$$

where

$$G_{s,b}(z) = (1+b)^s \left[\Phi(z, s, b) - b^{-s}\right] \tag{1.5}$$

and $*$ denotes the Hadamard product (or Convolution).

As special cases of $J_{s,b}(f)$, Srivastava and Attiya [18] introduced the following identities :

$$J_{0,b}(f)(z) = f(z), \tag{1.6}$$

$$J_{1,0}(f)(z) = \int_0^z \frac{f(t)}{t} dt = \mathcal{A}(f)(z), \tag{1.7}$$

$$J_{1,1}(f)(z) = \frac{2}{z} \int_0^z f(t)\ dt = \mathcal{L}(f)(z), \tag{1.8}$$

$$J_{1,\gamma}(f)(z) = \frac{1+\gamma}{z^{\gamma}} \int_0^z f(t)\ t^{\gamma-1}\ dt = \mathcal{L}_{\gamma}(f)(z) \quad (\gamma \text{ real};\ \gamma > -1), \tag{1.9}$$

and

$$J_{\sigma,1}(f)(z) = \frac{2^{\sigma}}{z\,\Gamma(\sigma)} \int_0^z \left(\log\left(\frac{z}{t}\right)\right)^{\sigma-1} f(t)\ dt = \mathcal{I}^{\sigma}(f)(z) \quad (\sigma \text{ real};\ \sigma > 0), \tag{1.10}$$

where, the operators $\mathcal{A}(f)$ and $\mathcal{L}(f)$ are the integral operators introduced earlier by Alexander [1] and Libera [11], respectively, $L_\gamma(f)$ is the generalized Bernardi operator, $\mathcal{L}_\gamma(f)$ $(\gamma \in \mathbb{N} = \{1, 2, ...\})$ introduced by Bernardi [3] and $\mathcal{I}^\sigma(f)$ is the Jung-Kim-Srivastava integral operator introduced by Jung et al. [8].

Moreover, Srivastava and Gaboury [20] (see also, Srivastava *et al.* [21]) extended the concept of $\Phi(z, s, a)$ by using the generalization of the Hurwitz–Lerch zeta function $\Phi^{(\rho_1,..,\rho_p,\sigma_1,...,\sigma q)}_{\lambda_1,...,\lambda_p,\mu_1,...,\mu_q}(z, s, a)$ which was introduced by [25, p. 503, Eq. (6.2)], to generalize the Srivastava-Attiya operator $J_{s,a}(f)$ as follows:

$$J^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(f)(z) : A \to A$$

defined by

$$J^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(f)(z) = G^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(z) * f(z),$$

the multiparameter function $G^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(z)$ is given by

$$G^{s,a,\lambda}_{(\lambda_p),(\mu_q),b}(z) := \frac{\lambda \Pi^q_{j=1}(\mu_j)\ \Gamma(s)\ (a+1)^s}{\Pi^q_{j=1}(\lambda_j)}.\Lambda(a+1, b, s, \lambda)^{-1}. \cdot\left[\Phi^{(1,..,1,1,...,1)}_{\lambda_1,...,\lambda_p,\mu_1,...,\mu_q}(z, s,\ a) - \frac{a^{-s}}{\lambda\Gamma(s)}\Lambda(a, b, s, \lambda)\right] \tag{1.11}$$

$$\left(\lambda_j \in \mathbb{C}\ (j = 1, ..., p)\ and\ \ \mu_j \in \mathbb{C}\backslash\mathbb{Z}_0^-, (j = 1, ..., q); p \le q+1; z \in \mathbb{U}\right)$$

with

$$\min\left(\mathrm{Re}(a), \mathrm{Re}(s)\right) > 0; \quad \lambda > 0\ if\ \ \mathrm{Re}(b) > 0$$

and

$$s \in \mathbb{C}, a \in \mathbb{C}\backslash\mathbb{Z}_0^-\ if\ \ b = 0$$

where

$$\Lambda(a, b, s, \lambda) := H^{2,0}_{0,2}\left[ab^{\frac{1}{\lambda}}\left|\overline{(s,1),\left(0,\frac{1}{\lambda}\right)}\right.\right]$$

and $H^{m,n}_{p,q}$ is the well-known *Fox's H-functio*n [14, Definition 1.1] (see also [22], [23]).

Now, we begin by the following lemma due to Srivastava and Attiya [18].

Lemma 1.1. *Let* $f(z) \in A$, *then*

$$zJ'_{s+1,b}(f)(z) = (1+b)J_{s,b}(f)(z) - bJ_{s+1,b}(f)(z) \tag{1.12}$$
$$\left(b \in \mathbb{C} \setminus \mathbb{Z}_0^-,\ s \in \mathbb{C},\ z \in \mathbb{U}\right)$$

Definition 1.3. *A function $f \in \mathcal{A}$ is said to be in the class $S^n_{s,b}(A,B,\zeta)$ if and only if*

$$1+\frac{1}{\zeta}\left(\frac{z(J^{(n+1)}_{s+1,b}(f)(z))}{J^{(n)}_{s+1,b}(f)(z)}-1+n\right)\prec\frac{1+Az}{1+Bz}, \tag{1.13}$$

where $n \in \mathbb{N}_0=\{0,1,...\}, -1\le B<A\le 1, \zeta\in\mathbb{C}^*:=\mathbb{C}\setminus\{0\}, s\in\mathbb{C}$ and $b\in\mathbb{C}\setminus\mathbb{Z}^-_0$.

We note that $S^0_{s-1,b}(A,B,1-\alpha)=H_{s,b,\alpha}(A,B)$ the class which introduced by Kutbi and Attiya [9], $S^0_{-1,b}(1,-1,1-\alpha)$ the well known class of starlike function of order α. Also, using special cases of n,b,A,B,ζ we have many various classes associated with Alexander operator, Libera operator, Bernardi and Jung-Kim-Srivastava operator.

Also, we use the following notations:

1. $S^n_{s,b}(-1,1,\zeta)=\mathcal{S}^n_{s,b}(\zeta)$.
2. $S^n_{s,b}(-1,1,1)=\mathcal{S}^n_{s,b}$.
3. $S^n_{0,0}(A,B,\zeta)=\mathcal{A}^n(A,B,\zeta)$
4. $S^n_{0,1}(A,B,\zeta)=\mathcal{L}^n(A,B,\zeta)$
5. $S^n_{0,\gamma}(A,B,\zeta)=\mathcal{L}^n_\gamma(A,B,\zeta)$ (γ real ; $\gamma>-1$)
6. $S^n_{\sigma,1}(A,B,\zeta)=\mathcal{I}^n_\sigma(A,B,\zeta)$ (σ real ; $\sigma>0$)

2. Majorization Problem for the Class $S^n_{s,b}(A,B,\zeta)$

In our investigation, we need the following lemma which we can prove it by using the induction and the virtue of Lemma 1.1.

Lemma 2.1. *Let $f(z)\in A$, then*

$$zJ^{(n+1)}_{s+1,b}(f)(z)=(1+b)J^{(n)}_{s,b}(f)(z)-(n+b)J^{(n)}_{s+1,b}(f)(z) \tag{2.1}$$
$$\left(n\in\mathbb{N}_0,\ b\in\mathbb{C}\setminus\mathbb{Z}^-_0,\ s\in\mathbb{C},\ z\in\mathbb{U}\right)$$

We begin by proving the following main result.

Theorem 2.1. *Let the function $f(z)\in\mathcal{A}$ and suppose that $g(z)\in S^n_{s,b}(A,B,\zeta)$, if*

$$J^{(n)}_{s+1,b}(f)(z)\ll J^{(n)}_{s+1,b}(g)(z),\quad (z\in\mathbb{U}), \tag{2.2}$$

then

$$|J^{(n)}_{s+1,b}(f)(z)|\le|J^{(n)}_{s+1,b}(g)(z)|\quad(|z|\le r_0), \tag{2.3}$$

where $r_0=r_0(\zeta,b,A,B)$ is the smallest positive root of the equation

$$r^3|\zeta(A-B)+(1+b)B|-[|1+b|+2|B|]r^2-[|\zeta(A-B)+(1+b)B|+2]r+|1+b|=0, \tag{2.4}$$

$$(-1\le B<A\le 1,\zeta\in\mathbb{C}^*,s\in\mathbb{C},b\in\mathbb{C}\setminus\mathbb{Z}^-_0),$$

Proof. Since $g(z)\in S^n_{s,b}(A,B,\zeta)$, we find from (1.13) that

$$1+\frac{1}{\zeta}\left(\frac{z(J^{(n+1)}_{s+1,b}(g)(z))}{J^{(n)}_{s+1,b}(g)(z)}-1+n\right)=\frac{1+A\,\omega(z)}{1+B\,\omega(z)}, \tag{2.5}$$

where $\omega(z)$ is analytic in $\mathbb{U}$ with

$$\omega(0) = 0 \;\; and \;\; |\omega(z)| < 1 \quad (z \in \mathbb{U}).$$

From (2.5), we get

$$\frac{z(J^{(n+1)}_{s+1,b}(g)(z))}{J^{(n)}_{s+1,b}(g)(z)} = \frac{(1-n) + [(1-n)B + \zeta(A-B)]\omega(z)}{1 + B\,\omega(z)}. \tag{2.6}$$

by virtue of Lemma 2.1 and (2.6), we get

$$|J^{(n)}_{s+1,b}(g)(z)| \leq \frac{(1+b)[1+|B||z|]}{(1+b) - |\zeta(A-B) + (1+b)B||z|}|J^{(n)}_{s,b}(g)(z)|. \tag{2.7}$$

Next, since $J^{(n)}_{s+1,b}(f)(z))$ is majorized by $J^{(n)}_{s+1,b}(g)(z)$, in the unit disk $\mathbb{U}$, from (2.2), we have

$$J^{(n)}_{s+1,b}(f)(z) = \varphi(z)J^{(n)}_{s+1,b}(g)(z), \tag{2.8}$$

where $|\varphi(z)| \leq 1$. Differentiating the above equation with respect to z and multiplying by z, we get

$$z(J^{(n+1)}_{s+1,b}(f)(z)) = z\varphi'(z)J^{(n)}_{s+1,b}(g)(z) + z\varphi(z)J^{(n+1)}_{s+1,b}(g)(z). \tag{2.9}$$

Using (2.6) in the above equation, it yields

$$J^{(n)}_{s,b}(f)(z)) = \frac{z\varphi'(z)}{(1+b)}J^{(n)}_{s+1,b}(g)(z) + \varphi(z)J^{(n)}_{s,b}(g)(z). \tag{2.10}$$

noting that $\varphi \in \mathcal{P}$ satisfying the inequality (See, e.g., Nehari [15])

$$|\varphi'(z)| \leq \frac{1-|\varphi(z)|^2}{1-|z|^2}, \quad (z \in \mathbb{U}), \tag{2.11}$$

and making use of (2.7) and (2.11) in(2.10), we get

$$|J^{(n)}_{s,b}(f)(z)| \leq \left(|\varphi(z)| + \frac{1-|\varphi(z)|^2}{1-|z|^2}\frac{[1+|B||z|]|z|}{|1+b| - |\zeta(A-B) + (1+b)B||z|}\right)|J^{(n)}_{s,b}(g)(z)|, \tag{2.12}$$

which upon setting

$$|z| = r \quad and \quad |\varphi(z)| = \rho \quad (0 \leq \rho \leq 1)$$

leads us to the inequality

$$|J^{(n)}_{s,b}(f)(z)| \leq \frac{\Phi(\rho)}{(1-r^2)\,[|1+b| - |\zeta(A-B) + (1+b)B|r]}|J^{(n)}_{s,b}(g)(z)|, \tag{2.13}$$

where

$$\Phi(\rho) = -r(1+|B|r)\rho^2 + (1-r^2)\,[|1+b| - |\zeta(A-B) + (1+b)B|r]\,\rho + r(1+|B|r), \tag{2.14}$$

takes its maximum value at $\rho = 1$, with $r_0 = r_0(A,B,S,b)$ where r_0 is the smallest positive root of (2.4). Furthermore, if $0 \leq \rho \leq r_0(A,B,s,b)$ then the function $\Psi(\rho)$ defined by

$$\Psi(\rho) = -\sigma(1+|B|\sigma)\rho^2 + (1-\sigma^2)\,[|1+b| - |\zeta(A-B) + (1+b)B|\sigma]\,\rho + \sigma(1+|B|\sigma),$$

is an increasing function on the interval $0 \leq \rho \leq 1$, so that

$$\Psi(\rho) \leq \Psi(1) = (1-\sigma^2)[|1+b| - |\zeta(A-B) + (1+b)B|\sigma], \tag{2.15}$$

$$(0 \leq \rho \leq 1; 0 \leq \sigma \leq r_0(A,B,s,b)).$$

Hence upon setting $\rho = 1$, in (2.14), we conclude that (2.3) of Theorem 2.1 holds true for

$$|z| \leq r_0 = r_0(A,B,s,b),$$

where r_0 is the smallest positive root of equation (2.4). This completes the proof of the Theorem 2.1. □

Setting $A = 1$ and $B = -1$ in Theorem 2.1, we get the following result.

Corollary 2.1. *Let the function $f(z) \in \mathcal{A}$ and suppose that $g(z) \in \mathcal{S}^n_{s,b}(\zeta)$, if*

$$J^{(n)}_{s+1,b}(f)(z) \ll J^{(n)}_{s+1,b}(g)(z), \quad (z \in \mathbb{U}), \tag{2.16}$$

then

$$|J^{(n)}_{s+1,b}(f)(z)| \leq |J^{(n)}_{s+1,b}(g)(z)| \quad (|z| \leq r_0), \tag{2.17}$$

where r_0 given by

$$r_0 = \begin{cases} \frac{m-\sqrt{m^2-4|b+1||2\zeta-b-1|}}{2|2\zeta-b-1|}, & \zeta \neq \frac{b+1}{2} \\ \frac{\sqrt{1+|b+1|(2+|b+1|)}-1}{2+|b+1|}, & \zeta = \frac{b+1}{2} \end{cases}, \tag{2.18}$$

$m = 2 + |b+1| + |2\zeta - b - 1|$, $\zeta \in \mathbb{C}^*, s \in \mathbb{C}$ *and* $b \in \mathbb{C} \setminus \mathbb{Z}_0^-$).

Setting $A = 1$, $B = -1$ and $\zeta = 1$ in Theorem 2.1, we get the following result.

Corollary 2.2. *Let the function $f(z) \in \mathcal{A}$ and suppose that $g(z) \in \mathcal{S}^n_{s,b}$, if*

$$J^{(n)}_{s+1,b}(f)(z) \ll J^{(n)}_{s+1,b}(g)(z), \quad (z \in \mathbb{U}), \tag{2.19}$$

then

$$|J^{(n)}_{s+1,b}(f)(z)| \leq |J^{(n)}_{s+1,b}(g)(z)| \quad (|z| \leq r_0), \tag{2.20}$$

where r_0 given by

$$r_0 = \begin{cases} \frac{m-\sqrt{m^2-4|b+1||1-b|}}{2|1-b|}, & b \neq 1 \\ \frac{1}{2}, & b = 1 \end{cases}, \tag{2.21}$$

$m = 2 + |b+1| + |b-1|$, $s \in \mathbb{C}$ *and* $b \in \mathbb{C} \setminus \mathbb{Z}_0^-$).

Letting $s = b = 0$, in Theorem 2.1, we get the following result.

Corollary 2.3. *Let the function $f(z) \in \mathcal{A}$ and suppose that $g(z) \in \mathcal{A}^n(A,B,\zeta)$, if*

$$\mathcal{A}^{(n)}(f)(z) \ll \mathcal{A}^{(n)}(g)(z), \quad (z \in \mathbb{U}), \tag{2.22}$$

then

$$|\mathcal{A}^{(n)}(f)(z)| \leq |\mathcal{A}^{(n)}(g)(z)| \quad (|z| \leq r_0), \tag{2.23}$$

where $r_0 = r_0(\zeta, A, B)$ is the smallest positive root of the equation

$$r^3|\zeta(A-B) + B| - [1 + 2|B|]r^2 - [|\zeta(A-B) + B| + 2]r + 1 = 0, \tag{2.24}$$

$$(-1 \leq B < A \leq 1, \zeta \in \mathbb{C}^*),$$

If we put $s = 0,\ b = 1$,in Theorem 2.1, then we have the following result.

Corollary 2.4. *Let the function* $f(z) \in \mathcal{A}$ *and suppose that* $g(z) \in \mathcal{L}^n(A, B, \zeta)$*, if*

$$\mathcal{L}^{(n)}(f)(z) \ll \mathcal{L}^{(n)}(g)(z), \quad (z \in \mathbb{U}), \tag{2.25}$$

then

$$|\mathcal{L}^{(n)}(f)(z)| \leq |\mathcal{L}^{(n)}(g)(z)| \quad (|z| \leq r_0), \tag{2.26}$$

where $r_0 = r_0(\zeta, A, B)$ *is the smallest positive root of the equation*

$$r^3|\zeta(A - B) + 2B| - 2[1 + |B|]r^2 - [|\zeta(A - B) + 2B| + 2]r + 2 = 0, \tag{2.27}$$

$(-1 \leq B < A \leq 1, \zeta \in \mathbb{C}^*)$.

Putting $s = 0$ and $b = \gamma > -1$ in Theorem 2.1, we get the following corollary.

Corollary 2.5. *Let the function* $f(z) \in \mathcal{A}$ *and suppose that* $g(z) \in \mathcal{L}^n_\gamma(A, B, \zeta)$*, if*

$$\mathcal{L}^{(n)}_\gamma(f)(z) \ll \mathcal{L}^{(n)}_\gamma(g)(z), \quad (z \in \mathbb{U},\ \gamma > -1), \tag{2.28}$$

then

$$|\mathcal{L}^{(n)}_\gamma(f)(z)| \leq |\mathcal{L}^{(n)}_\gamma(g)(z)| \quad (|z| \leq r_0), \tag{2.29}$$

where $r_0 = r_0(\zeta, b, A, B)$ *is the smallest positive root of the equation*

$$r^3|\zeta(A - B) + (1 + \gamma)B| - [1 + \gamma + 2|B|]r^2 - [|\zeta(A - B) + (1 + \gamma)B| + 2]r + (1 + \gamma) = 0, \tag{2.30}$$

$$(-1 \leq B < A \leq 1,\ \gamma > -1,\ \zeta \in \mathbb{C}^*, s \in \mathbb{C}),$$

Putting $s = \sigma$ (σ; real, $\sigma > 0$) and $b = 1$ in Theorem 2.1, we get the following corollary.

Corollary 2.6. *Let the function* $f(z) \in \mathcal{A}$ *and suppose that* $g(z) \in \mathcal{I}^n_\sigma(A, B, \zeta)$*, if*

$$\mathcal{I}^{(n)}_\sigma(f)(z) \ll \mathcal{I}^{(n)}_\sigma(g)(z), \quad (z \in \mathbb{U}; \sigma > 0), \tag{2.31}$$

then

$$|\mathcal{I}^{(n)}_\sigma(f)(z)| \leq |\mathcal{I}^{(n)}_\sigma(g)(z)| \quad (|z| \leq r_0), \tag{2.32}$$

where $r_0 = r_0(\zeta, A, B)$ *is the smallest positive root of the equation*

$$r^3|\zeta(A - B) + 2B| - 2[1 + |B|]r^2 - [|\zeta(A - B) + 2B| + 2]r + 2 = 0, \tag{2.33}$$

$$(-1 \leq B < A \leq 1, \zeta \in \mathbb{C}^*, s \in \mathbb{C}).$$

Acknowledgement. The authors would like to thank Professor H.M. Srivastava, university of Victoria, for his valuable suggestions.

References

[1] J.W. Alexander, Functions which map the interior of the unit circle upon simple region, Annals of Math. 17(1915), 12-22.
[2] A.A. Attiya and A. Hakami, Some subordination results associated with generalized Srivastava-Attiya operator, Adv. Difference Equ. 2013, 2013:105, 14 pp.
[3] S.D. Bernardi, Convex and starlike univalent functions, Trans. Amer. Math. Soc. 135(1969), 429-449.
[4] N.E. Cho, I.H. Kim and H.M. Srivastava, Sandwich-type theorems for multivalent functions associated with the Srivastava-Attiya operator, Appl. Math. Comput. 217 (2010), no. 2, 918–928.
[5] J. Choi, D.S. Jang and H.M. Srivastava, A generalization of the Hurwitz-Lerch Zeta function, Integral Transforms Spec. Funct. 19(2008), no. 1-2, 65-79.
[6] C. Ferreira and J.L. López, Asymptotic expansions of the Hurwitz-Lerch zeta function, J. Math. Anal. Appl. 298(2004), 210-224.
[7] P.L. Gupta, R.C. Gupta, S. Ong and H.M. Srivastava, A class of Hurwitz-Lerch zeta distributions and their applications in reliability, Appl. Math. Comput. 196 (2008), no. 2, 521–531.
[8] J.B. Jung, Y.C. Kim and H.M. Srivastava, The Hardy space of analytic functions associated with certain one-parameter families of integral operator, J. Math. Anal. Appl. 176(1993), 138-147.
[9] M.A. Kutbi and A.A. Attiya, Differential subordination result with the Srivastava-Attiya integral operator, J. Inequal. Appl. 2010(2010), 1-10.
[10] M.A. Kutbi and A.A. Attiya, Differential subordination results for certain integrodifferential operator and it's applications, Abs. Appl. Anal., 2012(2012), 13 pp.
[11] R.J. Libera, Some classes of regular univalent functions, Proc. Amer. Math. Soc. 16(1965), 755–758.
[12] Q.M. Luo and H.M. Srivastava, Some generalizations of the Apostol-Bernoulli and Apostol-Euler polynomials, J. Math. Anal. Appl. 308(2005), 290-302.
[13] T. H. MacGregor, Majorization by univalent functions, Duke Math. J. 34(1967), 95–102.
[14] A.M. Mathai, R.K. Saxena and H.J. Haubold, The H-Function: Theory and Applications, Springer, New York, Dordrecht, Heidelberg and London, 2010.
[15] Z. Nehari, Conformal Mapping, MacGra-Hill Book Company, New York, Toronto and London (1955).
[16] S. Owa and A.A. Attiya, An application of differential subordinations to the class of certain analytic functions, Taiwanese J. Math., 13(2009), no. 2A, 369-375.
[17] G. Schober, Univalent Functions: Selected Topics, Springer-Verlag, Berlin, Heidelberg, New York, 1975.
[18] H.M. Srivastava and A.A. Attiya, An integral operator associated with the Hurwitz-Lerch zeta function and differential subordination, Integral Transforms Spec. Funct. 18 (2007), no. 3-4, 207-216.
[19] H.M. Srivastava and J. Choi, Series Associated with the Zeta and Related Functions, Kluwer Academic Publishers, Dordrecht, 2001.
[20] H.M. Srivastava and S. Gaboury, A new class of analytic functions defined by means of a generalization of the Srivastava-Attiya operator, J. Inequal. Appl. 2015, 2015:39, 15 pp.
[21] H.M. Srivastava, S. Gaboury and F. Ghanim, A unified class of analytic functions involving a generalization of the Srivastava-Attiya operator, Appl. Math. Comput. 251(2015), 35–45.
[22] H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-Functions of One and Two Variables with Applications, South Asian Publishers, New Delhi, New Delhi and Madras, 1982.
[23] H.M. Srivastava and H.L. Manocha, A Treatise on Generating Functions, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester, Brisbane and Toronto, 1984.
[24] H.M. Srivastava, D. Răducanu, and G.S. Sălăgean, A new class of generalized close-to-starlike functions defined by the Srivastava-Attiya operator, Acta Math. Sin. (Engl. Ser.) 29 (2013), no. 5, 833-840.
[25] H.M. Srivastava, R.K. Saxena, T.K. Pogany and R. Saxena, Integral and computational representations of the extended Hurwitz–Lerch zeta function, Integral Transforms Spec. Funct. 22 (2011) 487–506.