Filomat 32:2 (2018), 439–455
https://doi.org/10.2298/FIL1802439V

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Global Existence and Stability Results for Mild Solutions of Random Impulsive Partial Integro-Differential Equations

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Abstract. In this paper, we discuss the global existence, uniqueness, continuous dependence and exponential stability of random impulsive partial integro-differential equations is investigated. The results are obtained by using the Leray-Schauder alternative fixed point theory and Banach Contraction Principle. Finally we give an example to illustrate our abstract results.

1. Introduction

Many real life processes are impulsive in nature. Impulsive differential equations are systems that are subject to rapid changes in the variables describing them. Impulsive control systems, impulsive differential equations play an important role in stability analysis (see[1, 2]). It is occurring at fixed times arise in the modelling of real world phenomena in which the state of the investigated process changes instantaneously at certain moments. Further, the investigation of these differential equations uses ideas in the qualitative theory of differential equations and probability theory (see [22–24]).

The study of global existence of solutions and its qualitative properties for partial differential equation, impulsive partial differential equation and partial integro-differential equation are very limited. E. Hernández et al. has studied the impulsive and non-impulsive global partial differential equations see[3–7] and the references there in. Tidke et al., [8, 9] studied the global existence and uniqueness for mixed integro differential equations.

The study of impulsive moments in random time is very limited. The existence, uniqueness and stability results were discussed in [13, 14] through Banach fixed point method for the system of differential equations with and without random impulsive effect. The stabilities like continuous dependence, existence and exponential stability for a random impulsive semilinear differential equations through the fixed point technique (see [11, 12] and the references therein). Ravi Agarwal, et al.[15], proved exponential stability for differential equations with random impulses at random times. For further study refer [10, 16–21] and references therein.

Motivated by the above mentioned works, we study the global existence, uniqueness and stability via continuous dependence and exponential stability of random impulsive differential equations.

2010 *Mathematics Subject Classification*. Primary 35R12, 34K20; Secondary 34K99
Keywords. Partial integro-differential equations, Random impulses, Leray-Schauder alternative fixed point theory, Contraction Principle.
Received: 09 July 2014; Accepted: 22 April 2018
Communicated by Dragan S. Djordjević
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The paper is organized as follows: In section 2, we recall briefly the notations, definitions, lemmas and preliminaries which are used throughout this paper. In section 3 and 4, we investigate the existence results and uniqueness of solutions of random impulsive PIDEs by relaxing the linear growth condition. In section 5, we extend the existence and uniqueness of solutions of random impulsive PIDEs globally. In section 6, we study the stability through continuous dependence on initial conditions of random impulsive PIDEs. The exponential stability under nonuniqueness of the mild solutions of random impulsive PIDEs are investigated in section 7. Finally in section 8, an example is given to illustrate our theoretical results.

2. Preliminaries

Let X be a real separable Hilbert space and Ω a nonempty set. Assume that τ_k is a random variable defined from Ω to $D_k \overset{def.}{=} (0, d_k)$ for $k = 1, 2, \cdots$, where $0 < d_k < +\infty$. Furthermore, assume that τ_k follow Erlang distribution, where $k = 1, 2, \ldots$ and let τ_i and τ_j are independent with each other as $i \neq j$ for $i, j = 1, 2, \cdots$. Let $\tau, T \in R$ be two constants satisfying $\tau < T$. For the sake of simplicity, we denote $\mathfrak{R}^+ = [0, +\infty)$; $\mathfrak{R}_\tau = [\tau, \infty)$.

We consider partial nonlinear integrodifferential equation of the form

$$\begin{cases} x'(t) &= Ax(t) + f(t, x_t, \int_0^t k(t, s, x_s)ds), t \neq \xi_k, \quad t \geq \tau, \\ x(\xi_k) &= b_k(\tau_k)x(\xi_k^-), \ \ k = 1, 2, \cdots, \\ x_{t_0} &= \varphi, \end{cases} \tag{2.1}$$

where A is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $S(t)$ in X; the functional $f : \mathfrak{R}_\tau \times C \times X \to X$, $C = C([-r, 0], X)$, $0 < r < \infty$ is the set of piecewise continuous functions mapping $[-r, 0]$ into X with $r > 0$; $k : \mathfrak{R}_\tau \times \mathfrak{R}_\tau \times X \to X$; x_t is a function when t is fixed, defined by $x_t(s) = x(t + s)$ for all $s \in [-r, 0]$; $\xi_0 = t_0$ and $\xi_k = \xi_{k-1} + \tau_k$ for $k = 1, 2, \cdots$;here $t_0 \in \mathfrak{R}_\tau$ is an arbitrary real number. Obviously, $t_0 = \xi_0 < \xi_1 < \xi_2 < \cdots < \lim_{k\to\infty} \xi_k = \infty$; $b_k : D_k \to \mathfrak{R}$ for each $k = 1, 2, \cdots$; $x(\xi_k^-) = \lim_{t\uparrow\xi_k} x(t)$ according to their paths with the norm $\|x\|_t = \sup_{t-r\leq s\leq t} |x(s)|$ for each t satisfying $t \geq \tau$. $\|\cdot\|$ is any given norm in X; φ is a function defined from $[-r, 0]$ to X.

Denote $\{B_t, t \geq 0\}$ the simple counting process generated by $\{\xi_n\}$, that is, $\{B_t \geq n\} = \{\xi_n \leq t\}$, and denote $\mathcal{F}_t$ the σ-algebra generated by $\{B_t, t \geq 0\}$. Then $(\Omega, P, \{\mathcal{F}_t\})$ is a probability space. Let $L_2 = L_2(\Omega, \mathcal{F}_t, X)$ denote the Hilbert space of all $\mathcal{F}_t$ - measurable square integrable random variables with values in X.

Let Γ denote the Banach space. $\Gamma\Big([t_0 - r, T], L_2\Big)$, the family of all $\mathcal{F}_t$-measurable, C-valued random variables ψ with the norm $\left\|\psi\right\|_\Gamma = \left(\sup_{t_0\leq t\leq T} E\left\|\psi\right\|_t^2\right)^{\frac{1}{2}}$.

Let $L_2^0(\Omega, \Gamma)$ denote the family of all $\mathcal{F}_0$ - measurable, Γ - valued random variable φ.

Definition 2.1. *A semigroup $\{S(t); t \geq t_0\}$ is said to be exponentially stable if there are positive constants $M \geq 1$ and $\gamma > 0$ such that $\|S(t)\| \leq Me^{-\gamma(t-t_0)}$ for all $t \geq t_0$, where $\|\cdot\|$ denotes the operator norm in $\mathcal{L}(X)$ (The Banach algebra of bounded linear operators from X into X). A semigroup $\{S(t),\ t \geq t_0\}$ is said to be uniformly bounded if $\|S(t)\| \leq M$ for all $t \geq t_0$, where $M \geq 1$ is some constant. If $M = 1$, then the semigroup is said to be contraction semigroup.*

Definition 2.2. *A map $f : \mathfrak{R}_\tau \times C \times X \to X$ is said to be $\mathcal{L}^2$-Caratheodory, if*
(i) $t \to f(t,u,v)$ is measurable for each $u \in C$;
(ii) $u,v \to f(t,u,v)$ is continuous for almost all $t \in [\tau, T]$;
(iii) for each positive integer $m > 0$, there exists $\alpha_m \in L^1([\tau, T], \mathfrak{R}^+)$ such that
$\sup_{\|x\|,\|y\|\leq m} \left\|f(t, x, y)\right\|^2 \leq \alpha_m(t)$*, for $t \in [\tau, T]$, a.e.*

Definition 2.3. *For a given $T \in (t_0, +\infty)$, a stochastic process $\{x(t) \in \Gamma,\ t_0 - r \le t \le T\}$ is called a mild solutions to equation (2.1) in $(\Omega, P, \{\mathcal{F}_t\})$, if*
(i) $x(t) \in X$ *is $\mathcal{F}_t$–adapted for $t \ge t_0$;*
(ii) $x(t_0 + s) = \varphi(s) \in L_2^0(\Omega, F)$, *when $s \in [-r, 0]$,*

$$\begin{aligned} x(t) &= \sum_{k=0}^{+\infty} \Big(\prod_{i=1}^{k} b_i(\tau_i) S(t-t_0)\varphi(0) + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t-s) f(s, x_s, \int_0^s k(s,\tau,x_\tau)d\tau)ds \\ &+ \int_{\xi_k}^{t} S(t-s) f(s, x_s, \int_0^s k(s,\tau,x_\tau)d\tau)ds \Big) I_{[\xi_k, \xi_{k+1})}(t), t \in [t_0, T], \end{aligned}$$

where $\prod_{j=m}^{n}(\cdot) = 1$ *as* $m > n$, $\prod_{j=i}^{k} b_j(\tau_j) = b_k(\tau_k) b_{k-1}(\tau_{k-1}) \cdots b_i(\tau_i)$, *and $I_A(\cdot)$ is the index function, i.e.,*

$$I_A(t) = \begin{cases} 1, & \text{if } \ t \in A, \\ 0, & \text{if } \ t \notin A. \end{cases}$$

Our existence and exponential stability theorems are based on the following theorem, which is a version of the topological transversality theorem.

Theorem 2.4 ([24] Lerray Schauder Alternative). *Let B be a convex subset of a Banach space E and assume that $0 \in B$. Let $F : B \to B$ be a completely continuous operator and let $U(F) = \{x \in B : x = \lambda\, Fx$ for some $0 < \lambda < 1\}$. Then either $U(F)$ is unbounded or F has a fixed point.*

3. Existence of mild solutions

In this section, we prove the existence theorem by using the following hypothesis.

(H_1) : There exists a continuous function $p : [t_0, T] \to R_+$ such that

$$E\|\int_0^t k(t,s,x_s)ds\|^2 \le p(t) E\|x\|_s^2,$$

for every $t, s \ge 0$ and $x \in X$.

(H_2) : There exists a continuous function $\delta : [t_0, T] \to R_+$ such that

$$E\|f(t,x,y)\|^2 \le \delta(t) H(E(\|x\|_s^2 + \|y\|_s^2)),$$

for every $t \in [t_0, T]$ and $x, y \in X$ where $H : R_+ \to (0, \infty)$ is a non-decreasing function satisfying $H(\alpha(t)x) \le \alpha(t)H(x)$..

(H_3) : $E\Big\{\max_{i,k} \prod_{j=i}^{k} \|b_j(\tau_j)\|\Big\}$ is uniformly bounded that there is $c > 0$ such that $E\Big\{\max_{i,k} \prod_{j=i}^{k} \|b_j(\tau_j)\|\Big\} \le c$ for all $\tau_j \in D_j, \quad j = 1, 2, \cdots$.

Theorem 3.1. *If the hypotheses $(H_1) - (H_3)$ hold, then system (2.1) has a mild solution x(t), defined on $[t_0, T]$ provided,*

$$\int_{t_0}^{T} M_1(s)ds \quad < \quad \int_{c_1}^{\infty} \frac{ds}{H(s)}, \tag{3.1}$$

where $M_1(t) = 2M^2 \max\{1, c^2\}(T - t_0)(1 + p(t))\delta(t)$, $c_1 = 2M^2c^2E\|\varphi\|^2$ *and* $Mc \ge \frac{1}{\sqrt{2}}$.

Proof 1. *Let T be an arbitrary number $t_0 < T < \infty$. We transform problem* (2.1) *into a fixed point problem. Consider the operator $F : \Gamma \to \Gamma$ defined by*

$$Fx(t) = \begin{cases} \varphi(t - t_0), \quad t \in [t_0 - r, t_0], \\ \sum_{k=0}^{+\infty} \Big(\prod_{i=1}^{k} b_i(\tau_i) S(t - t_0)\varphi(0) + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) ds \\ + \int_{\xi_k}^{t} S(t - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) ds \Big) I_{[\xi_k, \xi_{k+1})}(t), \quad t \in [t_0, T]. \end{cases}$$

In order to use the transversality theorem, first we establish a priori estimates for the solution of the integral solution and $\lambda \in (0, 1)$,

$$x(t) = \begin{cases} \lambda\varphi(t - t_0), \quad t \in [t_0 - r, t_0], \\ \lambda \sum_{k=0}^{+\infty} \Big(\prod_{i=1}^{k} b_i(\tau_i) S(t - t_0)\varphi(0) + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) ds \\ + \int_{\xi_k}^{t} S(t - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) ds \Big) I_{[\xi_k, \xi_{k+1})}(t), \quad t \in [t_0, T]. \end{cases}$$

$$\begin{aligned} \|x(t)\|^2 &\le \lambda^2 \Big\{ \sum_{k=0}^{+\infty} \Big[\| \prod_{i=1}^{k} b_i(\tau_i) \| \| S(t - t_0) \| \| \varphi(0) \| \\ &\quad + \sum_{i=1}^{k} \| \prod_{j=i}^{k} b_j(\tau_j) \| \int_{\xi_{i-1}}^{\xi_i} \| S(t - s) \| \| f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) \| ds \\ &\quad + \int_{\xi_k}^{t} \| S(t - s) \| \| f(s, x_s, \int_0^s k(s, \tau, x_\tau)) d\tau) \| ds \Big] I_{[\xi_k, \xi_{k+1})}(t) \Big\}^2 \\ &\le 2 \Big\{ \sum_{k=0}^{+\infty} \Big\{ \| \prod_{i=1}^{k} b_i(\tau_i) \|^2 \| S(t - t_0) \|^2 \| \varphi(0) \|^2 I_{[\xi_k, \xi_{k+1})}(t) \Big\} \\ &\quad + \Big\{ \sum_{k=0}^{+\infty} \Big[\sum_{i=1}^{k} \| \prod_{j=i}^{k} b_j(\tau_j) \| \int_{\xi_{i-1}}^{\xi_i} \| S(t - s) \| \| f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau \| ds \\ &\quad + \int_{\xi_k}^{t} \| S(t - s) \| \| f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) \| ds \Big] I_{[\xi_k, \xi_{k+1})}(t) \Big\}^2 \Big\} \\ &\le 2M^2 \max_k \{ \| \prod_{i=1}^{k} b_i(\tau_i) \|^2 \} \| \varphi(0) \|^2 + 2M^2 \Big[\max_{i,k} \{ 1, \prod_{j=i}^{k} \| b_j(\tau_j) \| \} \Big]^2 \Big(\int_{t_0}^{t} \| f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) \| ds \Big)^2. \end{aligned}$$

$$\begin{aligned} E\|x\|_t^2 &\le 2M^2 c^2 E\Big[\|\varphi\|^2 \Big] \\ &+ 2M^2 \max\{1, c^2\}(T - t_0) \int_{t_0}^{t} E\| f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) \|^2 ds \\ &\le 2M^2 c^2 E\Big[\|\varphi\|^2 \Big] + 2M^2 \max\{1, c^2\}(T - t_0) \int_{t_0}^{t} \delta(s) H\Big(E\Big[\|x\|s^2 + p(s)\|x\|_s^2 \Big] \Big) ds \\ &\le 2M^2 c^2 E\Big[\|\varphi\|^2 \Big] + 2M^2 \max\{1, c^2\}(T - t_0) \int_{t_0}^{t} \delta(s)(1 + p(s)) H\Big(E\Big[\|x\|_s^2 \Big] \Big) ds. \end{aligned}$$

Last term of the right hand side of above inequality also increases in t

$$\begin{aligned}\sup_{t_0\le v\le t} E\Big[\|x\|_v^2\Big] &\le 2M^2c^2E\Big[\|\varphi\|^2\Big] \\ &+ 2M^2\max\{1,c^2\}(T-t_0)\int_{t_0}^{t}\delta(s)(1+p(s))\sup_{t_0\le v\le t} H\Big(E\Big[\|x\|_v^2\Big]\Big)ds.\end{aligned}$$

Consider the function $l(t)$ *defined by* $l(t)=\sup_{t_0\le v\le t} E\Big[\|x\|_v^2\Big],\quad t\in[t_0,T]$. *For any* $t\in[t_0,T]$,

$$l(t) \le 2M^2c^2E\Big[\|\varphi\|^2\Big]+2M^2\max\{1,c^2\}(T-t_0)\int_{t_0}^{t}\delta(s)(1+p(s))H(l(s))ds. \tag{3.2}$$

Denoting by u(t) the right hand side of the (3.2) *we obtain that*

$$\begin{aligned} l(t) &\le u(t),\ \ t\in[t_0,T]\,;u(t_0)=2M^2c^2E\Big[\big\|\varphi\big\|^2\Big]=c_1.\\ u'(t) &= 2M^2(1+p(t))\max\{1,c^2\}(T-t_0)\delta(t)H(l(t))\\ &\le 2M^2(1+p(t))\max\{1,c^2\}(T-t_0)\delta(t)H(u(t)).\end{aligned}$$

$$\frac{u'(t)}{H(u(t))}\le M_1(t),\ \ t\in[t_0,T]\,. \tag{3.3}$$

Integrating (3.3) *from* t_0 *to T and making use of change of variable, we obtain*

$$\int_{c_1}^{u(t)}\frac{ds}{H(s)}\le\int_0^t M_1(s)ds\le\int_0^T M_1(s)ds<\int_{c_1}^{\infty}\frac{ds}{H(s)}, \tag{3.4}$$

where the last inequality is obtained by (3.1).

From (3.4) *and by mean value theorem there is a constant* η_1 *such that* $u(t)\le\eta_1$. *Hence* $l(t)\le\eta_1$. *Since* $\sup_{0\le v\le t} E\Big[\|x\|_v^2\Big]\le\eta_1$, *where* η_1 *only depends on T and the functions* δ *and H*

$$E\|x\|_\Gamma^2=\sup_{0\le v\le t}E\Big[\|x\|_v^2\Big]\le\eta_1.$$

We will prove that F is continuous and completely continuous.

Step-1 *We prove that F is continuous. Let* $\{x_n\}$ *be a convergent sequence of elements of* $x\in\Gamma$. *For each* $t\in[t_0,T]$, *we have*

$$\begin{aligned} Fx_n(t) &= \sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k}b_i(\tau_i)S(t-t_0)\varphi(0)+\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}S(t-s)f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau)ds\\ &+ \int_{\xi_k}^{t}S(t-s)f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau)ds\Big]I_{[\xi_k,\xi_{k+1})}(t),\ \ t\in[t_0,T].\end{aligned}$$

Then,

$$\begin{aligned} Fx_n(t)-Fx(t) &= \sum_{k=0}^{+\infty}\Big[\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}S(t-s)\Big\{f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau)-f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\Big\}ds\\ &+\int_{\xi_k}^{t}S(t-s)\Big\{f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau)-f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\Big\}\Big]I_{[\xi_k,\xi_{k+1})}(t).\end{aligned}$$

Therefore,

$$\begin{aligned} E\|Fx_n - Fx\|_t^2 &\leq M^2 \max\{1, c^2\}(T - t_0) \int_0^t E\|f(s, x_{n_s}, \int_0^s k(s, \tau, x_{n_\tau})d\tau) - f(s, x_s, \int_0^s k(s, \tau, x(\tau))d\tau)\|^2 ds \\ &\to 0 \quad as \quad n \to \infty. \end{aligned}$$

Thus F is continuous.

Step-2 *We prove that F is completely continuous. Denote $B_m = \left\{x \in \Gamma / \|x\|_\Gamma^2 \leq m\right\}$ for some $m \geq 0$.*

Step2.1 *We show that F maps B_m into an equicontinuous family. Let $y \in B_m$. Let $t_1, t_2 \in [t_0, T]$. If $t_0 < t_1 < t_2 < T$, then by using hypothesis (H_1) to (H_3), we have*

$$\begin{aligned} Fx(t_1) - Fx(t_2) &= \sum_{k=0}^{+\infty} \Bigg\{ \prod_{i=1}^{k} b_i(\tau_i) S(t_1 - t_0)\varphi(0) \\ &+ \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t_1 - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \\ &+ \int_{\xi_k}^{t_1} S(t_1 - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \Bigg\} I_{[\xi_k, \xi_{k+1})}(t_1) \\ &- \sum_{k=0}^{+\infty} \Bigg\{ \prod_{i=1}^{k} b_i(\tau_i) S(t_2 - t_0)\varphi(0) \\ &+ \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t_2 - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \\ &+ \int_{\xi_k}^{t_2} S(t_2 - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \Bigg\} I_{[\xi_k, \xi_{k+1})}(t_2) \\ &= \sum_{k=0}^{+\infty} \Bigg\{ \prod_{i=1}^{k} b_i(\tau_i) S(t_1 - t_0)\varphi(0) \\ &+ \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t_1 - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \\ &+ \int_{\xi_k}^{t_1} S(t_1 - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \Bigg\} \Big[I_{[\xi_k, \xi_{k+1})}(t_1) - I_{[\xi_k, \xi_{k+1})}(t_2) \Big] \\ &+ \sum_{k=0}^{+\infty} \Bigg\{ \prod_{i=1}^{k} b_i(\tau_i) \{S(t_1 - t_0) - S(t_2 - t_0)\}\varphi(0) \\ &+ \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} \{S(t_1 - s) - S(t_2 - s)\} f(s, x_s, \int_0^s k(s, \tau, x(\tau))d\tau)ds \\ &+ \int_{\xi_k}^{t_1} \{S(t_1 - s) - S(t_2 - s)\} f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \\ &+ \int_{t_1}^{t_2} S(t_2 - s) f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \Bigg\} I_{[\xi_k, \xi_{k+1})}(t_2). \end{aligned}$$

$$E\|Fx(t_1) - Fx(t_2)\|^2 \leq 2E\|I_1\|^2 + 2E\|I_2\|^2,$$

where

$$I_1 = \sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k} b_i(\tau_i)S(t_1-t_0)\varphi(0) + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} S(t_1-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds$$
$$+\int_{\xi_k}^{t_1} S(t_1-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds\Big]\Big(I_{[\xi_k,\xi_{k+1})}(t_1) - I_{[\xi_k,\xi_{k+1})}(t_2)\Big).$$

$$I_2 = \sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k} b_i(\tau_i)\{S(t_1-t_0)-S(t_2-t_0)\}\varphi(0)$$
$$+\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i}\{S(t_1-s)-S(t_2-s)\}f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds$$
$$+\int_{\xi_k}^{t_1}\{S(t_1-s)-S(t_2-s\}f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds$$
$$+\int_{t_1}^{t_2} S(t_2-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds\Big]I_{[\xi_k,\xi_{k+1})}(t_2).$$

$$\begin{aligned}
E\|I_1\|^2 \;\le\;& E\Big\{\sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k}\|b_i(\tau_i)\|\|S(t_1-t_0)\|\|\varphi(0)\| \\
&+\sum_{i=1}^{k}\prod_{j=i}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|S(t_1-s)\|\|f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|ds \\
&+\int_{\xi_k}^{t_1}\|S(t_1-s)\|\|f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|ds\Big]\Big[I_{[\xi_k,\xi_{k+1})}(t_1)-I_{[\xi_k,\xi_{k+1})}(t_2)\Big]\Big\}^2 \\
\le\;& 2M^2c^2E\|\varphi(0)\|^2E\|I_{[\xi_k,\xi_{k+1})}(t_1)-I_{[\xi_k,\xi_{k+1})}(t_2)\| \\
&+2\max\{1,c^2\}(t_1-t_0)\int_{t_0}^{t_1}\|S(t_1-s)\|^2M^*H(E\|x\|_s^2)dsE\|I_{[\xi_k,\xi_{k+1})}(t_1)-I_{[\xi_k,\xi_{k+1})}(t_2)\| \\
&\longrightarrow 0 \;\; as \;\; t_2\longrightarrow t_1,
\end{aligned}$$

where $M^* = \sup\{\delta(t)(1+p(t)) : t\in[t_0,T]\}$.

$$\begin{aligned}
E\|I_2\|^2 \;\le\;& 3c^2E\|S(t_1-t_0)-S(t_2-t_0)\|^2E\|\varphi(0)\|^2 \\
&+3\max\{1,c^2\}(t_1-t_0)E\int_{t_0}^{t_1}\|S(t_1-s)-S(t_2-s)\|^2\|f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2ds \\
&+3(t_2-t_1)E\int_{t_1}^{t_2}\|S(t_2-s)\|^2\|f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2ds \\
\le\;& 3c^2E\|S(t_1-t_0)-S(t_2-t_0)\|^2E\|\varphi(0)\|^2 \\
&+3\max\{1,c^2\}(t_1-t_0)\int_{t_0}^{t_1}E\|S(t_1-s)-S(t_2-s)\|^2\delta(s)H\Big[E\|x\|_s^2+p(s)E\|x\|_s^2\Big]ds \\
&+3(t_2-t_1)\int_{t_1}^{t_2}E\|S(t_2-s)\|^2\delta(s)H\Big[E\|x\|_s^2+p(s)E\|x\|_s^2\Big]ds.
\end{aligned}$$

Since there is $\kappa > 0$, *such that* $\|S(t_1)-S(t_2)\| \le \frac{\kappa}{t_2}\sqrt{t_1-t_2}$ *(see[22,proposition 1]) and the compactness of* $S(t)$ *for* $t > 0$ *implies the continuity in the uniform operator topology, we have* $\|S(t_1)-S(t_2)\|^2 \longrightarrow 0$, $\|S(t_1-s)-S(t_2-s)\|^2 \longrightarrow 0$ *as* $t_1 \longrightarrow t_2$.

Therefore $E\|I_2\|^2 \longrightarrow 0$ *as* $t_1 \longrightarrow t_2$. *Thus* F *maps* B_m *into an equicontinuous family of functions.*
Step2.2 *We show that* FB_m *is uniformly bounded.* $\|y\|_\Gamma^2 \leq m$, *from* (3.1) *and by* (H_1) - (H_3),

$$\begin{aligned}E\|Fx\|_t^2 &\leq 2M^2c^2E\|\varphi(0)\|^2 + 2M^2\max\{1,c^2\}(T-t_0)\int_{t_0}^{t} E\|f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2 ds\\ &\leq 2M^2c^2E\|\varphi(0)\|^2 + 2M^2\max\{1,c^2\}(T-t_0)\|\alpha_m\|.\end{aligned}$$

This yields that the set $\left\{Fx(t), \|y\|_\Gamma^2 \leq m\right\}$ *is uniformly bounded. So* $\{FB_m\}$ *is uniformly bounded. We have already shown that* FB_m *is an equicontinuous collection. Now it is sufficient, by the Arzela-Ascoli theorem, to show that* F *maps* B_m *into a precompact set in* X.
Step2.3 *We show that* FB_m *is compact. Let* $t_0 < t \leq T$ *be fixed. Let* ϵ *be a real number satisfying* $\epsilon \in (0, t-t_0)$, *for* $x \in B_m$. *Define*

$$\begin{aligned}F_\epsilon x(t) = &\sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k} b_i(\tau_i)S(t-t_0)\varphi(0)\\ &+\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} S(t-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)d\tau)ds\\ &+\int_{\xi_k}^{t-\epsilon} S(t-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds\Big]I_{[\xi_k,\xi_{k+1})}(t), t \in (t_0, t-\epsilon).\end{aligned}$$

Since $S(t)$ *is a compact operator, the set* $H_\epsilon(t) = \{F_\epsilon x(t) : x \in B_m\}$ *is precompact in* X *for every* $\epsilon \in (0, t-t_0)$. *Moreover, for every* $x \in B_m$, *we have*

$$\begin{aligned}Fx(t) - F_\epsilon x(t) = &\sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k} b_i(\tau_i)S(t-t_0)\varphi(0)\\ &+\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} S(t-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds\\ &+\int_{\xi_k}^{t} S(t-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds\Big]I_{[\xi_k,\xi_{k+1})}(t)\\ &-\sum_{k=0}^{+\infty}\Big[\prod_{i=1}^{k} b_i(\tau_i)S(t-t_0)\varphi(0)\\ &+\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} S(t-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds\\ &+\int_{\xi_k}^{t-\epsilon} S(t-s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)ds\Big]I_{[\xi_k,\xi_{k+1})}(t).\end{aligned}$$

By using (H_1)-(H_3), (3.1) *and* $\|x\|_\Gamma^2 \leq m$, *we obtain*

$$E\|Fx - F_\epsilon x\|_t^2 \leq M^2\max\{1,c^2\}(T-t_0)\int_{t-\epsilon}^{t} M^*H(m)ds.$$

There are precompact sets arbitrarily close to the set $\{Fx(t) : x \in B_m\}$. *Hence the set* $\{Fx(t) : x \in B_m\}$ *is precompact in* X. F *is a completely continuous operator. Moreover the set* $U(F) = \left\{x \in \Gamma : x = \lambda Fx \text{ for some } 0 < \lambda < 1\right\}$ *is bounded. By Theorem* 2.1, *the operator* F *has a fixed point in* Γ *and this fixed point is the mild solution of the system* (2.1).

4. Existence of unique solution:

To prove the existence results by Banach contraction principle, we need the following hypotheses.

(H_4) : There exists a constant $L > 0$ such that

$$E\|f(t, x_1, y_1) - f(t, x_2, y_2)\|^2 \leq LE\left\{\|x_1 - x_2\|_s^2 + \|y_1 - y_2\|_s^2\right\},$$

for all $t \in [t_0, T], x_i, y_i \in X, i = 1, 2.$

(H_5) : Let $k : [t_0, T] \times [t_0, T] \times X \to X$. There exists constants $M_k > 0$ such that

$$E\|\int_0^t [k(t, s, x_s) - k(t, s, y_s)]\, ds\|^2 \leq M_k E\|x - y\|_s^2,$$

for every $t, s \geq 0$ and $x, y \in X$.

Theorem 4.1. *If the hypotheses* (H_3)*-*(H_5) *holds then the system* (2.1) *has a unique mild solution on* $[t_0, T]$ *provided,*

$$\wp = 2M^2 \max\{1, c^2\}(T - t_0)^2 L(1 + M_k) < 1. \tag{4.1}$$

Proof. Consider the operator $F : \Gamma \to \Gamma$ defined as in Theorem 3.1, then

$$\begin{aligned}
E\|Fx - Fy\|_t^2 \leq{}& 2M^2 \max\{1, c^2\}(T - t_0) \int_0^t E\|f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau) \\
&- f(s, y_s, \int_0^s k(s, \tau, y_\tau)d\tau)\|^2 ds \\
&\leq 2M^2 \max\{1, c^2\}(T - t_0) \int_{t_0}^t LE\left\{\|x - y\|_s^2 + M_k\|x - y\|_s^2\right\} ds \\
&\leq 2M^2 \max\{1, c^2\}(T - t_0) \int_{t_0}^t L(1 + M_k)E\|x - y\|_s^2 ds.
\end{aligned}$$

Taking supremum over t, we get

$$\begin{aligned}
\|Fx(t) - Fy(t)\|_\Gamma^2 &\leq 2M^2 \max\{1, c^2\}(T - t_0)^2 L(1 + M_k)\|x - y\|_\Gamma^2 \\
&\leq \wp\|x - y\|_\Gamma^2.
\end{aligned}$$

From (4.1), F is a contraction on Γ. By the Banach contraction principle, there is a unique fixed point for F in space Γ and this fixed point is the mild solution of the system (2.1). □

5. Existence of global solutions

In this section, we study the global existence of solutions for

$$\begin{cases}
x'(t) = Ax(t) + f(t, x_t, \int_0^t k(t, s, x_s)ds), t \neq \xi_k, \quad t \in [t_0, \infty), \\
x(\xi_k) = b_k(\tau_k)x(\xi_k^-), \ k = 1, 2, \cdots, \\
x_{t_0} = \varphi.
\end{cases} \tag{5.1}$$

Definition 5.1. *A function* $x : [t_0, \infty) \to X$ *is called a mild solution of* (5.1)*, if* $x/_{[t_0,T]} \in \Gamma([t_0, T], X)$ *for every* $T \in (t_0, \infty)$,

$$\begin{aligned}
x(t) ={}& \sum_{k=0}^{+\infty} \Big(\prod_{i=1}^{k} b_i(\tau_i)S(t - t_0)\varphi(0) + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t - s)f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \\
&+ \int_{\xi_k}^t S(t - s)f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds\Big) I_{[\xi_k, \xi_{k+1})}(t), t \in [t_0, \infty).
\end{aligned} \tag{5.2}$$

In order to obtain our results, we need to introduce some additional notations, definitions and technical remarks. It follows that, $g : [t_0, \infty) \to R$ is a positive, continuous and nondecreasing function such that $g(t_0) = 1$ and $\lim_{t\to\infty} g(t) = \infty$. In this section, $\Gamma([t_0, \infty), X), C_0(X), C^0_g(X)$ and $\Gamma^0_g(X)$ are the spaces.

$\Gamma([t_0, \infty), L_2) = \left\{x : [t_0, \infty) \to X : x/_{[t_0,T]} \in \Gamma([t_0, T], X) \forall\, T \in [t_0, \infty), E\|x\|^2_\Gamma = \sup_{t\geq t_0} E\|x(t)\|^2 < \infty\right\};$

$C_0(X) = \left\{x \in C([t_0, \infty), X) : \lim_{t\to\infty} E\|x(t)\|^2 = 0\right\}; C^0_g(X) = \left\{x \in C([t_0, \infty), X) : \lim_{t\to\infty} \frac{E\|x(t)\|^2}{g(t)} = 0\right\};$

$\Gamma^0_g(X) = \left\{x \in \Gamma([t_0, \infty), X) : \lim_{t\to\infty} \frac{E\|x(t)\|^2}{g(t)} = 0\right\},$

endowed with the norms

$E\|x\|^2_\infty = \sup_{t\geq t_0} E\|x(t)\|^2; E\|x\|^2_0 = \sup_{t\geq t_0} E\|x(t)\|^2 ; E\|x\|^2_g = \sup_{t\geq t_0} \frac{E\|x(t)\|^2}{g(t)}$ and $E\|x\|^2_{\Gamma_g} = \sup_{t\geq t_0} \frac{E\|x(t)\|^2}{g(t)}$ respectively.

We recall here the following results of compactness in these spaces [3]. We omit the proof.

Lemma 5.2. *A set $B \subset C^0_g(X)$ is relatively compact in C^0_g if and only if,*
(a) B is equicontinuous;
(b) $\lim_{t\to\infty} \frac{E\|x(t)\|^2}{g(t)} = 0$, uniformly for $x \in B$;
(c) The set $B(t) = \{x(t) : x \in B\}$ is relatively compact in X, for every $t \geq t_0$.

Lemma 5.3. *A set $B \subset \Gamma^0_g(X)$ is relatively compact in $\Gamma^0_g(X)$ if and only if,*
(a) The set $B_T = \{x/_{[t_0,T]} : x \in B\}$ is relatively compact in $\Gamma([t_0, T]; X)$, for every $T \in (0, \infty)$.
(b) $\lim_{t\to\infty} \frac{E\|x(t)\|^2}{g(t)} = 0$, uniformly for $x \in B$.

Theorem 5.4. *Let the conditions (H_1),(H_2),(H_3) holds for every $T > 0$. Suppose, in addition, that the following conditions are verified*
(a) For every $t > t_0$, the set $\{S(t)f(s, x, y) : s \in [t_0, T], x, y \in B_m[0, X]$, (closed ball of radius $m > 0$ in Banach space X)$\}$ is relatively compact in X;
(b) For every $\iota > 0, \lim_{t\to\infty} \frac{1}{g(t)} \int_{t_0}^t M_1(s)H[\iota g(s)]ds = 0$;
(c) $\int_{t_0}^\infty M_1(s)ds < \int_{c_1}^\infty \frac{ds}{H(s)}$,
where $M_1(t) = 2M^2 \max\{1, c^2\}(T - t_0)(1 + p(t))\delta(t)$, $c_1 = 2M^2c^2E\|\varphi\|^2$ and $Mc \geq \frac{1}{\sqrt{2}}$. Then, there exists a mild solution for the system (5.1)*.*

Proof 2. *On the space $\Gamma^0_g(X)$, we define the operator*

$$\begin{aligned} Fx(t) &= \sum_{k=0}^{+\infty} \Big(\prod_{i=1}^{k} b_i(\tau_i)S(t - t_0)\varphi(0) + \sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \int_{\xi_{i-1}}^{\xi_i} S(t - s)f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds \\ &\quad + \int_{\xi_k}^t S(t - s)f(s, x_s, \int_0^s k(s, \tau, x_\tau)d\tau)ds\Big)I_{[\xi_k,\xi_{k+1})}(t), t \in [t_0, \infty). \end{aligned}$$

We observe that $E\|x(t)\|^2 \leq E\|x\|^2_{\Gamma_g} g(t)$.

$$\frac{E\|Fx(t)\|^2}{g(t)} \leq \frac{2M^2c^2E\left[\|\varphi\|^2\right]}{g(t)} + \frac{2M^2 \max\{1, c^2\}(T - t_0)}{g(t)} \int_{t_0}^t (1 + p(s))H(E\|x\|^2_{\Gamma_g} g(s))ds.$$

Next we show that F satisfies all the conditions in Theorem 2.1*.*

Let $(x_n)_{n\in N}$ be a sequence in $\Gamma^0_g(X)$ and $x \in \Gamma^0_g(X)$ such that $x_n \to x$ in $\Gamma^0_g(X)$. Let $\epsilon > 0$ be given. $\iota = \sup_{n\in N} E\|x_n\|^2_{\Gamma_g}$. From condition (b), there exists $L_1 > 0$ such that

$$\frac{2M^2 \max\{1, c^2\}(T - t_0)}{g(t)} \int_{t_0}^t \delta(s)(1 + p(s))H(\iota g(s))ds < \frac{\epsilon}{2}, \quad t \geq L_1.$$

From the Lebesgue-dominated convergence theorem, we infer that $N_\epsilon \in N$ such that

$$E\Big\{\int_0^{L_1}\|f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau)-f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2\Big\}ds<\frac{\epsilon}{2M^2\max\{1,c^2\}(T-t_0)},\ n\geq N_\epsilon.$$

For $t\in[0,L_1]$ and $n\in N_\epsilon$, we have

$$\frac{E\|Fx_n-Fx\|_t^2}{g(t)}\leq E\int_0^{L_1}\Big\{\|f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau)-f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2\Big\}ds<\epsilon.$$

Hence,

$$\sup\left\{\frac{E\|Fx_n(t)-Fx(t)\|^2}{g(t)}:t\in[0,L_1],n\geq N_\epsilon\right\}\quad\leq\quad\epsilon. \tag{5.3}$$

On the other hand, for $t\geq L_1$ and $n\geq N_\epsilon$, we find that

$$\begin{aligned}\frac{E\|Fx_n-Fx\|_t^2}{g(t)}&\leq\frac{2M^2\max\{1,c^2\}(T-t_0)}{g(t)}E\int_{t_0}^{L_1}\|f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau)-f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2ds\\&\quad+\frac{2M^2\max\{1,c^2\}(T-t_0)}{g(t)}E\int_{L_1}^{t}\|f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau)-f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2ds\\&\leq\frac{\epsilon}{2}+\frac{2M^2\max\{1,c^2\}(T-t_0)}{g(t)}\int_{L_1}^{t}\delta(s)HE\Big\{\|x_n-x\|_{\Gamma_g}^2g(s)+p(s)\|x_n-x\|_{\Gamma_g}^2g(s)\Big\}ds\\&\leq\frac{\epsilon}{2}+\frac{2M^2\max\{1,c^2\}(T-t_0)}{g(t)}\int_{L_1}^{t}\delta(s)(1+p(s))H(E\|x_n-x\|_{\Gamma_g}^2g(s))ds\\&\leq\frac{\epsilon}{2}+\frac{2M^2\max\{1,c^2\}(T-t_0)}{g(t)}\int_{L_1}^{t}\delta(s)(1+p(s))H(\iota g(s))ds,\end{aligned}$$

so that

$$\sup\Big\{\frac{E\|Fx_n-Fx\|_t^2}{g(t)}:t\geq L_1,n\geq N_\epsilon\Big\}\leq\epsilon. \tag{5.4}$$

From (5.3) *and* (5.4)*, we see that F is continuous. Next, we prove that F is completely continuous. Let $B_m=\{x\in\Gamma/\|x\|_\Gamma^2\leq m\}$. From the proof of Theorem* 3.1*, we establish that the set $F(B_m)|_{[t_0,T]}=\{x|_{[t_0,T]}\in B_m:x\in B_m\}$ is relatively compact in $\Gamma([t_0,T];X)$ for every $T\in(t_0,\infty)$. Moreover, for $x\in B_m$, we have that*

$$\frac{E\|Fx(t)\|^2}{g(t)}\quad\leq\quad\frac{2M^2c^2E\left[\|\varphi\|^2\right]}{g(t)}+\frac{2M^2\max\{1,c^2\}(T-t_0)}{g(t)}\int_{t_0}^{t}\delta(s)(1+p(s))H(E\|x\|_{\Gamma_g}^2g(s))ds$$

which from (*b*) *implies that $\frac{E\|Fx\|_t^2}{g(t)}\to0$, as $t\to\infty$, uniformly for $x\in B_m$. Now, Lemma* 5.2 *allows us to conclude that $F(B_m)$ is relatively compact in $\Gamma_g^0(X)$. Thus F is completely continuous.*

We establish the priori estimates for the equation (5.2)*. For $t\geq t_0$ we get*

$$E\|Fx\|_t^2\quad\leq\quad 2M^2c^2E\left[\|\varphi\|^2\right]+2M^2\max\{1,c^2\}(T-t_0)\int_{t_0}^{t}\delta(s)(1+p(s))H(E\|x\|_s^2)ds.$$

Denoting by $\hat{u}(t)$ the right hand side of the above equation, we obtain

$$\hat{u}'(t)\leq 2M^2(1+p(t))\max\{1,c^2\}(T-t_0)\delta(t)H(\hat{u}(t)),$$

and hence,

$$\int_{c_1}^{\hat{u}(t)} \frac{ds}{H(s)} \leq \int_{t_0}^{\infty} M_1(s)ds < \int_{c_1}^{\infty} \frac{ds}{H(s)}.$$

This inequality jointly with condition (*c*) *allows us to affirm that F is bounded in* $\Gamma_g^0(X)$*. By using Theorem* 2.1*, there exist a fixed point for F, and as a consequence the existence of a mild solution for* (5.1)*. The proof is complete.*

Theorem 5.5. *Let the condition* (H_3) *and* (H_5) *be satisfied for every* $T > t_0$*. Assume that there exists a continuous function* $L(t) > 0$ *such that*

(H_4') : $E\|f(t, x_1, y_1) - f(t, x_2, y_2)\|^2 \leq L(t)E\{\|x_1 - x_2\|_t^2 + \|y_1 - y_2\|_t^2\}$, *for all* $t \geq t_0$ *and* $x_i, y_i \in X, i = 1, 2$.

Then there exists a unique mild solution provided,

$$\hat{\wp} = 2M^2 \max\{1, c^2\}(T - t_0)(1 + M_k) \sup_{t \geq t_0} \frac{1}{g(t)} \int_{t_0}^{t} \|L(s)\|g(s)ds < 1. \tag{5.5}$$

Proof 3. *Let F be the operator defined as in Theorem 5.3 and using* (H_3),(H_4'),(H_5)

$$\begin{aligned}\frac{E\|Fx(t) - Fy(t)\|^2}{g(t)} &\leq \frac{2M^2 \max\{1, c^2\}(T - t_0)}{g(t)} \int_{t_0}^{t} L(s)E\left\{\|x - y\|_{\Gamma_g}^2 g(s) + M_k\|x - y\|_{\Gamma_g}^2 g(s)\right\} ds \\ &\leq \frac{2M^2 \max\{1, c^2\}(T - t_0)}{g(t)} \int_{t_0}^{t} L(s)(1 + M_k)E\|x - y\|_{\Gamma_g}^2 g(s)ds.\end{aligned}$$

$$\begin{aligned}\|Fx(t) - Fy(t)\|_{\Gamma_g}^2 &\leq 2M^2 \max\{1, c^2\}(T - t_0)(1 + M_k) \sup_{t \geq t_0} \frac{1}{g(t)} \int_{t_0}^{t} \|L(s)\|E\|x - y\|_{\Gamma_g}^2 g(s)ds \\ &\leq \hat{\wp}\|x - y\|_{\Gamma_g}^2.\end{aligned}$$

From (5.5)*, F is a contraction on* Γ_g^0*. Hence, there exist a unique global fixed point for F in space* Γ_g^0 *and this fixed point is the mild solution of the system* (5.1)*.*

6. Continuous Dependence

Theorem 6.1. *Let* $x(t)$ *and* $\bar{x}(t)$ *be mild solution of system* (5.1) *with initial values* $\varphi(0)$ *and* $\bar{\varphi}(0) \in \Gamma_g$ *respectively. If the assumption* (H_3), (H_4') *and* (H_5) *are satisfied then the mild solution of the system* (5.1) *is stable in the mean square.*

Proof 4. *By the assumptions x and* $\bar{x}$ *are the two mild solutions of the system* (5.1)*, for* $t \in [0, \infty)$*, then*

$$\begin{aligned}\sup_{t \geq t_0} \frac{E\|x - \bar{x}\|_t^2}{g(t)} &\leq \sup_{t \geq t_0} \frac{2M^2c^2E\|\varphi(0) - \bar{\varphi}(0)\|^2}{g(t)} \\ &\quad + 2M^2 \max\{1, c^2\}(T - t_0)(1 + M_k) \sup_{t \geq t_0} \frac{1}{g(t)} \int_{t_0}^{t} \|L(s)\|E\|x - \bar{x}\|_{\Gamma_g}^2 g(s)ds. \\ \|x - \bar{x}\|_{\Gamma_g}^2 &\leq 2M^2c^2E\|\varphi(0) - \bar{\varphi}(0)\|_{\Gamma_g}^2 \\ &\quad + 2M^2 \max\{1, c^2\}(T - t_0)(1 + M_k) \sup_{t \geq t_0} \frac{1}{g(t)} \int_{t_0}^{t} \|L(s)\|E\|x - \bar{x}\|_{\Gamma_g}^2 g(s)ds.\end{aligned}$$

By applying Grownwalls inequality we have

$$\|x-\bar{x}\|^2_{\Gamma_g} \le 2M^2c^2E\|\varphi(0)-\bar{\varphi}(0)\|^2_{\Gamma_g}\exp(2M^2\max\{1,c^2\}(T-t_0)(1+M_k)\sup_{t\ge t_0}\frac{1}{g(t)}\int_{t_0}^t\|L(s)\|g(s)ds).$$
$$\|x-\bar{x}\|^2_{\Gamma_g} \le \Im E\|\varphi(0)-\bar{\varphi}(0)\|^2_{\Gamma_g},$$

where $\Im = 2M^2c^2exp(2M^2\max\{1,c^2\}(T-t_0)(1+M_k)\sup_{t\ge t_0}\frac{1}{g(t)}\int_{t_0}^t\|L(s)\|g(s)ds)$.

Now given $\epsilon>0$ *choose* $\delta=\frac{\epsilon}{\Im}$ *such that* $E\|\varphi(0)-\bar{\varphi}(0)\|^2_{\Gamma_g}<\delta$. *Then* $\|x-\bar{x}\|^2_{\Gamma_g}\le\epsilon$.

This completes the proof.

7. Exponential Stability under nonuniqueness

In this section we will study the exponential stability of mild solution of the system (5.1). For any $\mathcal{F}_t$-adapted process $\phi(t):[-r,\infty)\to\Re$ is almost surely continuous in t. For the purposes of stability, we may assume that $f(t,0,0)=0$ for any $t\ge t_0$ so that the system (5.1) gives a trivial solution. Moreover $\phi(t)=\varphi(t-t_0)$ for $t\in[t_0-r,t_0]$ and $E\|\phi\|^2_t\to 0$ as $t\to\infty$.

Definition 7.1. *Eq.* (5.1) *is said to be exponentially stable in the quadratic mean if there exist positive constant* C_1 *and* $\lambda>0$ *such that*

$$E\|x(t)\|^2 \le C_1E\|\varphi\|^2e^{-\lambda(t-t_0)},\ t\ge t_0.$$

We now consider the following assumptions

(H_6) : $\mu H(\chi)\le H(\mu\chi)$, *for all* $\chi\in\Re^+$ *where* $\mu>1$.

(H_7) : $\|S(t)\|\le Me^{-\gamma(t-t_0)}$, $t\ge t_0$, *where* $M\ge 1$, $\gamma>0$.

Theorem 7.2. *Let the hypotheses of Theorem 5.3 and* (H_6)-(H_7) *hold. Then system* (5.1) *is exponentially stable in the quadratic mean if it satisfies the following,*
(a) For every $t>t_0$, *the set* $\{S(t)f(s,x,y): s\in[t_0,t], x,y\in B_m[0,X]\}$ *is relatively compact in* X;
(b) For every $\hat{\iota}>0$ $\lim_{t\to\infty}\frac{1}{g(t)}\int_{t_0}^t M_2(s)H[2\hat{\iota}g(s)]ds=0$.
(c) $\int_{t_0}^\infty M_2(s)ds<\int_{c_2}^\infty\frac{ds}{H(s)}$,
where, $M_2(t)=\frac{2M^2\max\{1,c^2\}(1+p(t))\delta(t)}{\gamma}$, $c_2=2M^2c^2E\|\varphi\|^2$, $Mc\ge\frac{1}{\sqrt{2}}$.

Proof 5. *The proof is similar to the proof of Theorem* 5.3, *we define the operator* F *on the space* $\Gamma^0_g(X)$ *and using* $(H_1)-(H_3)$, (H_6) *and* (H_7) *we get,*

$$E\|x\|^2_t \le 2M^2c^2e^{-\gamma(t-t_0)}E\|\varphi\|^2+\frac{2M^2\max\{1,c^2\}e^{-\gamma(t-t_0)}}{\gamma}\int_{t_0}^t e^{\gamma(s-t_0)}\delta(s)(1+p(s))H(E\|x\|^2_s)ds.$$

Then,

$$\sup_{t\ge t_0}e^{\gamma(t-t_0)}E\|x\|^2_t \le 2M^2c^2E\|\varphi\|^2+\frac{2M^2\max\{1,c^2\}}{\gamma}\sup_{t\ge t_0}\int_{t_0}^t\delta(s)(1+p(s))H(e^{\gamma(s-t_0)}E\|x\|^2_s)ds.$$

Consider $l_1(t)=\sup_{t\ge t_0}e^{\gamma(t-t_0)}E\|x\|^2_t$. *For any* $t\in[t_0,\infty)$,

$$l_1(t)\le 2M^2c^2E\|\varphi\|^2+\frac{2M^2\max\{1,c^2\}}{\gamma}\int_{t_0}^t\delta(s)(1+p(s))H(l_1(s))ds.$$

Denote the right hand side of above inequality by $u_1(t)$*, then* $l_1(t) \leq u_1(t)$*;* $u_1(t_0) = 2M^2c^2E\|\varphi\|^2 = c_2$*,* $u_1'(t) \leq \frac{2M^2\max\{1,c^2\}}{\gamma}\delta(t)(1+p(t))H(u_1(t))$*.*

Hence $\frac{u_1'(t)}{H(u_1(t))} \leq M_2(t)$*, integrating making use of a change of variable we obtain*

$$\int_{u_1(t_0)}^{u_1(t)} \frac{ds}{H(s)} \leq \int_{t_0}^{\infty} M_2(s)ds < \int_{c_2}^{\infty} \frac{ds}{H(s)}.$$

This inequality jointly with condition Theorem 7.1 (*c*) *allows us to affirm that F is bounded in* $\Gamma_g^0(X)$*.*

We will show that F is a completely continuous operator. First we prove that F is continuous.

Let $(x_n)_{n\in N}$ *be a sequence in* $\Gamma_g^0(X)$ *and* $x \in \Gamma_g^0(X)$ *such that* $x_n \to x$ *in* $\Gamma_g^0(X)$*. Let* $\epsilon > 0$ *be given.* $\hat{\iota} = \sup_{n\in N} E\|x_n\|_{\Gamma_g}^2$*. From condition Theorem* 7.1(*b*)*, there exists* $L_1 > 0$ *such that*

$$\frac{2M^2e^{-\gamma(t-t_0)}\max\{1,c^2\}}{\gamma\, g(t)} \int_{t_0}^{t} e^{\gamma(s-t_0)}\delta(s)(1+p(s))H(2\hat{\iota}g(s))ds < \frac{\epsilon}{2}, \quad t \geq L_1.$$

From the Lebesgue-dominated convergence theorem , we infer that $N_\epsilon \in N$ *such that*

$$E\Big\{\int_0^{L_1} \|f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau) - f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2\Big\}ds < \frac{\epsilon\,\gamma}{2M^2e^{-\gamma(t-t_0)}\max\{1,c^2\}}, n \geq N_\epsilon,$$

Consequently, for $t \in [0, L_1]$ *and* $n \geq N_\epsilon$*, we obtain that*

$$\begin{aligned}\frac{E\|Fx_n - Fx\|_t^2}{g(t)} &\leq \frac{2M^2e^{-\gamma(t-t_0)}\max\{1,c^2\}}{\gamma\, g(t)} \\ &\times \int_0^{L_1} e^{\gamma(s-t_0)}\Big\{E\|f(s,x_{n_s},\int_0^s k(s,\tau,x_{n_\tau})d\tau) - f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau)\|^2\Big\}ds < \epsilon,\end{aligned}$$

hence we get,

$$\sup\left\{\frac{E\|Fx_n(t) - Fx(t)\|^2}{g(t)} : t \in [0,L_1], n \geq N_\epsilon\right\} \leq \epsilon. \tag{7.1}$$

On the other hand, for $t \geq L_1$ *and* $n \geq N_\epsilon$ *we find that*

$$\begin{aligned}\frac{E\|Fx_n - Fx\|_t^2}{g(t)} &\leq \frac{\epsilon}{2} + \frac{2M^2e^{-\gamma(t-t_0)}\max\{1,c^2\}}{\gamma\, g(t)} \int_{L_1}^{t} e^{\gamma(s-t_0)}\delta(s)(1+p(s))H(E\|x_n - x\|_{\Gamma_g}^2 g(s))ds \\ &\leq \frac{\epsilon}{2} + \frac{2M^2e^{-\gamma(t-t_0)}\max\{1,c^2\}}{\gamma\, g(t)} \int_{L_1}^{t} e^{\gamma(s-t_0)}\delta(s)(1+p(s))H(2\hat{\iota}g(s))ds,\end{aligned}$$

so that

$$\sup\Big\{\frac{E\|Fx_n - Fx\|_t^2}{g(t)} : t \geq L_1, n \geq N_\epsilon\Big\} \leq \epsilon. \tag{7.2}$$

From (7.1) *and* (7.2)*, we see that F is continuous.*

Next, we prove that F is completely continuous. Let $B_m = \{x \in \Gamma / \|x\|_\Gamma^2 \leq m\}$*. From the proof of Theorem* 3.1 *we establish that the set* $F(B_m)|_{[t_0,T]} = \{x|_{[t_0,T]} \in B_m : x \in B_m\}$ *is relatively compact in* $\Gamma([t_0,T];X)$ *for every* $T \in (t_0,\infty)$*. Moreover, for* $x \in B_m$*, we have that*

$$\frac{E\|Fx(t)\|^2}{g(t)} \leq \frac{2M^2e^{-\gamma(t-t_0)}c^2E\left[\|\varphi\|^2\right]}{g(t)} + \frac{2M^2\max\{1,c^2\}e^{-\gamma(t-t_0)}}{\gamma g(t)} \int_{t_0}^{t} e^{\gamma(s-t_0)}\delta(s)(1+p(s))H(E\|x\|_{\Gamma_g}^2 g(s))ds,$$

which from Theorem 7.1(b) implies that $\frac{E\|Fx\|_t^2}{g(t)} \to 0$, *as* $t \to \infty$, *uniformly for* $x \in B_m$. *Now, Lemma 5.2 allows us to conclude that* $F(B_m)$ *is relatively compact in* $\Gamma_g^0(X)$. *Thus F is completely continuous. real number satisfying* $\epsilon \in (0, t - t_0)$.

By Theorem 2.1 the operator F has a fixed point in Γ_g^0. *Therefore the system (5.1) has a mild solution with* $\phi(t) = \varphi(t - t_0)$ *when* $t \in [t_0 - r, t_0]$ *and* $E\|\phi\|_t^2 \to 0$ *as* $t \to \infty$.
This completes the proof.

Theorem 7.3. *Let the hypotheses* (H_3), (H_4'), (H_5)-(H_7) *hold. Then system (5.1) is exponentially stable in the quadratic mean, provided*

$$\Xi = \max\{1, c^2\}M^2(1 + M_k) \sup_{t \geq t_0} \frac{1}{g(t)} \int_{t_0}^{t} e^{-\gamma(t-s)} \|L(s)\| g(s) ds < \gamma. \tag{7.3}$$

Proof 6. *We defined F as is in the Theorem 7.1, now we have to show that F is a contraction mapping. For any* $x, y \in \Gamma_g$, *we can obtain*

$$\begin{aligned} \frac{E\|F_x - F_y\|_t^2}{g(t)} &\leq \frac{\max\{1, c^2\}M^2}{g(t)\gamma} \int_{t_0}^{t} e^{-\gamma(t-s)} E\|f(s, x_s, \int_0^s k(s, \tau, x_\tau) d\tau) ds - f(s, y_s, \int_0^s k(s, \tau, y_\tau) d\tau)\|^2 ds \\ &\leq \frac{\max\{1, c^2\}M^2}{g(t)\gamma} \int_{t_0}^{t} e^{-\gamma(t-s)} (1 + M_k) L(s) E\|x - y\|_{\Gamma_g}^2 g(s) ds. \end{aligned}$$

$$\sup_{t \geq t_0} \frac{E\|F_x - F_y\|_t^2}{g(t)} \leq \frac{\max\{1, c^2\}M^2(1 + M_k)}{\gamma} \sup_{t \geq t_0} \frac{1}{g(t)} \int_{t_0}^{t} e^{-\gamma(t-s)} (L(s) E\|x - y\|_{\Gamma_g}^2 g(s)) ds.$$

Hence, we get

$$\|F_x - F_y\|_{\Gamma_g}^2 \leq \Xi \|x - y\|_{\Gamma_g}^2.$$

Thus by (7.3), this shows that F is a contraction mapping. Hence F has a unique fixed point $x(t) \in \Gamma_g$, *which is the solution of (5.1). This completes the proof.*

8. Example

Consider the random impulsive partial integro-differential equation with finite delay of the form,

$$\begin{aligned} \frac{\partial}{\partial t} v(t, x) &= \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} (a_{ij}(x) \frac{\partial}{\partial x_j} v(t, x)) - a_0 v(t, x) + \int_{-r}^{0} \beta(\theta) v\Big(t + \theta, x, \int_0^t a(t, s, v(s, x)) ds\Big) d\theta, \\ v(\xi_k, x) &= q(k) \tau_k v(\xi_k^-, x) \ \ a.s \ \ x \in \Delta, \\ v(t, x) &= 0, \ \ t \geq t_0, \ a.s \ \ x \in \partial\Delta, \\ v(\theta, x) &= \varphi(0, x), -r < \theta \leq 0, x \in \Delta, \end{aligned} \tag{8.1}$$

where a_0, r, are positive constants, Δ is an open bounded set in $\mathfrak{R}^n$ with a smooth boundary $\partial\Delta$, $\beta : [-r, 0] \to \mathfrak{R}$ is a positive function. Let τ_k be a random variable defined on $D_k \equiv (0, d_k)$ for $k = 1, 2, \ldots$, where $0 < d_k < +\infty$ and $\mu : [-r, 0] \to \mathfrak{R}$ is a positive function. Furthermore, assume that τ_k follow Erlang distribution, where $k = 1, 2, \ldots$ and τ_i and τ_j are independent with each other as $i \neq j$ for $i, j = 1, 2, \ldots$; q is a function of k; $\xi_0 = t_0$; $\xi_k = \xi_{k-1} + \tau_k$ for $k = 1, 2, \ldots$ and $t_0 \in \mathfrak{R}^+$ is an arbitrarily given real number.

The coefficients $a_{ij} \in L^{\infty}(\Delta)$ are symmetric and satisfy the ellipticity condition

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \quad \geq \ell\,|\xi|^2\,, \;\; x \in \Delta, \xi \in \Re^n,$$

for a positive constant ℓ.

In order to rewrite (8.1) in the abstract form, we introduce $X = L^2(\Delta)$ and we define the linear operator $A : D(A) \subset X \to X$ by

$$D(A) \quad = H^2(\Delta) \cap H_0^1(\Delta); A = -\sum_{i,j=1}^{n} \tfrac{\partial}{\partial x_i}(a_{ij}(x)\tfrac{\partial}{\partial x_j})$$

Here $H^1(\Delta)$ is the Sobolev space of functions $u \in L^2(\Delta)$ with distributional derivative $u' \in L^2(\Delta)$, $H_0^1(\Delta) = \{u \in H^1(\Delta); u = 0 \;\; on \;\; \partial\Delta\}$ and $H^2(\Delta) = \{u \in L^2(\Delta); u', u'' \in L^2(\Delta)\}$ ([6, 8]).

Then A generates a symmetric compact analytic semigroup $S(t)$ in X (see [12, 13]), and $S(t)\xi = \sum_{n=1}^{\infty} \exp\left(-n^2 t\right) \langle \xi, \xi_n \rangle\, \xi_n$ which satisfies $\|S(t)\| \leq \exp(-\pi^2(t - t_0)), t \geq t_0$.
Hence $S(t)$ is a contraction semigroup. Also, note that there exists a complete orthonormal set $\{\xi_n\}, n = 1, 2, \cdots$ of eigenvectors of A with $\xi_n(x) = \sqrt{\frac{2}{n}}\sin(nx)$. We assume the following conditions hold:
(i) The function $\beta(\cdot)$ is continuous with

$$\int_{-r}^{0} \beta(\theta)^2 d\theta < \infty.$$

(ii) $E\{\max_{i,k}(\prod_{j=i}^{k} \|q(j)(\tau_j)\|\|)\} < \infty$.
(iii) Let $\omega : [0, \infty) \times [0, \infty) \times X \to X$. There exists constants η_k such that $E\| \int_0^t [\sigma(t, s, x_s) - a(t, s, y_s)]ds\|^2 \leq \eta_k E\|x - y\|_s^2$, for every $t, s \geq 0$ and $x, y \in X$.
Assuming that condition (i), (ii) and (iii) are verified, then the problem (8.1) can be modeled as the abstract partial integro-differential equation with random impulsive perturbation of the form (5.1) with

$$f(t, x_t, \int_0^t k(t, s, x_s)ds) = \int_{-r}^{0} \beta(\theta)v\Big(t + \theta, x, \int_0^t a(t, s, v(s, x))ds\Big)d\theta, \text{and } b_k(\tau_k) = q(k)\tau_k.$$

The next results are consequences of Theorem 5.4 and Theorem 7.2.

Proposition 8.1. *Assume that the hypotheses* $(H_3), (H_4')$ *and* (H_5) *hold, then the system* (8.1) *has a unique, global mild solution* v*, provided,*

$$2\max\{1, c^2\}(T - t_0)(1 + \eta_k)\sup_{t \geq t_0} \frac{1}{g(t)} \int_{t_0}^{t} \|L(s)\|g(s)ds < 1.$$

Proof. Condition (i) implies that (H_4') holds with $L(t) = \displaystyle\int_{-r}^{0} \beta(\theta)^2 d\theta$ and (H_3),(H_5) follow from condition (ii) and (iii). □

Proposition 8.2. *Assume that the hypotheses* (H_3), (H_4') *and* $(H_5 - H_7)$ *hold, then the mild solution* v *for the system* (8.1) *is exponentially stable in the quadratic mean provided,*

$$\max\{1, c^2\}(1 + \eta_k)\sup_{t \geq t_0} \frac{1}{g(t)} \int_{t_0}^{t} \|L(s)\|g(s)ds < \pi^2,$$

is satisfied.

Proof. Since $(H_6), (H_7)$ holds. The condition (i) implies that (H_4') holds with $L(t) = \displaystyle\int_{-r}^{0} \beta(\theta)^2 d\theta$ and (H_3) and (H_5) follow from condition (ii) and (iii). □

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