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New Cartan's Tensors and Pseudotensors in a Generalized Finsler Space

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Abstract. In this work we defined a generalized Finsler space ($\mathbb{GF}_\mathbb{N}$) as $2N$-dimensional differentiable manifold with a non-symmetric basic tensor $g_{ij}(x, \dot{x})$, which applies that $g_{\underline{ij}\,\underset{\theta}{|}m}(x, \dot{x}) = 0$, $\theta = 1, 2$. Based on non-symmetry of basic tensor, we obtained ten Ricci type identities, comparing to two kinds of covariant derivative of a tensor in Rund's sense. There appear two new curvature tensors and fifteen magnitudes, we called "curvature pseudotensors" .

1. Introduction

Finsler geometry is a natural and fundamental generalization of Riemann geometry. It was first suggested by Riemann as early as 1854 [15], and studied systematically by Finsler in 1918 [5]. The name "*Finsler Geometry*" was first given by J. Taylor in 1927. The non-symmetric connection was interesting to many authors: K. Yano [1], A.C. Shamihoke [17], S.Minčić [8]-[10], S. Manoff [6], C. K. Mishra[11] and many others: [7], [20], [21].

Finsler [5] spaces $\mathbb{F}_\mathbb{N}$ are N-dimensional manifolds where the infinitesimal distance between two neighboring points x^i, $x^i + dx^i$ is given by:

$$ds = F(x^i, dx^i), \ i = 1, \ldots, N, \tag{1}$$

where F is required to satisfy some properties [16]:

$$\begin{aligned}
&1)\ F(x^i, dx^i) > 0;\\
&2)\ F(x^i, \lambda dx^i) = \lambda F(x, dx^i), \text{ for any } \lambda > 0;\\
&3)\ \text{The quadric form } \frac{\partial^2 F^2(x^i, dx^i)}{\partial dx^i \partial dx^j}\xi^i\xi^j > 0, \text{ for all vector } \xi^i \text{ and any } (x^i, dx^i).
\end{aligned} \tag{2}$$

Then, the metric tensor is defined from (1) as:

$$g_{ij} = \frac{1}{2}\frac{\partial^2 F^2(x^i, \dot{x}^i)}{\partial \dot{x}^i \partial \dot{x}^j}\ , \tag{3}$$

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where $\dot{x}^i = \dfrac{dx^i}{dt}$ are the tangent vectors to a curve $C:\ x^i = x^i(t)$ in the manifold space, or elements of the tangent space $T_n(x^i)$ at point x^i. Using the second condition in (2), g_{ij} are homogeneous of degree zero in the other set of variables and we can write:

$$ds^2 = g_{ij}(x^k, dx^k)dx^i dx^j\ . \tag{4}$$

Definition 1.1. *The generalized Finsler space* ($\mathbb{GF_N}$) *is a differentiable manifold with non-symmetric basic tensor* $g_{ij}(x,\dot{x})$ *, where*

$$g_{ij}(x,\dot{x}) \neq g_{ji}(x,\dot{x})\ , \quad (g = det(g_{ij}) \neq 0)\ . \tag{5}$$

Based on (5), it can be defined the symmetric, respectively, antisymmetric part of g_{ij} :

$$g_{\underline{ij}} = \frac{1}{2}(g_{ij} + g_{ji})\ , \qquad g_{\underset{\vee}{ij}} = \frac{1}{2}(g_{ij} - g_{ji})\ , \tag{6}$$

where, following [17], it is true that:

$$a)\ g_{\underline{ij}} = \frac{1}{2}\frac{\partial^2 F^2(x,\dot{x})}{\partial \dot{x}^i \partial \dot{x}^j}, \qquad b)\ \frac{\partial g_{\underset{\vee}{ij}}}{\partial \dot{x}^k} = 0\ , \tag{7}$$

where F is a metric function in $\mathbb{GF_N}$*, having the properties known from the theory of usual Finsler space* (2).

In the papers [8–10, 23, 24] we studied generalized Finsler space.

Introducing a Cartan tensor C_{ijk}, similar as in $\mathbb{F_N}$, we have:

$$C_{ijk}(x,\dot{x}) \overset{def}{=} \frac{1}{2}g_{ij,\dot{x}^k} \underset{(7b)}{=} \frac{1}{2}g_{\underline{ij},\dot{x}^k} = \frac{1}{4}F^2_{\dot{x}^i\dot{x}^j\dot{x}^k}\ , \tag{8}$$

where " $\underset{(7b)}{=}$ " signifies "equal based on (7b)". We can conclude that C_{ijk} is symmetric in relation to each pair of indices. Also, we have:

$$C^i_{jk} \overset{def}{=} h^{ip}C_{pjk} \underset{(8)}{=} h^{ip}C_{jpk} = h^{ip}C_{jkp} = C^i_{kj}\ . \tag{9}$$

In $\mathbb{GF_N}$ the next equations are valid:

$$C_{ijk}\dot{x}^i = C_{ijk}\dot{x}^j = C_{ijk}\dot{x}^k = 0\ . \tag{10}$$

One obtains coefficients of non-symmetric affine connections in the Cartan sense [2]:

$$\Gamma^{*i}_{jk} = \gamma^i_{jk} - h^{il}(C_{jlp}\Gamma^p_{ks} + C_{klp}\Gamma^p_{js} - C_{jkp}\Gamma^p_{ls})\dot{x}^s \neq \Gamma^{*i}_{kj}\ , \tag{11}$$

$$\Gamma^*_{i.jk} = \Gamma^{*r}_{jk}g_{\underline{ir}} = \gamma_{i.jk} - (C_{ijp}\Gamma^p_{ks} + C_{ikp}\Gamma^p_{js} - C_{jkp}\Gamma^p_{is})\dot{x}^s \neq \Gamma^*_{i.kj}. \tag{12}$$

We defined the coefficients:

$$\widetilde{\Gamma}^{*i}_{jk} = \gamma^i_{jk} - h^{il}(C_{klp}\Gamma^p_{sj} + C_{jlp}\Gamma^p_{sk} - C_{kjp}\Gamma^p_{sl})\dot{x}^s \neq \Gamma^{*i}_{kj}\ , \tag{13}$$

$$\widetilde{\Gamma}^*_{i.jk} = \widetilde{\Gamma}^{*r}_{jk}g_{\underline{ir}} = \gamma_{i.jk} - (C_{ikp}\Gamma^p_{sj} + C_{ijp}\Gamma^p_{sk} - C_{kjp}\Gamma^p_{si})\dot{x}^s \neq \widetilde{\Gamma}^*_{i.kj}\ . \tag{14}$$

Let us denote:

$$T^{*i}_{jk}(x,\dot{x}) = \Gamma^{*i}_{jk} - \Gamma^{*i}_{kj}\ , \quad \Gamma^i_{(jk)} = \Gamma^{*i}_{jk} + \Gamma^i_{kj}\ ,$$
$$\widetilde{T}^{*i}_{jk}(x,\dot{x}) = \widetilde{\Gamma}^{*i}_{jk} - \widetilde{\Gamma}^{*i}_{kj}\ , \quad \widetilde{\Gamma}^i_{(jk)} = \widetilde{\Gamma}^{*i}_{jk} + \widetilde{\Gamma}^i_{kj}\ ,$$

as **torsion tensors** of the connections $\Gamma^*, \widetilde{\Gamma}^*$, respectively.

Based on non-symmetry of the coefficients of the connection, it can be defined two kinds of h-covariant derivative:

$$\begin{aligned} T^i_{j\underset{1}{|}m} &= T^i_{j,m} + T^p_j \Gamma^{*i}_{pm} - T^i_p \Gamma^{*p}_{jm} - T^i_{j,\dot{s}} \Gamma^s_{rm} \dot{x}^r \ , \\ T^i_{j\underset{2}{|}m} &= T^i_{j,m} + T^p_j \widetilde{\Gamma}^{*i}_{mp} - T^i_p \widetilde{\Gamma}^{*p}_{mj} - T^i_{j,\dot{s}} \Gamma^s_{mr} \dot{x}^r \ . \end{aligned} \tag{15}$$

By the procedure that is similar in a Finsler space, it can be proved that covariant derivative (15) of a tensor also is a tensor.

Theorem 1.2. *For the tensor $g_{\underline{ij}}(x, \dot{x})$ based on both kinds of derivative (15) in $\mathbb{GF}_\mathbb{N}$ it is valid:*

$$g_{\underline{ij}\underset{\theta}{|}m}(x, \dot{x}) = 0 \ , \quad \theta = 1, 2. \tag{16}$$

Proof. Starting from (15), we get:

$$\begin{aligned} g_{\underline{ij}\underset{1}{|}m}(x, \dot{x}) &= g_{\underline{ij},m} - g_{\underline{ij},\dot{p}} \Gamma^p_{rm} \dot{x}^r - \Gamma^{*p}_{im} g_{\underline{pj}} - \Gamma^{*p}_{jm} g_{\underline{ip}} = \\ &\underset{(8,12)}{=} g_{\underline{ij},m} - 2C_{ijp} \Gamma^p_{rm} \dot{x}^r - (\Gamma^*_{i.jm} + \Gamma^*_{j.im}) = 0 \ . \end{aligned} \tag{17}$$

The same result one obtains for $g_{ij\underset{2}{|}m}$:

$$\begin{aligned} g_{\underline{ij}\underset{2}{|}m}(x, \dot{x}) &= g_{\underline{ij},m} - g_{\underline{ij},\dot{p}} \Gamma^p_{mr} \dot{x}^r - \widetilde{\Gamma}^{*p}_{mi} g_{\underline{pj}} - \widetilde{\Gamma}^{*p}_{mj} g_{\underline{ip}} = \\ &\underset{(8,12)}{=} g_{\underline{ij},m} - 2C_{ijp} \Gamma^p_{mr} \dot{x}^r - (\widetilde{\Gamma}^*_{j.mi} + \widetilde{\Gamma}^*_{i.mj}) = 0 \ , \end{aligned} \tag{18}$$

and we have proved (16). ∎

2. Ricci type identities for *h*-differentiation in $\mathbb{GF}_\mathbb{N}$

It is known that in Finsler space there is only one Ricci identity for h-differentiation, corresponding to alternated covariant derivative of the 2^{nd} order. In the case of non-symmetric affine connection there are 10 possibilities to form the difference:

$$a^{r_1 \dots r_u}_{t_1 \dots t_v \underset{\lambda}{|} m \underset{\mu}{|} n} - a^{r_1 \dots r_u}_{t_1 \dots t_v \underset{\nu}{|} m \underset{\omega}{|} n} \quad (\lambda, \mu, \nu, \omega = 1, 2) \ , \tag{19}$$

where $\underset{1}{|}, \underset{2}{|}$ denote two kinds of covariant derivative based on (1.17, 1.18), and we can obtain ten Ricci type identities and two tensors of curvature.

Corresponding identities in $\mathbb{GF}_\mathbb{N}$ may be proved by total induction method. The mentioned possibilities are obtained for these combinations:

$$\begin{aligned} (\lambda, \mu; \nu, \omega) \in \{&(1,1;1,1), (2,2;2,2), (1,2;1,2), (2,1;2,1), (1,1;2,2), \\ &(1,1;1,2), (1,1;2,1), (2,2;1,2), (2,2;2,1), (1,2;2,1)\}. \end{aligned} \tag{20}$$

For finding the general cases based on (2.1), we firstly observe the case of a tensor $a^i(x, \xi)$.

Let us obtain the case when the vector field ξ^l is stationary comparing to the first kind of covariant derivative, i.e.

$$\xi^l_{\underset{1}{|}h}(x, \xi) = 0, \quad \xi^l_{\underset{2}{|}h}(x, \xi) \neq 0 \ . \tag{21}$$

And we have:

$$\xi^l_{,h} = \frac{\partial \xi^l}{\partial x^h} = -\Gamma^{*l}_{rh}\xi^r = -\frac{\partial G^l(x,\xi)}{\partial \dot{x}^h} = G^l_{,\dot{h}}\ . \tag{22}$$

Then, we obtained, for example,

$$\begin{aligned}
a^i_{j\underset{1}{|}mn} =& (a^i_{j\underset{1}{|}m})_{,n} + (a^i_{j\underset{1}{|}m})_{,\dot s}\xi^s_{,n} + \Gamma^{*i}_{pn}a^p_{j\underset{1}{|}m} - \Gamma^{*p}_{jn}a^i_{p\underset{1}{|}m} - \Gamma^{*p}_{mn}a^i_{j\underset{1}{|}p} = \\
=& a^i_{j,mn} + a^i_{j,n\dot s}\xi^s_{,m} + a^i_{j,\dot s}\xi^s_{,mn} + \Gamma^{*i}_{pm,n}a^p_j + \Gamma^{*i}_{pm}a^p_{j,n} - \Gamma^{*p}_{jm,n}a^i_p - \Gamma^{*p}_{jm}a^i_{p,n} + \\
&+ a^i_{j,m\dot s}\xi^s_{,n} + a^i_{j,\dot l\dot s}\xi^l_{,m}\xi^s_{,n} + a^i_{j,\dot l}\xi^l_{,m\dot s}\xi^s_{,n} + \Gamma^{*i}_{pm,\dot s}a^p_j\xi^s_{,n} + \Gamma^{*i}_{pm}a^p_{j,\dot s}\xi^s_{,n} - \Gamma^{*p}_{jm,\dot s}a^i_p\xi^s_{,n} - \Gamma^{*p}_{jm}a^i_{p,\dot s}\xi^s_{,n} + \\
&+ \Gamma^{*i}_{pn}a^p_{j,m} + \Gamma^{*i}_{pn}a^p_{j,\dot s}\xi^s_{,m} + \Gamma^{*i}_{pn}\Gamma^{*p}_{sm}a^s_j - \Gamma^{*i}_{pn}\Gamma^{*s}_{jm}a^p_s - \\
&- \Gamma^{*p}_{jn}a^i_{p,m} - \Gamma^{*p}_{jn}a^i_{p,\dot s}\xi^s_{,m} - \Gamma^{*p}_{jn}\Gamma^{*i}_{sm}a^s_p + \Gamma^{*p}_{jn}\Gamma^{*s}_{pm}a^i_s - \Gamma^{*p}_{mn}a^i_{j\underset{1}{|}p}\ .
\end{aligned} \tag{23}$$

And similar:

$$\begin{aligned}
a^i_{j\underset{1}{|}m\underset{2}{|}n} =& a^i_{j,mn} + a^i_{j,n\dot s}\xi^s_{,m} + a^i_{j,\dot s}\xi^s_{,mn} + \Gamma^{*i}_{pm,n}a^p_j + \Gamma^{*i}_{pm}a^p_{j,n} - \Gamma^{*p}_{jm,n}a^i_p - \Gamma^{*p}_{jm}a^i_{p,n} + \\
&+ a^i_{j,m\dot s}\xi^s_{,n} + a^i_{j,\dot l\dot s}\xi^l_{,m}\xi^s_{,n} + a^i_{j,\dot l}\xi^l_{,m\dot s}\xi^s_{,n} + \Gamma^{*i}_{pm,\dot s}a^p_j\xi^s_{,n} + \Gamma^{*i}_{pm}a^p_{j,\dot s}\xi^s_{,n} - \Gamma^{*p}_{jm,\dot s}a^i_p\xi^s_{,n} - \Gamma^{*p}_{jm}a^i_{p,\dot s}\xi^s_{,n} + \\
&+ \widetilde{\Gamma}^{*i}_{np}a^p_{j,m} + \widetilde{\Gamma}^{*i}_{np}a^p_{j,\dot s}\xi^s_{,m} + \widetilde{\Gamma}^{*i}_{np}\Gamma^{*p}_{sm}a^s_j - \widetilde{\Gamma}^{*i}_{np}\Gamma^{*s}_{jm}a^p_s - \\
&- \widetilde{\Gamma}^{*p}_{nj}a^i_{p,m} - \widetilde{\Gamma}^{*p}_{nj}a^i_{p,\dot s}\xi^s_{,m} - \widetilde{\Gamma}^{*p}_{nj}\Gamma^{*i}_{sm}a^s_p + \widetilde{\Gamma}^{*p}_{nj}\Gamma^{*s}_{pm}a^i_s - \widetilde{\Gamma}^{*p}_{nm}a^i_{j\underset{1}{|}p}\ .
\end{aligned} \tag{24}$$

$$\begin{aligned}
a^i_{j\underset{2}{|}m\underset{1}{|}n} =& a^i_{j,mn} + a^i_{j,n\dot s}\xi^s_{,m} + a^i_{j,\dot s}\xi^s_{,mn} + \widetilde{\Gamma}^{*i}_{mp,n}a^p_j + \widetilde{\Gamma}^{*i}_{mp}a^p_{j,n} - \widetilde{\Gamma}^{*p}_{mj,n}a^i_p - \widetilde{\Gamma}^{*p}_{mj}a^i_{p,n} + \\
&+ a^i_{j,m\dot s}\xi^s_{,n} + a^i_{j,\dot l\dot s}\xi^l_{,m}\xi^s_{,n} + a^i_{j,\dot l}\xi^l_{,m\dot s}\xi^s_{,n} + \widetilde{\Gamma}^{*i}_{mp,\dot s}a^p_j\xi^s_{,n} + \widetilde{\Gamma}^{*i}_{mp}a^p_{j,\dot s}\xi^s_{,n} - \widetilde{\Gamma}^{*p}_{mj,\dot s}a^i_p\xi^s_{,n} - \widetilde{\Gamma}^{*p}_{mj}a^i_{p,\dot s}\xi^s_{,n} + \\
&+ \Gamma^{*i}_{pn}a^p_{j,m} + \Gamma^{*i}_{pn}a^p_{j,\dot s}\xi^s_{,m} + \Gamma^{*i}_{pn}\widetilde{\Gamma}^{*p}_{ms}a^s_j - \Gamma^{*i}_{pn}\widetilde{\Gamma}^{*s}_{mj}a^p_s - \\
&- \Gamma^{*p}_{jn}a^i_{p,m} - \Gamma^{*p}_{jn}a^i_{p,\dot s}\xi^s_{,m} - \Gamma^{*p}_{jn}\widetilde{\Gamma}^{*i}_{ms}a^s_p + \Gamma^{*p}_{jn}\widetilde{\Gamma}^{*s}_{mp}a^i_s - \Gamma^{*p}_{mn}a^i_{j\underset{2}{|}p}\ .
\end{aligned} \tag{25}$$

$$\begin{aligned}
a^i_{j\underset{2}{|}mn} =& a^i_{j,mn} + a^i_{j,n\dot s}\xi^s_{,m} + a^i_{j,\dot s}\xi^s_{,mn} + \widetilde{\Gamma}^{*i}_{mp,n}a^p_j + \widetilde{\Gamma}^{*i}_{mp}a^p_{j,n} - \widetilde{\Gamma}^{*p}_{mj,n}a^i_p - \widetilde{\Gamma}^{*p}_{mj}a^i_{p,n} + \\
&+ a^i_{j,m\dot s}\xi^s_{,n} + a^i_{j,\dot l\dot s}\xi^l_{,m}\xi^s_{,n} + a^i_{j,\dot l}\xi^l_{,m\dot s}\xi^s_{,n} + \widetilde{\Gamma}^{*i}_{mp,\dot s}a^p_j\xi^s_{,n} + \widetilde{\Gamma}^{*i}_{mp}a^p_{j,\dot s}\xi^s_{,n} - \widetilde{\Gamma}^{*p}_{mj,\dot s}a^i_p\xi^s_{,n} - \widetilde{\Gamma}^{*p}_{mj}a^i_{p,\dot s}\xi^s_{,n} + \\
&+ \widetilde{\Gamma}^{*i}_{np}a^p_{j,m} + \widetilde{\Gamma}^{*i}_{np}a^p_{j,\dot s}\xi^s_{,m} + \widetilde{\Gamma}^{*i}_{np}\widetilde{\Gamma}^{*p}_{ms}a^s_j - \widetilde{\Gamma}^{*i}_{np}\widetilde{\Gamma}^{*s}_{mj}a^p_s - \\
&- \widetilde{\Gamma}^{*p}_{nj}a^i_{p,m} - \widetilde{\Gamma}^{*p}_{nj}a^i_{p,\dot s}\xi^s_{,m} - \widetilde{\Gamma}^{*p}_{nj}\widetilde{\Gamma}^{*i}_{ms}a^s_p + \widetilde{\Gamma}^{*p}_{nj}\widetilde{\Gamma}^{*s}_{mp}a^i_s - \widetilde{\Gamma}^{*p}_{nm}a^i_{j\underset{2}{|}p}\ .
\end{aligned} \tag{26}$$

Also, we have

$$\begin{aligned}
a^i_{j,\dot s}\xi^s_{,mn} + a^i_{j,\dot l}\xi^l_{,m\dot s}\xi^s_{,n} &= -a^i_{j,\dot s}\frac{\partial}{\partial x^n}\Big(\frac{\partial G^s}{\partial \dot x^m}\Big) + a^i_{j,\dot l}\frac{\partial}{\partial \dot x^s}\Big(\frac{\partial G^l}{\partial \dot x^m}\Big)\frac{\partial G^s}{\partial \dot x^n} = \\
&= -a^i_{j,\dot s}\Big(\frac{\partial \Gamma^{*s}_{rm}}{\partial x^n}\dot x^r - \Big(\Gamma^{*s}_{lm} + \frac{\partial \Gamma^{*s}_{rm}}{\partial \dot x^l}\dot x^r\Big)G^l_{,\dot n}\Big) = \\
&= -a^i_{j,\dot s}\Big(\Gamma^{*s}_{rm,n} - \Gamma^{*s}_{lm}\Gamma^{*l}_{rn} - \Gamma^{*l}_{rm,\dot l}G^l_{,\dot n}\Big)\dot x^r.
\end{aligned} \tag{27}$$

1. We considered the difference:

$$
\begin{aligned}
a^i_{j\underset{1}{|}mn} - a^i_{j\underset{1}{|}nm} &= a^i_{j,mn} + a^i_{j,n\dot{s}}\xi^s_{,m} + a^i_{j,\dot{s}}\xi^s_{,mn} + \Gamma^{*i}_{pm,n}a^p_j + \Gamma^{*i}_{pm}a^p_{j,n} - \Gamma^{*p}_{jm,n}a^i_p - \Gamma^{*p}_{jm}a^i_{p,n} + \\
&+ a^i_{j,m\dot{s}}\xi^s_{,n} + a^i_{j,l\dot{s}}\xi^l_{,m}\xi^s_{,n} + a^i_{j,\dot{l}}\xi^l_{,m\dot{s}}\xi^s_{,n} + \Gamma^{*i}_{pm,\dot{s}}a^p_j\xi^s_{,n} + \Gamma^{*i}_{pm}a^p_{j,\dot{s}}\xi^s_{,n} - \Gamma^{*p}_{jm,\dot{s}}a^i_p\xi^s_{,n} - \Gamma^{*p}_{jm}a^i_{p,\dot{s}}\xi^s_{,n} + \\
&+ \Gamma^{*i}_{pn}a^p_{j,m} + \Gamma^{*i}_{pn}a^p_{j,\dot{s}}\xi^s_{,m} + \Gamma^{*i}_{pn}\Gamma^{*p}_{sm}a^s_j - \Gamma^{*i}_{pn}\Gamma^{*s}_{jm}a^p_s - \\
&- \Gamma^{*p}_{jn}a^i_{p,m} - \Gamma^{*p}_{jn}a^i_{p,\dot{s}}\xi^s_{,m} - \Gamma^{*p}_{jn}\Gamma^{*i}_{sm}a^s_p + \Gamma^{*p}_{jn}\Gamma^{*s}_{pm}a^i_s - \Gamma^{*p}_{mn}a^i_{j\underset{1}{|}p} - \\
&- a^i_{j,nm} - a^i_{j,m\dot{s}}\xi^s_{,n} - a^i_{j,\dot{s}}\xi^s_{,nm} - \Gamma^{*i}_{pn,m}a^p_j - \Gamma^{*i}_{pn}a^p_{j,m} + \Gamma^{*p}_{jn,m}a^i_p + \Gamma^{*p}_{jn}a^i_{p,m} - \\
&- a^i_{j,n\dot{s}}\xi^s_{,m} - a^i_{j,l\dot{s}}\xi^l_{,n}\xi^s_{,m} - a^i_{j,\dot{l}}\xi^l_{,n\dot{s}}\xi^s_{,m} - \Gamma^{*i}_{pn,\dot{s}}a^p_j\xi^s_{,m} - \Gamma^{*i}_{pn}a^p_{j,\dot{s}}\xi^s_{,m} + \Gamma^{*p}_{jn,\dot{s}}a^i_p\xi^s_{,m} + \Gamma^{*p}_{jn}a^i_{p,\dot{s}}\xi^s_{,m} - \\
&- \Gamma^{*i}_{pm}a^p_{j,n} - \Gamma^{*i}_{pm}a^p_{j,\dot{s}}\xi^s_{,n} - \Gamma^{*i}_{pm}\Gamma^{*p}_{sn}a^s_j + \Gamma^{*i}_{pm}\Gamma^{*s}_{jn}a^p_s + \\
&+ \Gamma^{*p}_{jm}a^i_{p,n} + \Gamma^{*p}_{jm}a^i_{p,\dot{s}}\xi^s_{,n} + \Gamma^{*p}_{jm}\Gamma^{*i}_{sn}a^s_p - \Gamma^{*p}_{jm}\Gamma^{*s}_{pn}a^i_s + \Gamma^{*p}_{nm}a^i_{j\underset{1}{|}p} = \\
&= \underset{1}{K}{}^i{}_{pmn}a^p_j - \underset{1}{K}{}^p{}_{jmn}a^i_p - a^i_{j,\dot{s}}\underset{1}{K}{}^s{}_{rmn}\dot{x}^r - T^{*p}_{mn}a^i_{j\underset{1}{|}p} \ ,
\end{aligned}
\tag{28}
$$

where

$$
\underset{1}{K}{}^i{}_{pmn} = \Gamma^{*i}_{pm,n} - \Gamma^{*i}_{pn,m} + \Gamma^{*s}_{pm}\Gamma^{*i}_{sn} - \Gamma^{*s}_{pn}\Gamma^{*i}_{sm} - \Gamma^{*i}_{pm,\dot{s}}G^s_{,\dot{n}} + \Gamma^{*i}_{pn,\dot{s}}G^s_{,\dot{m}} \ . \tag{29}
$$

Theorem 2.1. *In the generalized Finsler space $\mathbb{GF}_{\mathbb{N}}$, for h-differentiation, the first Ricci type identity is expressed:*

$$
a^{\cdots}_{\cdots\underset{1}{|}mn} - a^{\cdots}_{\cdots\underset{1}{|}nm} = \sum_{\alpha=1}^{u} \underset{1}{K}{}^{r_\alpha}{}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} \underset{1}{K}{}^{p}{}_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^s{}_{rmn}\dot{x}^r - T^{*p}_{mn}a^{\cdots}_{\cdots\underset{1}{|}p} \ , \tag{30}
$$

where $\underset{1}{K}$ given by (29) and

$$
\binom{p}{r_\alpha}a^{\cdots}_{\cdots} = a^{r_1\ldots r_{\alpha-1}pr_{\alpha+1}\ldots r_u}_{t_1\ldots t_v} \ , \qquad \binom{t_\beta}{p}a^{\cdots}_{\cdots} = a^{r_1\ldots r_u}_{t_1\ldots t_{\beta-1}pr_{\beta+1}\ldots t_v} \ . \tag{31}
$$

Using (2.6) it is easy to prove that:

$$
\underset{1}{K}{}^i{}_{pmn}\dot{x}^r = \frac{\partial^2 G^i}{\partial x^n \partial \dot{x}^m} - \frac{\partial^2 G^i}{\partial x^m \partial \dot{x}^n} + G^i_{sm}\frac{\partial G^s}{\partial \dot{x}^n} - G^i_{sn}\frac{\partial G^s}{\partial \dot{x}^m} = G^i_{,n\dot{m}} - G^i_{,\dot{n}m} + G^i_{sm}G^s_{,\dot{n}} - G^i_{sn}G^s_{,\dot{m}} \ , \tag{32}
$$

where

$$
G^i_{mn} = \frac{\partial^2 G^i}{\partial \dot{x}^n \partial \dot{x}^m} = G^i_{nm} \ .
$$

We considered:

$$
\begin{aligned}
a^i_{j\underset{1}{|}mn} - a^i_{j\underset{1}{|}nm} &= \underset{1}{K}{}^i{}_{pmn}a^p_j - \underset{1}{K}{}^p{}_{jmn}a^i_p - a^i_{j,\dot{s}}\underset{1}{K}{}^s{}_{rmn}\dot{x}^r - T^{*p}_{mn}a^i_{j\underset{1}{|}p} = \\
&= (\underset{1}{K}{}^i{}_{pmn} + A^i_{ps}\underset{1}{K}{}^s{}_{rmn}l^r)a^p_j - (\underset{1}{K}{}^p{}_{jmn} + A^p_{js}\underset{1}{K}{}^s{}_{rmn}l^r)a^i_p - a^i_j|_s\underset{1}{K}{}^s{}_{rmn}l^r - T^{*p}_{mn}a^i_{j\underset{1}{|}p} \ ,
\end{aligned}
\tag{33}
$$

and finally, we get:

$$
a^i_{j\underset{1}{|}mn} - a^i_{j\underset{1}{|}nm} = \underset{1}{R}{}^i{}_{pmn}a^p_j - \underset{1}{R}{}^p{}_{jmn}a^p_j - a^i_j|_s\underset{1}{K}{}^s{}_{rmn}l^r - T^{*p}_{mn}a^i_{j\underset{1}{|}p} \ , \tag{34}
$$

where we denoted the third tensor of Cartan, our the first "new" curvature tensor (see Rund):

$$
\begin{aligned}
\underset{1}{R}{}^i{}_{pmn} &= \underset{1}{K}{}^i{}_{pmn} + C^i_{ps}\underset{1}{K}{}^s{}_{rmn}\dot{x}^r = \\
&= \Gamma^{*i}_{pm,n} - \Gamma^{*i}_{pn,m} + \Gamma^{*s}_{pm}\Gamma^{*i}_{sn} - \Gamma^{*s}_{pn}\Gamma^{*i}_{sm} - \Gamma^{*i}_{pm,\dot{s}}G^s_{,\dot{n}} + \Gamma^{*i}_{pn,\dot{s}}G^s_{,\dot{m}} + C^i_{ps}\left(G^s_{,n\dot{m}} - G^s_{,\dot{n}m} + G^i_{qm}G^q_{,\dot{n}} - G^s_{qn}G^q_{,\dot{m}}\right) ,
\end{aligned}
\tag{35}
$$

and also it is valid:

$$\underset{1}{R}{}^{i}{}_{pmn}\dot{x}^{p} = \underset{1}{K}{}^{i}{}_{pmn}\dot{x}^{p}\ . \tag{36}$$

2. For the next difference, we got:

$$\begin{aligned} a^{i}_{j\underset{2}{|}mn} - a^{i}_{j\underset{2}{|}nm} &= \underset{2}{K}{}^{i}{}_{pmn}a^{p}_{j} - \underset{2}{K}{}^{p}{}_{jmn}a^{i}_{p} + a^{i}_{j,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^{r} + \widetilde{T}^{*p}_{mn}a^{i}_{j\underset{2}{|}p} = \\ &= (\underset{2}{K}{}^{i}{}_{pmn} + A^{i}_{ps}\underset{1}{K}{}^{s}{}_{rmn}l^{r})a^{p}_{j} - (\underset{2}{K}{}^{p}{}_{jmn} + A^{p}_{js}\underset{1}{K}{}^{s}{}_{rmn}l^{r})a^{i}_{p} - a^{i}_{j}|_{s}\underset{1}{K}{}^{s}{}_{rmn}l^{r} + \widetilde{T}^{*p}_{mn}a^{i}_{j\underset{2}{|}p} = \\ &= \underset{2}{R}{}^{i}{}_{pmn}a^{p}_{j} - \underset{2}{R}{}^{p}{}_{jmn}a^{p}_{j} - a^{i}_{j}|_{s}\underset{1}{K}{}^{s}{}_{rmn}l^{r} + \widetilde{T}^{*p}_{mn}a^{i}_{j\underset{2}{|}p}\ , \end{aligned} \tag{37}$$

where

$$\underset{2}{K}{}^{i}{}_{jmn} = \widetilde{\Gamma}^{*i}_{mj,n} - \widetilde{\Gamma}^{*i}_{nj,m} + \widetilde{\Gamma}^{*p}_{mj}\widetilde{\Gamma}^{*i}_{np} - \widetilde{\Gamma}^{*p}_{nj}\widetilde{\Gamma}^{*i}_{mp} - \widetilde{\Gamma}^{*p}_{mj,\dot{s}}G^{s}_{,\dot{n}} + \widetilde{\Gamma}^{*p}_{nj,\dot{s}}G^{s}_{,\dot{m}}\ , \tag{38}$$

and the second curvature tensor is given with:

$$\begin{aligned} \underset{2}{R}{}^{i}{}_{pmn} &= \underset{2}{K}{}^{i}{}_{pmn} + C^{i}_{ps}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^{r} = \\ &= \widetilde{\Gamma}^{*i}_{mj,n} - \widetilde{\Gamma}^{*i}_{nj,m} + \widetilde{\Gamma}^{*p}_{mj}\widetilde{\Gamma}^{*i}_{np} - \widetilde{\Gamma}^{*p}_{nj}\widetilde{\Gamma}^{*i}_{mp} - \widetilde{\Gamma}^{*p}_{mj,\dot{s}}G^{s}_{,\dot{n}} + \widetilde{\Gamma}^{*p}_{nj,\dot{s}}G^{s}_{,\dot{m}} + C^{i}_{ps}\left(G^{s}_{,n\dot{m}} - G^{s}_{,\dot{n}m} + G^{i}_{qm}G^{q}_{,\dot{n}} - G^{s}_{qn}G^{q}_{,\dot{m}}\right). \end{aligned} \tag{39}$$

It is easy to prove that:

$$\underset{2}{R}{}^{i}{}_{pmn}\dot{x}^{p} = \underset{2}{K}{}^{i}{}_{pmn}\dot{x}^{p}\ . \tag{40}$$

Theorem 2.2. *In the generalized Finsler space $\mathbb{GF}_{\mathbb{N}}$, for h-differentiation, the second Ricci type identity is expressed:*

$$a^{\cdots}_{\cdots\underset{2}{|}mn} - a^{\cdots}_{\cdots\underset{2}{|}nm} = \sum_{\alpha=1}^{u}\underset{2}{K}{}^{r_\alpha}{}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}\underset{2}{K}{}^{p}{}_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^{r} + \widetilde{T}^{*p}_{mn}a^{\cdots}_{\cdots\underset{2}{|}p}\ , \tag{41}$$

where $\underset{2}{K}$ given by (38).

3. For the difference, we can get:

$$\begin{aligned} a^{i}_{j\underset{1}{|}m\underset{2}{|}n} - a^{i}_{j\underset{1}{|}n\underset{2}{|}m} &= \underset{1}{A}{}^{i}{}_{pmn}a^{p}_{j} - \underset{2}{A}{}^{p}{}_{jmn}a^{i}_{p} + 2a^{i}_{j<\underset{\vee}{m}n>} + 2a^{i}_{j\leqslant \underset{\vee}{m}n\geqslant} - a^{i}_{,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^{r} + \widetilde{T}^{*p}_{mn}a^{i}_{j\underset{1}{|}p} = \\ &= (\underset{1}{A}{}^{i}{}_{pmn} + A^{i}_{ps}\underset{1}{K}{}^{s}{}_{rmn}l^{r})a^{p}_{j} - (\underset{2}{A}{}^{p}{}_{jmn} + A^{p}_{js}\underset{1}{K}{}^{s}{}_{rmn}l^{r})a^{i}_{p} - a^{i}_{j}|_{s}\underset{1}{K}{}^{s}{}_{rmn}l^{r} + \\ &+ 2a^{i}_{j<\underset{\vee}{m}n>} + 2a^{i}_{j\leqslant \underset{\vee}{m}n\geqslant} - a^{i}_{,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^{r} + \widetilde{T}^{*p}_{mn}a^{i}_{j\underset{1}{|}p} = \\ &= \underset{1}{B}{}^{i}{}_{pmn}a^{p}_{j} - \underset{2}{B}{}^{p}{}_{jmn}a^{p}_{j} - a^{i}_{j}|_{s}\underset{1}{K}{}^{s}{}_{rmn}l^{r} + 2a^{i}_{j<\underset{\vee}{m}n>} + 2a^{i}_{j\leqslant \underset{\vee}{m}n\geqslant} - a^{i}_{,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^{r} + \widetilde{T}^{*p}_{mn}a^{i}_{j\underset{1}{|}p}\ , \end{aligned} \tag{42}$$

where

$$\underset{1}{A}{}^{i}{}_{pmn} = \Gamma^{*i}_{pm,n} - \Gamma^{*i}_{pn,m} + \Gamma^{*s}_{pm}\widetilde{\Gamma}^{*i}_{ns} - \Gamma^{*s}_{pn}\widetilde{\Gamma}^{*i}_{ms} - \Gamma^{*i}_{pm,\dot{s}}G^{s}_{,\dot{n}} + \Gamma^{*i}_{pn,\dot{s}}G^{s}_{,\dot{m}}\ , \tag{43}$$

$$\underset{2}{A}{}^{i}{}_{pmn} = \Gamma^{*i}_{pm,n} - \Gamma^{*i}_{pn,m} + \widetilde{\Gamma}^{*s}_{mp}\Gamma^{*i}_{sn} - \widetilde{\Gamma}^{*s}_{np}\Gamma^{*i}_{sm} - \Gamma^{*i}_{pm,\dot{s}}G^{s}_{,\dot{n}} + \Gamma^{*i}_{pn,\dot{s}}G^{s}_{,\dot{m}}\ , \tag{44}$$

$$\underset{1}{B}{}^{i}{}_{pmn} = \underset{1}{A}{}^{i}{}_{pmn} + C^{i}_{ps}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^{r}\ , \qquad \underset{2}{B}{}^{i}{}_{pmn} = \underset{2}{A}{}^{i}{}_{pmn} + C^{i}_{ps}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^{r}, \tag{45}$$

$$a^{i}_{j<mn>} = M^{*i}_{pm}(a^{p}_{j,n} + a^{p}_{j,\dot{s}}\xi^{s}_{,n}) - M^{*p}_{jm}(a^{i}_{p,n} + a^{i}_{p,\dot{s}}\xi^{s}_{,n})\ , \tag{46}$$

$$a^i_{j\leqslant mn\geqslant} = (\widetilde{\Gamma}^{*i}_{mp}\Gamma^{*s}_{jn} - \Gamma^{*i}_{pm}\widetilde{\Gamma}^{*s}_{nj})a^p_s \ . \tag{47}$$

It is easy to prove that:

$$\underset{1}{B}{}^i_{pmn}\dot{x}^p = \underset{1}{A}{}^i_{pmn}\dot{x}^p \ , \quad \underset{2}{B}{}^i_{pmn}\dot{x}^p = \underset{2}{A}{}^i_{pmn}\dot{x}^p \ . \tag{48}$$

Theorem 2.3. *In $\mathbb{GF}_\mathbb{N}$, for h-differentiation, the 3^{rd} Ricci type identity is expressed by:*

$$\begin{aligned} a^{\cdots}_{\cdots\underset{1}{|}m\underset{2}{|}n} - a^{\cdots}_{\cdots\underset{1}{|}n\underset{2}{|}m} = {} & \sum_{\alpha=1}^{u} \underset{1}{A}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} \underset{2}{A}{}^{p}_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^s_{rmn}\dot{x}^r + \\ & + 2a^{\cdots}_{\cdots<\underset{\vee}{mn}>} + 2a^{\cdots}_{\cdots\leqslant\underset{\vee}{mn}\geqslant} + \widetilde{T}^{*p}_{mn}a^{\cdots}_{\cdots\underset{1}{|}p} \ , \end{aligned} \tag{49}$$

where $\underset{1}{A}$ and $\underset{2}{A}$ are given by equations (43, 44) and

$$a^{\cdots}_{\cdots<mn>} = \sum_{\alpha=1}^{u} M^{*r_\alpha}_{pm}\binom{p}{r_\alpha}(a^{\cdots}_{\cdots,n} + a^{\cdots}_{\cdots,\dot{s}}\xi^s_{,n}) - \sum_{\beta=1}^{u} M^{*p}_{t_\beta m}\binom{t_\beta}{p}(a^{\cdots}_{\cdots,n} + a^{\cdots}_{\cdots,\dot{s}}\xi^s_{,n}) \ , \tag{50}$$

$$\begin{aligned} a^{r_1\ldots r_u}_{t_1\ldots t_v\leqslant mn\geqslant} = {} & \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{u-1}\sum_{\beta=2}^{u} \Gamma^{*r_\alpha}_{[\breve{p}m}\Gamma^{*r_\beta}_{\breve{n}s]}\binom{p}{r_\alpha}\binom{s}{r_\beta}a^{\cdots}_{\cdots} - \sum_{\alpha=1}^{u}\sum_{\beta=1}^{v} \Gamma^{*r_\alpha}_{[\breve{p}m}\Gamma^{*s}_{\breve{n}t_\beta]}\binom{p}{r_\alpha}\binom{t_\beta}{s}a^{\cdots}_{\cdots} + \\ & + \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{v-1}\sum_{\beta=1}^{v} \Gamma^{*p}_{[\breve{t}_\alpha m}\Gamma^{*s}_{\breve{n}t_\beta]}\binom{t_\alpha}{p}\binom{t_\beta}{s}a^{\cdots}_{\cdots} \ , \end{aligned} \tag{51}$$

where

$$\binom{p}{r_\alpha}\binom{t_\beta}{s}a^{\cdots}_{\cdots} = a^{r_1\ldots r_{\alpha-1}pr_{\alpha+1}\ldots r_u}_{t_1\ldots t_{\beta-1}sr_{\beta+1}\ldots t_v} \ .$$

4. We also considered the difference:

$$a^i_{j\underset{2}{|}m\underset{1}{|}n} - a^i_{j\underset{2}{|}n\underset{1}{|}m} = \underset{3}{A}{}^i_{pmn}a^p_j - \underset{4}{A}{}^p_{jmn}a^i_p - 2a^i_{j<\underset{\vee}{mn}>} - 2a^i_{j\leqslant\underset{\vee}{mn}\geqslant} + a^i_{,\dot{s}}\underset{1}{K}{}^s_{rmn}\dot{x}^r - T^{*p}_{mn}a^i_{j\underset{2}{|}p} \ , \tag{52}$$

where

$$\underset{3}{A}{}^i_{pmn} = \widetilde{\Gamma}^{*i}_{mp,n} - \widetilde{\Gamma}^{*i}_{np,m} + \widetilde{\Gamma}^{*s}_{mp}\Gamma^{*i}_{sn} - \widetilde{\Gamma}^{*s}_{np}\Gamma^{*i}_{sm} - \widetilde{\Gamma}^{*i}_{mp,\dot{s}}G^s_{,\dot{n}} + \widetilde{\Gamma}^{*i}_{np,\dot{s}}G^s_{,\dot{m}} \ , \tag{53}$$

$$\underset{4}{A}{}^i_{pmn} = \widetilde{\Gamma}^{*i}_{mp,n} - \widetilde{\Gamma}^{*i}_{np,m} + \Gamma^{*s}_{pm}\widetilde{\Gamma}^{*i}_{ns} - \Gamma^{*s}_{pn}\widetilde{\Gamma}^{*i}_{ms} - \widetilde{\Gamma}^{*i}_{mp,\dot{s}}G^s_{,\dot{n}} + \widetilde{\Gamma}^{*i}_{np,\dot{s}}G^s_{,\dot{m}} \ . \tag{54}$$

Theorem 2.4. *Applying the two kinds of covariant derivatives in an inverse order than usual, we obtained the 4^{th} Ricci type identity in $\mathbb{GF}_\mathbb{N}$ for h-differentiation:*

$$\begin{aligned} a^{\cdots}_{\cdots\underset{2}{|}m\underset{1}{|}n} - a^{\cdots}_{\cdots\underset{2}{|}n\underset{1}{|}m} = {} & \sum_{\alpha=1}^{u} \underset{3}{A}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} \underset{4}{A}{}^{p}_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^s_{rmn}\dot{x}^r - \\ & - 2a^{\cdots}_{\cdots<\underset{\vee}{mn}>} - 2a^{\cdots}_{\cdots\leqslant\underset{\vee}{mn}\geqslant} - T^{*p}_{mn}a^{\cdots}_{\cdots\underset{2}{|}p} \ , \end{aligned} \tag{55}$$

where $\underset{3}{A}$ and $\underset{4}{A}$ are given by (53, 54).

5. We considered the difference:

$$a^i_{\underset{1}{j|}mn} - a^i_{\underset{2}{j|}nm} = \underset{5}{A}{}^i_{pmn}a^p_j - \underset{6}{A}{}^p_{jmn}a^i_p + 2a^i_{j<\underline{mn}>} + 2a^i_{j\lessdot\underline{mn}\gtrdot} - a^i_{,\dot s}\underset{1}{K}{}^s_{rmn}\dot x^r - \Gamma^{*p}_{mn}a^i_{\underset{1}{j|p}} + \widetilde{\Gamma}^{*p}_{mn}a^i_{\underset{2}{j|p}}\ , \tag{56}$$

where

$$\underset{5}{A}{}^i_{pmn} = \Gamma^{*i}_{pm,n} - \widetilde{\Gamma}^{*i}_{np,m} + \Gamma^{*s}_{pm}\Gamma^{*i}_{sn} - \widetilde{\Gamma}^{*s}_{np}\widetilde{\Gamma}^{*i}_{ms} - \Gamma^{*i}_{pm,\dot s}G^s_{,\dot n} + \widetilde{\Gamma}^{*i}_{np,\dot s}G^s_{,\dot m}\ , \tag{57}$$

$$\underset{6}{A}{}^i_{pmn} = \Gamma^{*i}_{pm,n} - \widetilde{\Gamma}^{*i}_{np,m} + \widetilde{\Gamma}^{*s}_{mp}\widetilde{\Gamma}^{*i}_{ns} - \Gamma^{*s}_{pn}\Gamma^{*i}_{sm} - \Gamma^{*i}_{pm,\dot s}G^s_{,\dot n} + \widetilde{\Gamma}^{*i}_{np,\dot s}G^s_{,\dot m}\ . \tag{58}$$

Theorem 2.5. *In $\mathbb{GF_N}$, for h-differentiation, there is the 5^{th} Ricci type identity:*

$$\begin{aligned} a^{\cdots}_{\underset{1}{\cdots|}mn} - a^{\cdots}_{\underset{2}{\cdots|}nm} = &\sum_{\alpha=1}^{u}\underset{5}{A}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}\underset{6}{A}{}^{p}_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot s}\underset{1}{K}{}^s_{rmn}\dot x^r + \\ &+ 2a^{\cdots}_{\cdots<\underline{mn}>} + 2a^{\cdots}_{\cdots\lessdot\underline{mn}\gtrdot} - \Gamma^{*p}_{mn}a^{\cdots}_{\underset{1}{\cdots|p}} + \widetilde{\Gamma}^{*p}_{mn}a^{\cdots}_{\underset{2}{\cdots|p}}\ , \end{aligned} \tag{59}$$

where $\underset{5}{A}$ and $\underset{6}{A}$ are given by (57, 58).

$$\begin{aligned} a^{r_1\ldots r_u}_{t_1\ldots t_v\lessdot mn\gtrdot} = &\sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{u-1}\sum_{\beta=2}^{u}\Gamma^{*r_\alpha}_{[\breve{p}m}\Gamma^{*r_\beta}_{\breve{s}n]}\binom{p}{r_\alpha}\binom{s}{t_\beta}a^{\cdots}_{\cdots} - \sum_{\alpha=1}^{u}\sum_{\beta=1}^{v}\Gamma^{*r_\alpha}_{[\breve{p}m}\Gamma^{*s}_{\breve{t}_\beta n]}\binom{p}{r_\alpha}\binom{t_\beta}{s}a^{\cdots}_{\cdots} + \\ &+ \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{v-1}\sum_{\beta=2}^{v}\Gamma^{*p}_{[\breve{t}_\alpha m}\Gamma^{*s}_{\breve{t}_\beta n]}\binom{t_\alpha}{p}\binom{t_\beta}{s}a^{\cdots}_{\cdots}\ . \end{aligned} \tag{60}$$

6. In the next difference we got:

$$a^i_{\underset{1}{j|}mn} - a^i_{\underset{1\ 2}{j|n|}m} = \underset{7}{A}{}^i_{pmn}a^p_j - \underset{8}{A}{}^p_{jmn}a^i_p + 2a^i_{j<mn>} - 2a^i_{j\lessdot\underline{mn}\gtrdot} + a^i_{,\dot s}\underset{1}{K}{}^s_{rmn}\dot x^r - M^{*p}_{mn}a^i_{\underset{1}{j|p}}\ , \tag{61}$$

where

$$\underset{7}{A}{}^i_{pmn} = \Gamma^{*i}_{pm,n} - \Gamma^{*i}_{pn,m} + \Gamma^{*s}_{pm}\Gamma^{*i}_{sn} - \Gamma^{*s}_{pn}\widetilde{\Gamma}^{*i}_{ms} - \Gamma^{*i}_{pm,\dot s}G^s_{,\dot n} + \Gamma^{*i}_{pn,\dot s}G^s_{,\dot m}\ , \tag{62}$$

$$\underset{8}{A}{}^i_{pmn} = \Gamma^{*i}_{pm,n} - \Gamma^{*i}_{pn,m} + \widetilde{\Gamma}^{*s}_{mp}\Gamma^{*i}_{sn} - \Gamma^{*s}_{pn}\Gamma^{*i}_{sm} - \Gamma^{*i}_{pm,\dot s}G^s_{,\dot n} + \Gamma^{*i}_{pn,\dot s}G^s_{,\dot m}\ . \tag{63}$$

Theorem 2.6. *In $\mathbb{GF_N}$ for h-differentiation, there is the 6^{th} Ricci type identity:*

$$a^{\cdots}_{\underset{1}{\cdots|}mn} - a^{\cdots}_{\underset{1\ 2}{\cdots|n|}m} = \sum_{\alpha=1}^{u}\underset{7}{A}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}\underset{8}{A}{}^{p}_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots} - 2a^{\cdots}_{\cdots,\dot s}\underset{1}{K}{}^s_{rmn}\dot x^r + 2a^{\cdots}_{\cdots<mn>} + 2a^{\cdots}_{\cdots\lessdot mn\gtrdot}\ , \tag{64}$$

where $\underset{7}{A}$ and $\underset{8}{A}$ are given by equations (62, 63) and

$$\begin{aligned} a^{r_1\ldots r_u}_{t_1\ldots t_v\lessdot mn\gtrdot} = &\sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{u-1}\sum_{\beta=2}^{u}(\Gamma^{*r_\alpha}_{[pm]}\Gamma^{*r_\beta}_{sn} + \Gamma^{*r_\alpha}_{pn}\Gamma^{*r_\beta}_{[sm]})\binom{p}{r_\alpha}\binom{s}{r_\beta}a^{\cdots}_{\cdots} - \\ &- \sum_{\alpha=1}^{u}\sum_{\beta=1}^{v}(\Gamma^{*r_\alpha}_{[pm]}\Gamma^{*s}_{t_\beta n} + \Gamma^{*r_\alpha}_{pn}\Gamma^{*s}_{[t_\beta n]})\binom{p}{r_\alpha}\binom{t_\beta}{s}a^{\cdots}_{\cdots} + \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{v-1}\sum_{\beta=2}^{v}(\Gamma^{*p}_{[t_\alpha m]}\Gamma^{*s}_{t_\beta n} + \Gamma^{*p}_{t_\alpha n}\Gamma^{*s}_{[t_\beta n]})\binom{t_\alpha}{p}\binom{t_\beta}{s}a^{\cdots}_{\cdots}\ . \end{aligned} \tag{65}$$

7. Then, we considered the difference:

$$a^i_{j\underset{1}{|}mn} - a^i_{j\underset{2}{|}n\underset{1}{|}m} = \underset{9}{A}{}^i_{pmn}a^p_j - \underset{10}{A}{}^p_{jmn}a^i_p + 2a^i_{j<nm>} - 2a^i_{j\lessdot\underline{\underline{nm}}\gtrdot} + a^i_{,\dot{s}}\underset{1}{K}{}^s_{rmn}\dot{x}^r - \Gamma^{*p}_{mn}a^i_{j\underset{1}{|}p} + \Gamma^{*p}_{nm}a^i_{j\underset{2}{|}p}\ , \tag{66}$$

where

$$\underset{9}{A}{}^i_{pmn} = \Gamma^{*i}_{pm,n} - \widetilde{\Gamma}^{*i}_{np,m} + \Gamma^{*s}_{pm}\Gamma^{*i}_{sn} - \widetilde{\Gamma}^{*s}_{np}\Gamma^{*i}_{sm} - \Gamma^{*i}_{pm,\dot{s}}G^s_{,\dot{n}} + \widetilde{\Gamma}^{*i}_{np,\dot{s}}G^s_{,\dot{m}}\ , \tag{67}$$

$$\underset{10}{A}{}^i_{pmn} = \Gamma^{*i}_{pm,n} - \widetilde{\Gamma}^{*i}_{np,m} + \Gamma^{*s}_{pm}\widetilde{\Gamma}^{*i}_{ns} - \Gamma^{*s}_{pn}\Gamma^{*i}_{sm} - \Gamma^{*i}_{pm,\dot{s}}G^s_{,\dot{n}} + \widetilde{\Gamma}^{*i}_{np,\dot{s}}G^s_{,\dot{m}}\ . \tag{68}$$

Theorem 2.7. *In $\mathbb{GF}_\mathbb{N}$, for h-differentiation, the 7^{th} Ricci type identity is valid:*

$$\begin{aligned} a^{\cdots}_{\cdots\underset{1}{|}mn} - a^{\cdots}_{\cdots\underset{2}{|}n\underset{1}{|}m} = &\sum_{\alpha=1}^{u}\underset{9}{A}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}\underset{10}{A}{}^p_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^s_{rmn}\dot{x}^r + \\ &+ 2a^{\cdots}_{\cdots<nm>} + 2a^{\cdots}_{\cdots\lessdot nm\gtrdot} - (\Gamma^{*p}_{mn}a^{\cdots}_{\cdots\underset{1}{|}p} - \Gamma^{*p}_{nm}a^{\cdots}_{\cdots\underset{2}{|}p})\ , \end{aligned} \tag{69}$$

where $\underset{9}{A}$ and $\underset{10}{A}$ are given by equations (67, 68).

8. And, we considered the difference:

$$a^i_{j\underset{2}{|}mn} - a^i_{j\underset{1}{|}n\underset{2}{|}m} = \underset{11}{A}{}^i_{pmn}a^p_j - \underset{12}{A}{}^p_{jmn}a^i_p - 2a^i_{j<nm>} + 2a^i_{j\lessdot\underline{\underline{nm}}\gtrdot} + a^i_{,\dot{s}}\underset{1}{K}{}^s_{rmn}\dot{x}^r + \widetilde{\Gamma}^{*p}_{mn}a^i_{j\underset{1}{|}p} - \widetilde{\Gamma}^{*p}_{nm}a^i_{j\underset{2}{|}p}\ , \tag{70}$$

where

$$\underset{11}{A}{}^i_{pmn} = \widetilde{\Gamma}^{*i}_{mp,n} - \Gamma^{*i}_{pn,m} + \widetilde{\Gamma}^{*s}_{mp}\widetilde{\Gamma}^{*i}_{ns} - \Gamma^{*s}_{pn}\widetilde{\Gamma}^{*i}_{ms} - \widetilde{\Gamma}^{*i}_{mp,\dot{s}}G^s_{,\dot{n}} + \Gamma^{*i}_{pn,\dot{s}}G^s_{,\dot{m}}\ , \tag{71}$$

$$\underset{12}{A}{}^i_{pmn} = \widetilde{\Gamma}^{*i}_{mp,n} - \Gamma^{*i}_{pn,m} + \widetilde{\Gamma}^{*s}_{mp}\Gamma^{*i}_{sn} - \widetilde{\Gamma}^{*s}_{np}\widetilde{\Gamma}^{*i}_{ms} - \widetilde{\Gamma}^{*i}_{mp,\dot{s}}G^s_{,\dot{n}} + \Gamma^{*i}_{pn,\dot{s}}G^s_{,\dot{m}}\ . \tag{72}$$

Theorem 2.8. *In $\mathbb{GF}_\mathbb{N}$, for h-differentiation, there is the 8^{th} Ricci type identity:*

$$\begin{aligned} a^{\cdots}_{\cdots\underset{2}{|}mn} - a^{\cdots}_{\cdots\underset{1}{|}n\underset{2}{|}m} = &\sum_{\alpha=1}^{u}\underset{11}{A}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha}a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v}\underset{12}{A}{}^p_{t_\beta mn}\binom{t_\beta}{p}a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^s_{rmn}\dot{x}^r - \\ &- 2a^{\cdots}_{\cdots<nm>} + 2a^{\cdots}_{\cdots\lessdot mn\gtrdot} + \widetilde{\Gamma}^{*p}_{mn}a^{\cdots}_{\cdots\underset{1}{|}p} - \widetilde{\Gamma}^{*p}_{nm}a^{\cdots}_{\cdots\underset{2}{|}p}\ , \end{aligned} \tag{73}$$

where $\underset{11}{A}$ and $\underset{12}{A}$ are given by equations (71, 72) and

$$\begin{aligned} a^{r_1\ldots r_u}_{t_1\ldots t_v\lessdot mn\gtrdot} = &\sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{u-1}\sum_{\beta=2}^{u}(\Gamma^{*r_\alpha}_{mp}\Gamma^{*r_\beta}_{[ns]} + \Gamma^{*r_\alpha}_{[np]}\Gamma^{*r_\beta}_{ms})\binom{p}{r_\alpha}\binom{s}{r_\beta}a^{\cdots}_{\cdots} - \\ &- \sum_{\alpha=1}^{u}\sum_{\beta=1}^{v}(\Gamma^{*r_\alpha}_{mp}\Gamma^{*s}_{[nt_\beta]} + \Gamma^{*r_\alpha}_{[np]}\Gamma^{*s}_{mt_\beta})\binom{p}{r_\alpha}\binom{t_\beta}{s}a^{\cdots}_{\cdots} + \sum_{\substack{\alpha=1\\(\alpha<\beta)}}^{v-1}\sum_{\beta=1}^{v}(\Gamma^{*p}_{mt_\alpha}\Gamma^{*s}_{[nt_\beta]} + \Gamma^{*p}_{[nt_\alpha]}\Gamma^{*s}_{mt_\beta})\binom{t_\alpha}{p}\binom{t_\beta}{s}a^{\cdots}_{\cdots}\ . \end{aligned} \tag{74}$$

9. In the next difference, we got:

$$a^i_{j\underset{2}{|}mn} - a^i_{j\underset{2}{|}n\underset{1}{|}m} = \underset{13}{A}{}^i_{pmn}a^p_j - \underset{14}{A}{}^p_{jmn}a^i_p - 2a^i_{j<mn>} + 2a^i_{j\lessdot\underline{\underline{nm}}\gtrdot} + a^i_{,\dot{s}}\underset{1}{K}{}^s_{rmn}\dot{x}^r + M^{*p}_{nm}a^i_{j\underset{2}{|}p}\ , \tag{75}$$

where

$$\underset{13}{A}{}^i_{pmn} = \widetilde{\Gamma}^{*i}_{mp,n} - \widetilde{\Gamma}^{*i}_{np,m} + \widetilde{\Gamma}^{*s}_{mp}\widetilde{\Gamma}^{*i}_{ns} - \widetilde{\Gamma}^{*s}_{np}\Gamma^{*i}_{sm} - \widetilde{\Gamma}^{*i}_{mp,\dot{s}}G^s_{,\dot{n}} + \widetilde{\Gamma}^{*i}_{np,\dot{s}}G^s_{,\dot{m}}\ , \tag{76}$$

$$\underset{14}{A}{}^i_{pmn} = \widetilde{\Gamma}^{*i}_{mp,n} - \widetilde{\Gamma}^{*i}_{np,m} + \Gamma^{*s}_{pm}\widetilde{\Gamma}^{*i}_{ns} - \widetilde{\Gamma}^{*s}_{np}\widetilde{\Gamma}^{*i}_{ms} - \widetilde{\Gamma}^{*i}_{mp,\dot{s}}G^s_{,\dot{n}} + \widetilde{\Gamma}^{*i}_{np,\dot{s}}G^s_{,\dot{m}}\ . \tag{77}$$

Theorem 2.9. *In $\mathbb{GF_N}$, for h-differentiation, there is the 9^{th} Ricci type identity:*

$$\begin{aligned} a^{\cdots}_{\cdots\underset{2}{|}mn} - a^{\cdots}_{\cdots\underset{2}{|}n\underset{1}{|}m} = \sum_{\alpha=1}^{u} \underset{13}{A}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha} a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} \underset{14}{A}{}^{p}_{t_\beta mn}\binom{t_\beta}{p} a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^r - \\ - 2a^{\cdots}_{\cdots<mn>} + 2a^{\cdots}_{\cdots\leqslant nm\geqslant} + M^{*p}_{nm} a^{i}_{j\underset{2}{|}p}\ , \end{aligned} \tag{78}$$

where $\underset{13}{A}$ and $\underset{14}{A}$ are given by equations (76, 77).

10. At last, we considered the difference:

$$a^{i}_{j\underset{1}{|}m\underset{2}{|}n} - a^{i}_{j\underset{2}{|}n\underset{1}{|}m} = \underset{15}{A}{}^{i}_{pmn}a^{p}_{j} - \underset{15}{A}{}^{p}_{jmn}a^{i}_{p} + a^{i}_{,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^r + \Gamma^{*p}_{nm}a^{i}_{j\underset{2}{|}p} - \widetilde{\Gamma}^{*p}_{nm}a^{i}_{j\underset{1}{|}p}\ , \tag{79}$$

where

$$\underset{15}{A}{}^{i}_{pmn} = \Gamma^{*i}_{pm,n} - \widetilde{\Gamma}^{*i}_{np,m} + \Gamma^{*s}_{pm}\widetilde{\Gamma}^{*i}_{ns} - \widetilde{\Gamma}^{*s}_{np}\Gamma^{*i}_{sm} - \Gamma^{*i}_{pm,\dot{s}}G^{s}_{,\dot{n}} + \widetilde{\Gamma}^{*i}_{np,\dot{s}}G^{s}_{,\dot{m}}\ . \tag{80}$$

Theorem 2.10. *In $\mathbb{GF_N}$, for h-differentiation, the 10^{th} Ricci type identity is valid:*

$$a^{\cdots}_{\cdots\underset{1}{|}m\underset{2}{|}n} - a^{\cdots}_{\cdots\underset{2}{|}n\underset{1}{|}m} = \sum_{\alpha=1}^{u} \underset{15}{A}{}^{r_\alpha}_{pmn}\binom{p}{r_\alpha} a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} \underset{15}{A}{}^{p}_{t_\beta mn}\binom{t_\beta}{p} a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^r - \widetilde{\Gamma}^{*p}_{nm}a^{\cdots}_{\cdots\underset{1}{|}p} - \Gamma^{*p}_{nm}a^{\cdots}_{\cdots\underset{2}{|}p}\ , \tag{81}$$

where $\underset{15}{A}$ is given by equation (80).

The identity (2.64) can be written in another form:

$$a^{r_1\ldots r_u}_{t_1\ldots t_v\underset{1}{|}m\underset{2}{|}n} - a^{r_1\ldots r_u}_{t_1\ldots t_v\underset{2}{|}n\underset{1}{|}m} = \sum_{\alpha=1}^{u} \underset{3}{K}{}^{r_\alpha}{}_{pmn}\binom{p}{r_\alpha} a^{\cdots}_{\cdots} - \sum_{\beta=1}^{v} \underset{3}{K}{}^{p}{}_{t_\beta mn}\binom{t_\beta}{p} a^{\cdots}_{\cdots} - a^{\cdots}_{\cdots,\dot{s}}\underset{1}{K}{}^{s}{}_{rmn}\dot{x}^r\ ,$$

where

$$\begin{aligned} \underset{3}{K}{}^{i}{}_{jmn} = \underset{15}{A}{}^{i}{}_{jmn} + \Gamma^{*p}_{nm}T^{*i}_{pj} = \Gamma^{*i}_{jm,n} - \Gamma^{*i}_{nj,m} + \Gamma^{*p}_{jm}\Gamma^{*i}_{np} - \Gamma^{*p}_{nj}\Gamma^{*i}_{pm} + \Gamma^{*p}_{nm}(\Gamma^{*i}_{pj} - \Gamma^{*i}_{jp}) + \\ + P^{*p}_{jm,\dot{s}}\xi^{s}_{,n} - \Gamma^{*p}_{nj,\dot{s}}\xi^{s}_{,m}\ . \end{aligned} \tag{82}$$

3. Conclusion

Based on non-symmetry of basic tensor in generalized Finsler space, using h-differentiation, we defined two kinds of covariant derivative of a tensor in Rund's sense and obtained ten Ricci type identities, two new curvature tensors and fifteen magnitudes, we called "pseudotensors".

Apart from attempt at a theoretical unification of gravitation and electromagnetic phenomena in a single geometrical framework, Finsler spaces were also considered either as formal propositions of new theoretical structures and field equations, or more directly concerned with exploring possible observational consequences.

Finsler geometry does have many fields of applications, besides geometrical extensions of theories of gravity. Also, computer algebra can be very helpful to give Finsler's expressions from a chosen metric or connection.

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