Filomat 29:9 (2015), 1941–1951
DOI 10.2298/FIL1509941S

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Some Relations in Non-Symmetric Affine Connection Spaces with Regard to a Special almost Geodesic Mappings of the Third Type

Mića S. Stanković[a], Nenad O. Vesić[a]

[a]*Faculty of Science and Mathematics Niš*

Abstract. We investigate two kinds of special almost geodesic mappings of the third type in this paper. We also find some relations for curvature tensors of almost geodesic mappings of the third type.

1. Introduction

A lot of research papers and monographs [1]-[30] are dedicated to the theory of geodesic mappings of Riemannian spaces, affine connected ones and their generalizations. Sinyukov [22] and Mikeš [1], [2] [12], [13], [29] gave some other significant contributions to the study of almost geodesic mappings of affine connected spaces.

Let GA_N be an N-dimensional space with an affine connection L given with the aid of components L^i_{jk} in each local map V on a differentiable manifold. Generally, it is $L^i_{jk} \neq L^i_{kj}$.

Generalizing conception of a geodesic mappings for Riemannian and affine connected spaces, Sinyukov introduced [22] the following notations:

A curve $l : x^h = x^h(t)$ is an almost geodesic line if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$\overline{\lambda}^h_{(2)} = \overline{a}(t)\lambda^h + \overline{b}(t)\overline{\lambda}^h_{(1)}, \quad \overline{\lambda}^h_{(1)} = \lambda^h_{||\alpha}\lambda^\alpha, \quad \overline{\lambda}^h_{(2)} = \overline{\lambda}^h_{(1)||\alpha}\lambda^\alpha,$$

where $\overline{a}(t)$ and $\overline{b}(t)$ are functions of a parameter t, and $||$ denotes the covariant derivative with respect to the connection in $\overline{A}_N$.

A mapping f of an affine connected space A_N onto a space $\overline{A}_N$ is an almost geodesic mapping if it any geodesic line of the space A_N turns into an almost geodesic line of the space $\overline{A}_N$.

Sinyukov [22] singled out the three types, π_1, π_2, π_3, of almost geodesic mappings for spaces without torsion. We investigate mappings of the type π_3 for spaces with torsion in the present article. In the case of a differentiable manifold with non-symmetric affine connection L^i_{jk}, there exist two kinds of the covariant derivative of a vector λ^h defined as follows:

$$\lambda^h_{\underset{1}{|}m} = \lambda^h_{,m} + L^h_{pm}\lambda^p, \quad \lambda^h_{\underset{2}{|}m} = \lambda^h_{,m} + L^h_{mp}\lambda^p.$$

2010 *Mathematics Subject Classification*. Primary 53B05; Secondary 53B20, 53C15
Keywords. Almost geodesic mappings, affine connected space, almost geodesic mappings of the third type, property of reciprocity
Received: 15 March 2014; Accepted: 16 June 2014
Communicated by Ljubica Velimirović
Research supported by project 174012 of Serbian Ministry of Education, Science and Technological Development
Email addresses: stmica@ptt.rs (Mića S. Stanković), vesko1985@pmf.ni.ac.rs (Nenad O. Vesić)

For this reason, in the case of a space with non-symmetric affine connection we can define two kinds of almost geodesic lines on this space and we can also define two kinds of almost geodesic mappings between two spaces.

There are five linearly independent curvature tensors into space GA_N [16]. We are specially interested in the following ones [30] in this paper:

$$\begin{aligned}
\underset{1}{K}{}^h_{ijk} =& L^h_{ij,k} - L^h_{ik,j} + L^\alpha_{ij}L^h_{\alpha k} - L^\alpha_{ik}L^h_{\alpha j};\\
\underset{2}{K}{}^h_{ijk} =& \frac{1}{2}\left(L^h_{ij,k} - L^h_{ik,j} + L^h_{ji,k} - L^h_{ki,j} + L^\alpha_{ij}L^h_{\alpha k} - L^\alpha_{ik}L^i_{j\alpha} - L^\alpha_{ki}L^h_{\alpha j}\right);\\
\underset{3}{K}{}^h_{ijk} =& L^h_{ij,k} - L^h_{ki,j} + L^\alpha_{ij}L^i_{k\alpha} - L^\alpha_{ki}L^i_{\alpha j} + 2L^\alpha_{kj}L^h_{\underset{\vee}{\alpha i}};\\
\underset{4}{K}{}^h_{ijk} =& \frac{1}{2}\left(L^h_{ij,k} - L^h_{ik,j} + L^h_{ji,k} - L^h_{ki,j} + L^\alpha_{ji}L^h_{\alpha k} + L^\alpha_{ij}L^h_{k\alpha} - L^\alpha_{ik}L^h_{\alpha j} - L^\alpha_{ki}L^h_{j\alpha}\right);\\
\underset{5}{K}{}^h_{ijk} =& \frac{1}{2}\left(L^h_{ij,k} - L^h_{ik,j} + L^h_{ji,k} - L^h_{ki,j} + L^\alpha_{\underline{ij}}L^h_{k\alpha} - L^\alpha_{\underline{ki}}L^h_{j\alpha} + L^\alpha_{\underline{ji}}L^h_{\alpha k} - L^\alpha_{\underline{ki}}L^h_{\alpha j} + L^\alpha_{\underset{\vee}{kj}}L^h_{\underset{\vee}{\alpha i}}\right),
\end{aligned} \tag{1.1}$$

where "," denotes prime derivative of a magnitude (affine connection coefficients in our case).

Curvature tensors (1.1) may be expressed in the form

$$\begin{aligned}
\underset{1}{K}{}^h_{ijk} &= R^h_{ijk} + L^h_{\underset{\vee}{ij};k} - L^h_{\underset{\vee}{ik};j} + L^\alpha_{\underset{\vee}{ij}}L^h_{\underset{\vee}{\alpha k}} - L^\alpha_{\underset{\vee}{ik}}L^h_{\underset{\vee}{\alpha j}};\\
\underset{2}{K}{}^h_{ijk} &= R^h_{ijk} - L^\alpha_{\underset{\vee}{ij}}L^h_{\underset{\vee}{\alpha k}} + L^\alpha_{\underset{\vee}{ik}}L^h_{\underset{\vee}{\alpha j}};\\
\underset{3}{K}{}^h_{ijk} &= R^h_{ijk} + L^h_{\underset{\vee}{ij};k} + L^h_{\underset{\vee}{ik};j} - L^\alpha_{\underset{\vee}{ij}}L^h_{\underset{\vee}{\alpha k}} + L^\alpha_{\underset{\vee}{ik}}L^h_{\underset{\vee}{\alpha j}} - 2L^\alpha_{\underset{\vee}{jk}}L^h_{\underset{\vee}{\alpha i}};\\
\underset{4}{K}{}^h_{ijk} &= R^h_{ijk} - L^\alpha_{\underset{\vee}{ij}}L^h_{\underset{\vee}{\alpha k}} - L^\alpha_{\underset{\vee}{ik}}L^h_{\underset{\vee}{\alpha j}};\\
\underset{5}{K}{}^h_{ijk} &= R^h_{ijk} - \frac{1}{2}L^\alpha_{\underset{\vee}{jk}}L^h_{\underset{\vee}{\alpha i}}.
\end{aligned} \tag{1.2}$$

where

$$R^h_{ijk} = L^h_{\underline{ij},k} - L^h_{\underline{ik},j} + L^\alpha_{\underline{ij}}L^h_{\underline{\alpha k}} - L^\alpha_{\underline{ik}}L^h_{\underline{\alpha j}} \tag{1.3}$$

is Riemannnian-Christofel curvature tensor and ";" denotes the covariant derivative of a magnitude with respect to the affine connection of the associated space $\mathbb{A}_N$ of the space $\mathbb{GA}_N$.

2. Almost geodesic mappings of non-symmetric affine connected spaces

One can define four kinds of a covariant derivative [14], [15] in the space GA_N with non-symmetric affine connection L^i_{jk}. These four kinds of covariant derivative become two ones in the case of a vector λ^h.

Signify by $\underset{\theta}{|}, \underset{\theta}{\|}$ a covariant derivative of the kind θ $(\theta = 1, 2)$ in GA_N and $G\overline{A}_N$ respectively. For example, for a tensor a^i_j in GA_N we have

$$a^i_{j\underset{1}{|}m} = a^i_{j,m} + L^i_{\alpha m}a^\alpha_j - L^\alpha_{jm}a^i_\alpha \quad \text{and} \quad a^i_{j\underset{2}{|}m} = a^i_{j,m} + L^i_{m\alpha}a^\alpha_j - L^\alpha_{mj}a^i_\alpha.$$

A curve $\ell = \ell(t)$ is the first kind almost geodesic line if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$\underset{1}{\overline{\lambda}}{}^h_{(2)} = \underset{1}{\overline{a}}(t)\lambda^h + \underset{1}{\overline{b}}(t)\underset{1}{\overline{\lambda}}{}^h_{(1)}, \quad \underset{1}{\overline{\lambda}}{}^h_{(1)} = \lambda^h_{\underset{1}{\|}\alpha}\lambda^\alpha, \quad \underset{1}{\overline{\lambda}}{}^h_{(2)} = \underset{1}{\overline{\lambda}}{}^h_{(1)\underset{1}{\|}\alpha}\lambda^\alpha, \tag{2.1}$$

where $\underset{1}{\overline{a}}(t)$ and $\underset{1}{\overline{b}}(t)$ are functions of a parameter t. This curve is the second kind almost geodesic line if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$\underset{2}{\overline{\lambda}}{}^h_{(2)} = \underset{2}{\overline{a}}(t)\lambda^h + \underset{2}{\overline{b}}(t)\underset{2}{\overline{\lambda}}{}^h_{(1)}, \quad \underset{2}{\overline{\lambda}}{}^h_{(1)} = \lambda^h_{\underset{2}{\|}\alpha}\lambda^\alpha, \quad \underset{2}{\overline{\lambda}}{}^h_{(2)} = \underset{2}{\overline{\lambda}}{}^h_{(1)\underset{2}{\|}\alpha}\lambda^\alpha, \tag{2.2}$$

where $\underset{2}{\overline{a}}(t)$ and $\underset{2}{\overline{b}}(t)$ are functions of a parameter t.

A mapping f of the space GA_N onto a space with non-symmetric affine connection $G\overline{A}_N$ is the first kind almost geodesic mapping if any geodesic line of the space GA_N turns into the first kind almost geodesic line of the space $G\overline{A}_N$. This mapping is the second kind almost geodesic mapping if any geodesic line of the space GA_N turns into the second kind almost geodesic line of the space $G\overline{A}_N$.

We put

$$P^h_{ij} = \overline{L}^h_{ij}(x) - L^h_{ij}(x), \tag{2.3}$$

where $L^h_{ij}(x)$ and $\overline{L}^h_{ij}(x)$ are connection coefficients of the space GA_N and $G\overline{A}_N$, $(N > 2)$. Magnitude P^h_{ij} is deformation tensor. The following theorems are satisfied [24]–[27]:

Theorem 2.1. *A mapping f of the space GA_N onto $G\overline{A}_N$ is an almost geodesic mapping of the first kind if and only if the deformation tensor P^h_{ij} satisfies the conditions*

$$(P^h_{\alpha\beta\underset{1}{|}\gamma} + P^h_{\delta\alpha}P^\delta_{\beta\gamma})\lambda^\alpha\lambda^\beta\lambda^\gamma = \underset{1}{b}P^h_{\alpha\beta}\lambda^\alpha\lambda^\beta + \underset{1}{a}\lambda^h \tag{2.4}$$

identically, where $\underset{1}{a}$ and $\underset{1}{b}$ are invariants.

Analogously it is satisfied the next one:

Theorem 2.2. *A mapping f of the space GA_N onto $G\overline{A}_N$ is an almost geodesic mapping of the second kind if and only if the deformation tensor P^h_{ij} satisfies the conditions*

$$(P^h_{\alpha\beta\underset{2}{|}\gamma} + P^h_{\alpha\delta}P^\delta_{\beta\gamma})\lambda^\alpha\lambda^\beta\lambda^\gamma = \underset{2}{b}P^h_{\alpha\beta}\lambda^\alpha\lambda^\beta + \underset{2}{a}\lambda^h \tag{2.5}$$

identically, where $\underset{1}{a}$ and $\underset{1}{b}$ are invariant.

Almost geodesic mappings of the space A_N without torsion are investigated by Sinyukov in [22].

The third type almost geodesic mappings of the first kind is determined by a condition for the function $\underset{1}{b}(x;\lambda)$ from the equation (2.4):

$$\underset{1}{b} = \frac{\underset{1}{b}_{\alpha\beta\gamma}\lambda^\alpha\lambda^\beta\lambda^\gamma}{\sigma_{\varepsilon\delta}\lambda^\varepsilon\lambda^\delta}, \tag{2.6}$$

where $\sigma_{\varepsilon\delta}\lambda^\varepsilon\lambda^\delta \neq 0$. Then, deformation tensor P^h_{ij} satisfies the following equation [25]

$$P^h_{ij}(x) = \psi_i\delta^h_j + \psi_j\delta^h_i + \sigma_{ij}(x)\varphi^h(x) + \xi^h_{ij}(x). \tag{2.7}$$

Magnitude ξ^h_{ij} in the equation (2.7) is an anti-symmetric tensor of the type $\binom{1}{2}$. From (2.7) and (2.4) we have

$$(\varphi^h_{\underset{1}{|}\gamma} + \xi^h_{\varepsilon\gamma}\varphi^\varepsilon)\lambda^\gamma = (\nu_\gamma\varphi^h + \mu\delta^h_\gamma)\lambda^\gamma , \tag{2.8}$$

wherefrom

$$\varphi^h_{\underset{1}{|}m} + \xi^h_{\varepsilon m}\varphi^\varepsilon = \nu_m\varphi^h + \mu\delta^h_m . \tag{2.9}$$

In the equation (2.9), magnitude ν_m is a covariant vector, magnitude μ is an invariant one and magnitude ξ^h_{ij} is an anti-symmetric tensor.

The equations (2.7) and (2.9) characterize the third type almost geodesic mapping of the first kind. We denote that mapping as $\underset{1}{\pi}_3$.

Almost geodesic mapping of the second kind of a space $\mathbb{G}\mathbb{A}_N$ into a space $\mathbb{G}\overline{\mathbb{A}}_N$ is *the third kind* $\underset{2}{\pi}_3$ if the function $\underset{2}{b}(x;\ \lambda)$ in (2.5) has the form

$$\underset{2}{b} = \frac{\underset{2}{b}_{\alpha\beta\gamma}\lambda^\alpha\lambda^\beta\lambda^\gamma}{\sigma_{\varepsilon\delta}\lambda^\varepsilon\lambda^\delta}, \tag{2.10}$$

where is $\sigma_{\varepsilon\delta}\lambda^\varepsilon\lambda^\delta \neq 0$. Deformation tensor P^h_{ij} has the form (2.5) in this case. Analogously as in the case of $\underset{1}{\pi}_3$, we obtain it holds

$$\varphi^h_{\underset{2}{|}m} + \xi^h_{m\varepsilon}\varphi^\varepsilon = \nu_m\varphi^h + \mu\delta^h_m . \tag{2.11}$$

In this case, as in the previous one, a magnitude ν_m is a covariant vector, magnitude μ is an invariant one and magnitude ξ^h_{ij} is an antisymmetric tensor.

The equations (2.7) and (2.11) characterize the third type almost geodesic mapping of the first kind. That mapping we denote as $\underset{1}{\pi}_3$.

Let a mapping $\underset{1}{\pi}_3 :\ GA_N \to G\overline{A}_N$ has *the property of reciprocity*, i.e. let its inverse mapping be a mapping of the type $\underset{1}{\widetilde{\pi}}_3$ type also. Then, it is satisfied

$$\varphi^h_{\underset{1}{\|}m} - \xi^h_{\alpha m}\varphi^\alpha = \tilde{\nu}_m\varphi^h + \tilde{\mu}\delta^h_m , \tag{2.12}$$

where $\tilde{\nu}_m$ is a vector and $\tilde{\mu}$ is an invariant.

From this equation (see [25]) it follows the condition

$$\xi^h_{\alpha m}\varphi^\alpha = \theta_m\varphi^h + \overline{\rho}\delta^h_m , \tag{2.13}$$

where

$$\theta_m = \nu_m - \tilde{\nu}_m + \sigma_{\alpha m}\varphi^\alpha + \psi_m, \qquad \overline{\rho} = \mu - \tilde{\mu} + \psi_\alpha\varphi^\alpha.$$

Magnitude $\overline{\rho}$ in this equation is an invariant, and magnitude θ_m is a vector.

Suppose the conditions (2.13) are satisfied identically with respect to φ^h. Then, there exists a special class of almost geodesic mappings of the type $\underset{1}{\widetilde{\pi}}_3$. Basic equations which characterize mappings from this class [25] has the form

$$\overline{L}^h_{ij} = L^h_{ij} + \psi_i\delta^h_j + \psi_j\delta^h_i + \sigma_{ij}\varphi^h + \theta_j\delta^h_i - \theta_i\delta^h_j , \tag{2.14}$$

$$\varphi^h_{\underset{1}{|}m} = \eta_m\varphi^h + \rho\delta^i_m , \tag{2.15}$$

where magnitudes θ_i, η_i are vectors and magnitude ρ is an invariant.

Basic equations which characterize almost geodesic mappings of the type $\underset{2}{\widetilde{\pi}}_3$ are

$$\overline{L}^h_{ij} = L^h_{ij} + \psi_i\delta^h_j + \psi_j\delta^h_i + \sigma_{ij}\varphi^h + \theta_j\delta^h_i - \theta_i\delta^h_j, \tag{2.16}$$

$$\varphi^h_{\underset{2}{|}m} = \eta_m\varphi^h + \rho\delta^i_m, \tag{2.17}$$

where θ_i, η_i are vectors and ρ is an invariant.

3. Curvature tensors and $\widetilde{\pi}_3$-mappings

Our purpose is discovering change formulas of curvature tensors under the almost geodesic mappings of the types $\underset{1}{\widetilde{\pi}}_3$ and $\underset{2}{\widetilde{\pi}}_3$. Let prove the following propositions at the start.

Proposition 3.1. *Covariant derivative $\varphi^h_{;k}$ of a vector φ^h from the space $\mathbb{GA}_N$ based on the connection of the associated space $\mathbb{A}_N$, in the case of $\varphi^h_{\underset{1}{|}k}$ is*

$$\varphi^h_{;k} = \rho\delta^h_k + \eta_k\varphi^h - L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha. \tag{3.1}$$

Covariant derivative $\varphi^h_{;k}$ of a vector φ^h from the space $\mathbb{GA}_N$ based on the connection of the associated space $\mathbb{A}_N$, in the case of $\varphi^h_{\underset{2}{|}k}$ is

$$\varphi^h_{;k} = \rho\delta^h_k + \eta_k\varphi^h + L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha. \tag{3.2}$$

Magnitudes ρ, η_i and φ^i in the equations (3.1), (3.2) *are an invariant, a covariant vector and a contravariant one, respectively.*

Proof. Validity of this proposition holds directly from the equations (2.15), (2.16) and the definition of the covariant derivatives of the first and the second kind. □

Proposition 3.2. *Let $f : \mathbb{GA}_N \to \mathbb{G\overline{A}}_N$ be an almost geodesic mapping of the third type between spaces $\mathbb{GA}_N$ and $\mathbb{G\overline{A}}_N$. Affine connection coefficients L^h_{ij} and $\overline{L}^h_{ij}$ of these spaces satisfy the equation*

$$\overline{L}^h_{ij;k} = L^h_{ij;k} + \sigma_{ij;k}\varphi^h + \sigma_{ij}\eta_k\varphi^h - \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha + \psi_{i;k}\delta^h_j + \psi_{j;k}\delta^h_i + \sigma_{ij}\rho\delta^h_k + \theta_{j;k}\delta^h_i - \theta_{i;k}\delta^h_j. \tag{3.3}$$

Proof. Covariant derivative of the equation (2.14) by x^k with respect to the space $\mathbb{A}_N$ returns

$$\overline{L}^h_{ij;k} = L^h_{ij;k} + \psi_{i;k}\delta^h_j + \psi_{j;k}\delta^h_i + \sigma_{ij;k}\varphi^h + \sigma_{ij}\varphi^h_{;k} + \theta_{j;k}\delta^h_i - \theta_{i;k}\delta^h_j.$$

If we interchange the result (3.1) in this equation, we obtain

$$\overline{L}^h_{ij;k} = L^h_{ij;k} + \psi_{i;k}\delta^h_j + \psi_{j;k}\delta^h_i + \sigma_{ij;k}\varphi^h + \sigma_{ij}\rho\delta^h_k + \sigma_{ij}\eta_k\varphi^h - \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha + \theta_{j;k}\delta^h_i - \theta_{i;k}\delta^h_j,$$

which proves this proposition. □

Proposition 3.3. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an almost geodesic mapping of the third type. Affine connection coefficients L^h_{ij} and $\overline{L}^h_{ij}$ of these spaces satisfy the equation*

$$\overline{L}^{\alpha}_{\underset{\vee}{ij}}\overline{L}^{h}_{\underset{\vee}{\alpha k}} = L^{\alpha}_{\underset{\vee}{ij}}L^{h}_{\underset{\vee}{\alpha k}} + L^{h}_{\underset{\vee}{ij}}\theta_k + L^{h}_{ik}\theta_j + \theta_j\theta_k\delta^h_i - L^{h}_{\underset{\vee}{jk}}\theta_i - L^{\alpha}_{\underset{\vee}{ij}}\theta_\alpha\delta^h_k - \theta_i\theta_k\delta^h_j. \tag{3.4}$$

Proof. From the equation (2.14) we get

$$\overline{L}^h_{\underset{\vee}{ij}} = L^h_{\underset{\vee}{ij}} + \theta_j\delta^h_i - \theta_i\delta^h_j.$$

Based on the this result, we obtain

$$\begin{aligned}\overline{L}^{\alpha}_{\underset{\vee}{ij}}\overline{L}^{h}_{\underset{\vee}{\alpha k}} =& L^{\alpha}_{\underset{\vee}{ij}}L^{h}_{\underset{\vee}{\alpha k}} + L^{\alpha}_{\underset{\vee}{ij}}\theta_k\delta^h_\alpha - L^{\alpha}_{\underset{\vee}{ij}}\theta_\alpha\delta^h_k + L^{h}_{\underset{\vee}{\alpha k}}\theta_j\delta^\alpha_i + \theta_j\theta_k\delta^h_i - \theta_j\theta_\alpha\delta^\alpha_i\delta^h_k - L^{h}_{\underset{\vee}{\alpha k}}\theta_i\delta^\alpha_j - \theta_i\theta_k\delta^h_j + \theta_i\theta_\alpha\delta^\alpha_j\delta^h_k\\ =& L^{\alpha}_{\underset{\vee}{ij}}L^{h}_{\underset{\vee}{\alpha k}} + L^{h}_{\underset{\vee}{ij}}\theta_k - L^{\alpha}_{\underset{\vee}{ij}}\theta_\alpha\delta^h_k + L^{h}_{\underset{\vee}{ik}}\theta_j + \theta_j\theta_k\delta^h_i - \theta_j\theta_i\delta^h_k - L^{h}_{\underset{\vee}{jk}}\theta_i - \theta_i\theta_k\delta^h_j + \theta_i\theta_j\delta^h_k\end{aligned}$$

which, after ordering of this equation, proves the proposition. □

The following lemma is useful in our research presented below.

Lemma 3.4. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an almost geodesic mapping of the type $\underset{1}{\widetilde{\pi}}_3$. Riemann-Christoffel curvature tensor defined with respect to the connection coefficients $L^h_{\underline{ij}} = \frac{1}{2}\left(L^h_{ij} + L^h_{ji}\right)$*

$$R^h_{ijk} = L^h_{\underline{ij},k} - L^h_{\underline{ik},j} + L^\alpha_{\underline{ij}}L^h_{\underline{\alpha k}} - L^\alpha_{\underline{ik}}L^h_{\underline{\alpha j}} \tag{3.5}$$

of the associated space $\mathbb{A}_N$ and the corresponding one $\overline{R}^h_{ijk}$ of the space $\overline{\mathbb{A}}_N$ satisfy the equation

$$\begin{aligned}\overline{R}^h_{ijk} =& R^h_{ijk} + \left(\psi_{i;j} - \psi_i\psi_j - \sigma_{ij}(\rho + \varphi^\alpha\psi_\alpha)\right)\delta^h_k - (\psi_{i;k} - \psi_i\psi_k - \sigma_{ik}(\rho + \varphi^\alpha\psi_\alpha))\,\delta^h_j + \left(\psi_{j;k} - \psi_{k;j}\right)\delta^h_i\\ &+ \left(\sigma_{ij;k} - \sigma_{ik;j} + \eta_k\sigma_{ij} - \eta_j\sigma_{ik} + \sigma_{ij}\sigma_{kp}\varphi^p - \sigma_{ik}\sigma_{jp}\varphi^p\right)\varphi^h - \sigma_{ij}L^h_{\underset{\vee}{pk}}\varphi^p + \sigma_{ik}L^h_{\underset{\vee}{pj}}\varphi^p.\end{aligned} \tag{3.6}$$

Proof. With respect to the equation (2.14) we have it holds

$$\overline{L}^h_{\underline{ij}} = L^h_{\underline{ij}} + \psi_i\delta^h_j + \psi_j\delta^h_i + \sigma_{ij}\varphi^h$$

and

$$\overline{L}^h_{\underline{ij},k} = L^h_{\underline{ij},k} + \psi_{i,k}\delta^h_j + \psi_{i,j}\delta^h_k + \sigma_{ij,k}\varphi^h + \sigma_{ij}\varphi^h_{,k},$$

which causes the equation

$$\overline{L}^h_{\underline{ij},k} - \overline{L}^h_{\underline{ik},j} = L^h_{\underline{ij},k} - L^h_{\underline{ik},j} + \psi_{i,k}\delta^h_j - \psi_{i,j}\delta^h_k + (\psi_{j,k} - \psi_{k,j})\delta^h_i + (\sigma_{ij,k} - \sigma_{ik,j})\varphi^h + \sigma_{ij}\varphi^h_{,k} - \sigma_{ik}\varphi^h_{,j}.$$

which proves this lemma. □

If we denote expressions in brackets from the equation (3.6) as

$$\begin{aligned}\psi_{ij} &= \psi_{i;j} - \psi_i\psi_j - \sigma_{ij}(\rho + \varphi^\alpha\psi_\alpha)\\ \sigma_{ijk} &= \sigma_{ij;k} - \sigma_{ik;j} + \sigma_{ij}\eta_k - \sigma_{ik}\eta_j + \sigma_{ij}\sigma_{k\alpha}\varphi^\alpha - \sigma_{ik}\sigma_{j\alpha}\varphi^\alpha,\end{aligned} \tag{3.7}$$

this equation becomes

$$\overline{R}^h_{ijk} = R^h_{ijk} + \psi_{ij}\delta^h_k - \psi_{ik}\delta^h_j - \psi_{[jk]}\delta^h_i - \sigma_{ijk}\varphi^h - \sigma_{ij}\underset{\vee}{L^h_{\alpha k}}\varphi^\alpha + \sigma_{ik}\underset{\vee}{L^h_{\alpha j}}\varphi^\alpha. \tag{3.8}$$

Theorem 3.5. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an almost geodesic mapping of the type $\underset{1}{\widetilde{\pi}}_3$. Curvature tensors $\underset{1}{K}{}^h_{ijk}$ and $\underset{1}{\overline{K}}{}^h_{ijk}$ satisfy the equation*

$$\underset{1}{\overline{K}}{}^h_{ijk} = \underset{1}{K}{}^h_{ijk} + \underset{1}{\psi}_{ij}\delta^h_k - \underset{1}{\psi}_{ik}\delta^h_j + \underset{1}{\psi}_{[jk]}\delta^h_i + \sigma_{ijk}\varphi^h - \sigma_{ij}\underset{\vee}{L^h_{\alpha k}}\varphi^\alpha + \sigma_{ik}\underset{\vee}{L^h_{\alpha j}}\varphi^\alpha - \underset{\vee}{L^\alpha_{ij}}\theta_\alpha\delta^h_k + \underset{\vee}{L^\alpha_{ik}}\theta_\alpha\delta^h_j - 2\underset{\vee}{L^h_{jk}}\theta_i, \tag{3.9}$$

where

$$\underset{1}{\psi}_{ij} = \psi_{ij} + \theta_{i;j} + \theta_i\theta_j, \tag{3.10}$$

and ψ_{ik} and σ_{ijk} are given by (3.7).

Proof. Firstly, based on the equation (3.3), we obtain it holds

$$\underset{\vee}{\overline{L}^h_{ij;k}} - \underset{\vee}{\overline{L}^h_{ik;j}} = \underset{\vee}{L^h_{ij;k}} - \underset{\vee}{L^h_{ik;j}} + \left(\theta_{j;k} - \theta_{k;j}\right)\delta^h_i - \theta_{i;k}\delta^h_j + \theta_{i;j}\delta^h_k.$$

Based on Proposition 3.3, we get

$$\underset{\vee}{\overline{L}^\alpha_{ij}}\underset{\vee}{\overline{L}^h_{\alpha k}} - \underset{\vee}{\overline{L}^\alpha_{ik}}\underset{\vee}{\overline{L}^h_{\alpha j}} = \underset{\vee}{L^\alpha_{ij}}\underset{\vee}{L^h_{\alpha k}} - \underset{\vee}{L^\alpha_{ik}}\underset{\vee}{L^h_{\alpha j}} + \left(\underset{\vee}{L^\alpha_{ik}}\theta_\alpha - \theta_i\theta_k\right)\delta^h_j - \left(\underset{\vee}{L^\alpha_{ij}}\theta_\alpha - \theta_i\theta_j\right)\delta^h_k - 2\underset{\vee}{L^h_{jk}}\theta_i. \tag{3.11}$$

The first of the equations in (1.2) is

$$\underset{1}{K}{}^h_{ijk} = R^h_{ijk} + \underset{\vee}{L^h_{ij;k}} - \underset{\vee}{L^h_{ik;j}} + \underset{\vee}{L^\alpha_{ij}}\underset{\vee}{L^h_{\alpha k}} - \underset{\vee}{L^\alpha_{ik}}\underset{\vee}{L^h_{\alpha j}}.$$

Based on it, after the involving of results obtained in this proof and result presented in Lemma 2.1, we prove this theorem is valid. □

Theorem 3.6. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an almost geodesic mapping of the type $\underset{1}{\widetilde{\pi}}_3$. Curvature tensors $\underset{2}{K}{}^h_{ijk}$ and $\underset{2}{\overline{K}}{}^h_{ijk}$ satisfy the equation*

$$\underset{2}{\overline{K}}{}^h_{ijk} = \underset{2}{K}{}^h_{ijk} + \underset{2}{\psi}_{ij}\delta^h_k - \underset{2}{\psi}_{ik}\delta^h_j + \underset{2}{\psi}_{[jk]}\delta^h_i + \sigma_{ijk}\varphi^h - \sigma_{ij}\underset{\vee}{L^h_{\alpha k}}\varphi^\alpha + \sigma_{ik}\underset{\vee}{L^h_{\alpha j}}\varphi^\alpha + \underset{\vee}{L^\alpha_{ij}}\theta_\alpha\delta^h_k - \underset{\vee}{L^\alpha_{ik}}\theta_\alpha\delta^h_j + 2\underset{\vee}{L^h_{jk}}\theta_i, \tag{3.12}$$

where

$$\underset{2}{\psi}_{ij} = \psi_{ij} - \theta_i\theta_j \tag{3.13}$$

and ψ_{ik} and σ_{ijk} are given by (3.7).

Proof. Based on the second of the equations (1.2) which is

$$\underset{2}{K}{}^h_{ijk} = R^h_{ijk} - \underset{\vee}{L}{}^\alpha_{ij}\underset{\vee}{L}{}^h_{\alpha k} + \underset{\vee}{L}{}^\alpha_{ik}\underset{\vee}{L}{}^h_{\alpha j},$$

the equation (3.1) and the result (3.11) we obtain it holds

$$\begin{aligned}\underset{2}{\overline{K}}{}^h_{ijk} = \underset{2}{K}{}^h_{ijk} &+ \Big(\psi_{i;j} - \psi_i\psi_j - \sigma_{ij}(\rho + \varphi^\alpha\psi_\alpha) + \underset{\vee}{L}{}^\alpha_{ij}\theta_\alpha - \theta_i\theta_j\Big)\delta^h_k\\
&- \Big(\psi_{i;k} - \psi_i\psi_k - \sigma_{ik}(\rho + \varphi^\alpha\psi_\alpha) + \underset{\vee}{L}{}^\alpha_{ik}\theta_\alpha - \theta_i\theta_k\Big)\delta^h_j\\
&+ \Big(\psi_{j;k} - \psi_{k;j}\Big)\delta^h_i + \Big(\sigma_{ij;k} - \sigma_{ik;j} + \eta_k\sigma_{ij} - \eta_j\sigma_{ik} + \sigma_{ij}\sigma_{k\alpha}\varphi^\alpha - \sigma_{ik}\sigma_{j\alpha}\varphi^\alpha\Big)\varphi^h\\
&- \sigma_{ij}\underset{\vee}{L}{}^h_{\alpha k}\varphi^\alpha + \sigma_{ik}\underset{\vee}{L}{}^h_{\alpha j}\varphi^\alpha + 2\underset{\vee}{L}{}^h_{jk}\theta_i,\end{aligned}$$

which proves this theorem. □

Theorem 3.7. *Let $f : \mathbb{GA}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an almost geodesic mapping of the type π_3. Curvature tensors $\underset{3}{K}{}^h_{ijk}$ and $\underset{3}{\overline{K}}{}^h_{ijk}$ satisfy the equation*

$$\begin{aligned}\underset{3}{\overline{K}}{}^h_{ijk} = \underset{3}{K}{}^h_{ijk} &+ \underset{3}{\psi}_{ij}\delta^h_k - \underset{3}{\psi}_{ik}\delta^h_j + \underset{3}{\psi}_{[jk]}\delta^h_i + \sigma_{ijk}\varphi^h - \sigma_{ij}\underset{\vee}{L}{}^h_{\alpha k}\varphi^\alpha + \sigma_{ik}\underset{\vee}{L}{}^h_{\alpha j}\varphi^\alpha\\
&- 2\theta_{i;j}\delta^h_k + 2\underset{\vee}{L}{}^\alpha_{jk}\theta_\alpha\delta^h_i - 2\underset{\vee}{L}{}^h_{ik}\theta_j + 2\underset{\vee}{L}{}^h_{ij}\theta_k\end{aligned} \tag{3.14}$$

where

$$\underset{3}{\psi}_{ij} = \psi_{ij} + \theta_{i;j} + \theta_i\theta_j, \tag{3.15}$$

and ψ_{ik} and σ_{ijk} are given by (3.7).

Proof. As we can conclude from the second and the third equation in (1.2), it holds

$$\underset{3}{K}{}^h_{ijk} = \underset{2}{K}{}^h_{ijk} + \underset{\vee}{L}{}^h_{ij;k} + \underset{\vee}{L}{}^h_{ik;j} - 2\underset{\vee}{L}{}^\alpha_{jk}\underset{\vee}{L}{}^h_{\alpha i}.$$

Based on the equation (3.4), we conclude it holds

$$\underset{\vee}{\overline{L}}{}^\alpha_{jk}\underset{\vee}{\overline{L}}{}^h_{\alpha i} = \underset{\vee}{L}{}^\alpha_{jk}\underset{\vee}{L}{}^h_{\alpha i} - \theta_i\theta_j\delta^h_k + \theta_i\theta_k\delta^h_j - \underset{\vee}{L}{}^\alpha_{jk}\theta_\alpha\delta^h_i + \underset{\vee}{L}{}^h_{jk}\theta_i - \underset{\vee}{L}{}^h_{ij}\theta_k + \underset{\vee}{L}{}^h_{ik}\theta_j. \tag{3.16}$$

It also holds

$$\underset{\vee}{\overline{L}}{}^h_{ij;k} + \underset{\vee}{\overline{L}}{}^h_{ik;j} = \underset{\vee}{L}{}^h_{ij;k} + \underset{\vee}{L}{}^h_{ik;j} + \Big(\theta_{j;k} + \theta_{k;j}\Big)\delta^h_i - \theta_{i;k}\delta^h_j - \theta_{i;j}\delta^h_k,$$

which combined with $\underset{2}{K}{}^h_{ijk}$ and the equation (3.16) proves this theorem. □

Theorem 3.8. *Let $f:\mathbb{G}\mathbb{A}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an almost geodesic mapping of the type π_3. Curvature tensors $\underset{4}{K}{}^h_{ijk}$ and $\underset{4}{\overline{K}}{}^h_{ijk}$ satisfy the equation*

$$\begin{aligned}\underset{4}{\overline{K}}{}^h_{ijk} &= \underset{4}{K}{}^h_{ijk} + \underset{4}{\psi}_{ij}\delta^h_k - \underset{4}{\psi}_{ik}\delta^h_j + \underset{4}{\psi}_{[jk]}\delta^h_i \\ &+ \sigma_{ijk}\varphi^h - \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha + \sigma_{ik}L^h_{\underset{\vee}{\alpha j}}\varphi^\alpha + L^\alpha_{\underset{\vee}{ij}}\theta_\alpha\delta^h_k + 2\left(L^\alpha_{\underset{\vee}{ik}}\theta_\alpha + \theta_i\theta_k\right)\delta^h_j - 2\theta_j\theta_k\delta^h_i - 2L^h_{\underset{\vee}{ij}}\theta_k - 2L^h_{\underset{\vee}{ik}}\theta_j,\end{aligned} \tag{3.17}$$

where

$$\underset{4}{\psi}_{ij} = \psi_{ij} + \theta_i\theta_j, \tag{3.18}$$

and ψ_{ij} and σ_{ijk} given by (3.7).

Proof. After the symmetrization without division of the equation (3.4) by indices j and k, we obtain

$$\overline{L}^\alpha_{\underset{\vee}{ij}}\overline{L}^h_{\underset{\vee}{\alpha k}} + \overline{L}^\alpha_{\underset{\vee}{ik}}\overline{L}^h_{\underset{\vee}{\alpha j}} = L^\alpha_{\underset{\vee}{ij}}L^h_{\underset{\vee}{\alpha k}} + L^\alpha_{\underset{\vee}{ik}}L^h_{\underset{\vee}{\alpha j}} - \left(L^\alpha_{\underset{\vee}{ik}}\theta_\alpha + \theta_i\theta_k\right)\delta^h_j - \left(L^\alpha_{\underset{\vee}{ij}}\theta_\alpha + \theta_i\theta_j\right)\delta^h_k + 2\theta_j\theta_k\delta^h_i + 2L^h_{\underset{\vee}{ij}}\theta_k + 2L^h_{\underset{\vee}{ik}}\theta_j.$$

Based on the equation

$$\underset{4}{K}{}^h_{ijk} = R^h_{ijk} - L^\alpha_{\underset{\vee}{ij}}L^h_{\underset{\vee}{\alpha k}} - L^\alpha_{\underset{\vee}{ik}}L^h_{\underset{\vee}{\alpha j}}$$

(the forth one in the equation 1.2) and the last obtained one we conclude the equation (3.17) is valid. □

After involving of (3.16) in the fifth equation in (1.2) we have the next theorem:

Theorem 3.9. *Let $f:\mathbb{G}\mathbb{A}_N \to \mathbb{G}\overline{\mathbb{A}}_N$ be an almost geodesic mapping of the type π_3. Curvature tensors $\underset{5}{K}{}^h_{ijk}$ and $\underset{5}{\overline{K}}{}^h_{ijk}$ satisfy the equation*

$$\begin{aligned}\underset{5}{\overline{K}}{}^h_{ijk} &= \underset{5}{K}{}^h_{ijk} + \underset{5}{\psi}_{ij}\delta^h_k - \underset{5}{\psi}_{ik}\delta^h_j + \underset{5}{\psi}_{[jk]}\delta^h_i + \sigma_{ijk}\varphi^h - \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha + \sigma_{ik}L^h_{\underset{\vee}{\alpha j}}\varphi^\alpha \\ &+ \frac{1}{2}L^\alpha_{\underset{\vee}{jk}}\theta_\alpha\delta^h_i - \frac{1}{2}L^h_{\underset{\vee}{jk}}\theta_i + \frac{1}{2}L^h_{\underset{\vee}{ij}}\theta_k - \frac{1}{2}L^h_{\underset{\vee}{ik}}\theta_j,\end{aligned} \tag{3.19}$$

where

$$\underset{5}{\psi}_{ij} = \psi_{ij} + \frac{1}{2}\theta_i\theta_j, \tag{3.20}$$

and ψ_{ij} and σ_{ijk} given by (3.7).

Proof. The validity of this theorem holds directly from the equation (1.2), the fifth one which is

$$\underset{5}{K}{}^h_{ijk} = R^h_{ijk} - \frac{1}{2}L^\alpha_{\underset{\vee}{jk}}L^h_{\underset{\vee}{\alpha i}},$$

and the equation (3.16). □

Based on the equation (3.2), we obtain Riemann-Christofel curvature tensor R^h_{ijk} of the associated space $\mathbb{A}_N$ of a space $\mathbb{GA}_N$ after application of an almost geodesic mapping $\underset{2}{\widetilde{\pi}}_3$ satisfies the equation

$$\begin{aligned}\overline{R}^h_{ijk} &= R^h_{ijk} + \left(\psi_{i;j} - \psi_i\psi_j - \sigma_{ij}(\rho + \varphi^\alpha\psi_\alpha)\right)\delta^h_k - \left(\psi_{i;k} - \psi_i\psi_k - \sigma_{ik}(\rho + \varphi^\alpha\psi_\alpha)\right)\delta^h_j + \left(\psi_{j;k} - \psi_{k;j}\right)\delta^h_i\\ &+ \left(\sigma_{ij;k} - \sigma_{ik;j} + \eta_k\sigma_{ij} - \eta_j\sigma_{ik} + \sigma_{ij}\sigma_{k\alpha}\varphi^\alpha - \sigma_{ik}\sigma_{j\alpha}\varphi^\alpha\right)\varphi^h + \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha - \sigma_{ik}L^h_{\underset{\vee}{\alpha j}}\varphi^\alpha.\end{aligned} \tag{3.21}$$

As result of the equation (3.21), we obtain it holds

$$\underset{1}{\overline{K}}{}^h_{ijk} = \underset{1}{K}{}^h_{ijk} + \underset{1}{\psi}_{ij}\delta^h_k - \underset{1}{\psi}_{ik}\delta^h_j + \underset{1}{\psi}_{[jk]}\delta^h_i + \sigma_{ijk}\varphi^h + \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha - \sigma_{ik}L^h_{\underset{\vee}{\alpha j}}\varphi^\alpha - L^\alpha_{\underset{\vee}{ij}}\theta_\alpha\delta^h_k + L^\alpha_{\underset{\vee}{ik}}\theta_\alpha\delta^h_j - 2L^h_{\underset{\vee}{jk}}\theta_i, \tag{3.22}$$

$$\underset{2}{\overline{K}}{}^h_{ijk} = \underset{2}{K}{}^h_{ijk} + \underset{2}{\psi}_{ij}\delta^h_k - \underset{2}{\psi}_{ik}\delta^h_j + \underset{2}{\psi}_{[jk]}\delta^h_i + \sigma_{ijk}\varphi^h + \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha - \sigma_{ik}L^h_{\underset{\vee}{\alpha j}}\varphi^\alpha + L^\alpha_{\underset{\vee}{ij}}\theta_\alpha\delta^h_k - L^\alpha_{\underset{\vee}{ik}}\theta_\alpha\delta^h_j + 2L^h_{\underset{\vee}{jk}}\theta_i, \tag{3.23}$$

$$\begin{aligned}\underset{3}{\overline{K}}{}^h_{ijk} &= \underset{3}{K}{}^h_{ijk} + \underset{3}{\psi}_{ij}\delta^h_k\\ &- \underset{3}{\psi}_{ik}\delta^h_j + \underset{3}{\psi}_{[jk]}\delta^h_i + \sigma_{ijk}\varphi^h + \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha - \sigma_{ik}L^h_{\underset{\vee}{\alpha j}}\varphi^\alpha - 2\theta_{i;j}\delta^h_k + 2L^\alpha_{\underset{\vee}{jk}}\theta_\alpha\delta^h_i - 2L^h_{\underset{\vee}{ik}}\theta_j + 2L^h_{\underset{\vee}{ij}}\theta_k\end{aligned} \tag{3.24}$$

$$\begin{aligned}\underset{4}{\overline{K}}{}^h_{ijk} &= \underset{4}{K}{}^h_{ijk} + \underset{4}{\psi}_{ij}\delta^h_k - \underset{4}{\psi}_{ik}\delta^h_j + \underset{4}{\psi}_{[jk]}\delta^h_i\\ &+ \sigma_{ijk}\varphi^h + \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha - \sigma_{ik}L^h_{\underset{\vee}{\alpha j}}\varphi^\alpha + L^\alpha_{\underset{\vee}{ij}}\theta_\alpha\delta^h_k + 2\left(L^\alpha_{\underset{\vee}{ik}}\theta_\alpha + \theta_i\theta_k\right)\delta^h_j - 2\theta_j\theta_k\delta^h_i - 2L^h_{\underset{\vee}{ij}}\theta_k - 2L^h_{\underset{\vee}{ik}}\theta_j,\end{aligned} \tag{3.25}$$

$$\begin{aligned}\underset{5}{\overline{K}}{}^h_{ijk} &= \underset{5}{K}{}^h_{ijk} + \underset{5}{\psi}_{ij}\delta^h_k\\ &- \underset{5}{\psi}_{ik}\delta^h_j + \underset{5}{\psi}_{[jk]}\delta^h_i + \sigma_{ijk}\varphi^h + \sigma_{ij}L^h_{\underset{\vee}{\alpha k}}\varphi^\alpha - \sigma_{ik}L^h_{\underset{\vee}{\alpha j}}\varphi^\alpha + \frac{1}{2}L^\alpha_{\underset{\vee}{jk}}\theta_\alpha - \frac{1}{2}L^h_{\underset{\vee}{jk}}\theta_i + \frac{1}{2}L^h_{\underset{\vee}{ij}}\theta_k - \frac{1}{2}L^h_{\underset{\vee}{ik}}\theta_j,\end{aligned} \tag{3.26}$$

for $\underset{r}{\psi}_{ij}$, ψ_{ij}, σ_{ijk} defined as above.

References

[1] Berezovski, V.E., Mikeš, J. *On the classification of almost geodesic mapings of affine connection spaces,* Acta Univ. Palacki. Olomouc, Fac. Rer. Nat. Math. 35 (1996), 21-24.

[2] Berezovski, V.E., Mikeš, J. Vanžurová, A., it Fundamental PDE's of the Canonical Almost Geodesic Mappings of Type $\tilde{\pi}_1$, Bull. Malaysian Math. Sci. Soc. (2014) (in print), .

[3] Hall, G. S., Lonie, D. P., *The principle of equivalence and projective structure in spacetimes*, Class. Quantum Grav. 24 (2007), 3617-3636.

[4] Hall, G. S., Lonie, D. P., *The principle of equivalence and cosmological metrics*, J. Math. Phys. 49, 022502 (2008).

[5] Hall, G. S., Lonie, D. P., *Projective equivalence of Einstein spaces in general relativity*, Class. Quantum Grav. 26 (2009) 125009.

[6] Hinterleitner I., *Geodesic mappings of compact Riemannian manifolds with conditions on sectional curvature*, Publ. Inst. Math. (Beograd) (N.S), 94(108) (2013), 125-130.

[7] Hinterleitner, I. and Mikeš, J., *Geodesic mappings and Einstein Spaces*, Geometric Methods in Physics, Trends in Mathematics, (2013), pp 331-335.

[8] Hinterleitner, I. and Mikeš, J., *Geodesic mappings of (pseudo-) Riemannian manifolds preserve class of differentiability*, Miskolc Mathematical Notes, Vol. 14 (2013), No. 2, pp. 575-582.

[9] Mikeš, J., *Geodesic mappings of Einstein spaces*, Math. Notes 28, 922-923 (1981); translation from Mat. Zametki 28, (1980) 935-938.

[10] Mikeš J. *Geodesic mappings of affine-connected and Riemannian spaces*, J. Math. Sci., New York 78, 3 (1996), 311-333.

[11] Mikeš, J., *Holomorphically projective mappings and their generalizations*, J. Math. Sci., New York, 89, 3 (1998), 1334-1353.

[12] Mikeš, J., Pokorná, O., Starko, G., *On almost geodesic mappings $\pi_2(e)$ onto Riemannian spaces*, Rendiconti del circolo matematico di Palermo, Serie II, Suppl.72 (2004), 151-157.
[13] Mikeš, J., Vanžurová, A., Hinterleitner, I., *Geodesic Mappings and Some Generalizations*, Olomouc, 2009.
[14] Minčić, S.M., *Ricci identities in the space of non-symmetric affine connexion*, Mat. Vesnik, 10(25) Sv. 2, (1973), 161-172.
[15] Minčić, S.M., *New commutation formulas in the non-symetric affine connexion space*, Publ. Inst. Math. (Beograd) (N.S), 22(36), (1977), 189-199.
[16] S. M. Minčić, *Independent curvature tensors and pseudotensors of spaces with non-symmetric affine connexion*, Colloquia Mathematica Societatis János Bolayai, 31, Differential Geometry, Budapest (Hungary), (1979), 445–460.
[17] Minčić, S.M., Stanković, M.S., *On geodesic mappings of general affine connexion spaces and of generalized Riemannian spaces*, Mat. Vesnik, 49, (1997), 27–33.
[18] Minčić, S.M., Stanković, M.S., *Equitorsion geodesic mapings of generalized Riemannian spaces*, Publ. Inst. Math. (Beograd) (N.S), 61 (75), (1997), 97–104.
[19] Minčić, S. M., Stanković, M. S., Velimirović, Lj. S., *Generalized Kahlerian Spaces*, Filomat 15 (2001), 167–174.
[20] S. M. Minčić, M. S. Stanković, Lj. S. Velimirović, *Generalized Riemannian spaces and spaces of non-symmetric affine connection*, Niš, 2013.
[21] Minčić, S. M., Velimirović, Lj.S., Stanković, M.S., *New integrability conditions of derivational equations of a submanifold in a generalized Riemannian space*, Filomat 24:2 (2010), 137–146.
[22] Sinyukov, N.S., *Geodesic mappings of Riemannian spaces*, "Nauka" Moscow, (1979) (in Russian).
[23] Stanković, M.S., Minčić, S.M., *New special geodesic mappings of generalized Riemannian space*, Publ. Inst. Math. (Beograd), (N.S), 67(81), (2000), 92–102.
[24] Stanković, M.S., *Almost geodesic mappings of the first type of affine spaces*, Novi Sad J. Math., 29 No. 3, (1999), 313–323.
[25] Stanković, M.S., *On a canonic almost geodesic mappings of the second type of affine spaces*, FILOMAT 13, (1999), 105-114.
[26] Stanković, M. S., Minčić, S. M., *New special geodesic mappings of generalized Riemannian space*, Publ. Inst. Math. (Beograd) (N.S), 67(81), (2000), 92–102.
[27] Stanković, M.S., *On a special almost geodesic mappings of the third type of affine spaces*, Novi Sad J. Math., 31 No. 2, (2001), 125–135.
[28] Stanković, M. S. , Minčić, S. M., Velimirović, Lj. S., *On equitorsion holomorphically projective mappings of generalized Kählerian spaces*, Czechoslovak Mathematical Journal, 54(129), (2004), 701–715.
[29] Vavříková, H., Mikeš, J., Pokorná, O., Starko, G., *On fundamental equations of almost geodesic mappings of type $\pi_2(e)$*, Iz. VUZ Matematika, Vol 51 No. 1, (2007), 8–12.
[30] M. Lj. Zlatanović, *New projective tensors for equitorsion geodesic mappings*, Applied Mathematics Letters, 25, (2012) 890–897.