# One Application of Discrete Mean Square Approximation For Prediction Of Voluntary Pension Fund Development 

Branislav Randjelovića, Ivan Radojkovićc ${ }^{\text {b }}$, Boban Gajićc<br>${ }^{a}$ University of Nis, Faculty of Electrical Engineering, Departmant of Mathematics, Nis, Serbia<br>b"Dunav" Voluntary Pension Fund, Nis, Serbia<br>c"Dunav Insurance", Belgrade, Serbia


#### Abstract

In this paper we apply discrete mean square approximation to analysis of relation between average salary in Serbia, size of parameter FONDEX and number of voluntary pension fund members. We obtain two different approximation functions, that gives relation between those values. We use those approximation functions for prediction of the number of voluntary pension fund members, we present conclusions and point to some possible ways for further research.


## 1. Introduction

Discrete mean square approximation is one of, so called, best approximations, and it is very good tool for approximation of various experimental data or data obtained by measuring. This approximation is simple, easy for calculation, and it is also very useful for prediction of behavior of observed values in a range that is not covered by experimental measuring (see [6], [7]).

Prediction of the number of voluntary pension fund members in one country and related tendencies are very important for successful operation of those funds (see [9]). In this field, values that affects most to fund members are average salary and value of parameter FONDEX. That is the main reason why we discuss relation between fund members, average salary and FONDEX, in this paper.

## 2. Theoretical background

Let function $f:(a, b) \rightarrow R$ is given with a set of pairs $\left\{\left(x_{j}, f_{j}\right)\right\}_{j=0,1, \ldots, m}$ where $f_{j} \equiv f\left(x_{j}\right)$. We can approximate this function with a linear approximation function

$$
\begin{equation*}
\varphi(x)=\sum_{i=0}^{n} a_{i} \varphi_{i}(x) \quad(n<m) \tag{1}
\end{equation*}
$$

[^0]where our demand is to minimize a norm
\[

$$
\begin{equation*}
\left\|\delta_{n}\right\|_{2}=\left\|\delta_{n}\right\|_{2, p}=\left(\sum_{j=0}^{m} p\left(x_{j}\right) \delta_{n}\left(x_{j}\right)^{2}\right)^{1 / 2} \tag{2}
\end{equation*}
$$

\]

where $p:[a, b] \rightarrow R_{+}$is a given weight function and $\delta_{n}$ is defined with (see [6])

$$
\begin{equation*}
\delta_{n}(x)=f(x)-\sum_{i=0}^{n} a_{i} \varphi_{i}(x), \tag{3}
\end{equation*}
$$

We can describe everything using matrix notation

$$
\begin{align*}
& X=\left[\begin{array}{cccc}
\varphi_{0}\left(x_{0}\right) & \varphi_{1}\left(x_{0}\right) & \ldots & \varphi_{n}\left(x_{0}\right) \\
\varphi_{0}\left(x_{1}\right) & \varphi_{1}\left(x_{1}\right) & & \varphi_{n}\left(x_{1}\right) \\
\vdots & & & \\
\varphi_{0}\left(x_{m}\right) & \varphi_{1}\left(x_{m}\right) & & \varphi_{n}\left(x_{m}\right)
\end{array}\right] \quad \vec{f}=\left[\begin{array}{c}
f_{0} \\
f_{1} \\
\vdots \\
f_{m}
\end{array}\right] \quad \vec{a}=\left[\begin{array}{c}
a_{0} \\
a_{1} \\
\vdots \\
a_{n}
\end{array}\right]  \tag{4}\\
& P=\left[\begin{array}{ccc}
p\left(x_{0}\right) & & \\
& p\left(x_{1}\right) & \\
& & \vdots \\
& & p\left(x_{m}\right)
\end{array}\right] \tag{5}
\end{align*}
$$

Square of this norm, defined with (2), can be presented in a form ([6],[7])

$$
\begin{equation*}
F=\left\|\delta_{n}^{2}\right\|=\left\|\delta_{n}^{2}\right\|_{2}=\sum_{j=0}^{m} p\left(x_{j}\right) \delta_{n}\left(x_{j}\right)^{2}=\vec{v}^{T} P \vec{v} \tag{6}
\end{equation*}
$$

We can determine best discrete mean square approximation, if we minimize value $F$, from conditions

$$
\begin{equation*}
\frac{\partial F}{\partial a_{i}}=2 \sum_{j=0}^{m} p\left(x_{j}\right) \delta_{n}\left(x_{j}\right) \frac{\partial \delta_{n}\left(x_{j}\right)}{\partial a_{i}}=0(i=0,1, \ldots, n) \tag{7}
\end{equation*}
$$

and we have, so called, normal system of equations:

$$
\begin{equation*}
\sum_{j=0}^{m} p\left(x_{j}\right) \delta_{n}\left(x_{j}\right) \varphi_{i}\left(x_{j}\right)=0 \quad(i=0,1, \ldots, n) \tag{8}
\end{equation*}
$$

to determine parameters $a_{i} \quad(i=0,1, \ldots, n)$.
In accordance with this notation, system of equations could be represented in matrix form

$$
\begin{equation*}
X^{T} P \vec{v}=\overrightarrow{0} \tag{9}
\end{equation*}
$$

i.e.

$$
\begin{equation*}
X^{T} P X \vec{a}=X^{T} P \vec{f} \tag{10}
\end{equation*}
$$

Note that system of equations (8), i.e. (10), can be obtained from system of equations represented in matrix form

$$
\begin{equation*}
X \vec{a}=\vec{f} \tag{11}
\end{equation*}
$$

by multiplication from left side with matrix $X^{T} P$.

Diagonal matrix $P$, so called "weight matrix", has role to ensure bigger weights $p_{j} \equiv p\left(x_{j}\right)$ to values of function $f_{j}$ with higher precision. This is especially important for approximation of experimental data, obtained during measuring.

Most common situation is that all weights are equal, i.e. $P$ is identity matrix of dimension $m+1$. In that case (10) is

$$
\begin{equation*}
X^{T} X \vec{a}=X^{T} \vec{f} \tag{12}
\end{equation*}
$$

Vector with coefficients $a$ can be obtained from (10) or (12). For example, from (12) we have

$$
\begin{equation*}
\vec{a}=\left(X^{T} X\right)^{-1} X^{T} \vec{f} \tag{13}
\end{equation*}
$$

In case that sytem of base functions is $\varphi_{i}(x)=x^{i} \quad(i=0,1, \ldots, n)$ then we have

$$
X=\left[\begin{array}{ccccc}
1 & x_{0} & x_{0}^{2} & \cdots & x_{0}^{n}  \tag{14}\\
1 & x_{1} & x_{1}^{2} & & x_{1}^{n} \\
\vdots & & & & \\
1 & x_{m} & x_{m}^{2} & & x_{m}^{n}
\end{array}\right]
$$

It is especially interesting case $n=1$, i.e. approximation function is $\varphi(x)=a_{0}+a_{1} x$. So system of equations (10) becomes

$$
\left[\begin{array}{ll}
s_{11} & s_{12}  \tag{15}\\
s_{21} & s_{22}
\end{array}\right] \cdot\left[\begin{array}{l}
a_{0} \\
a_{1}
\end{array}\right]=\left[\begin{array}{l}
b_{0} \\
b_{1}
\end{array}\right],
$$

where

$$
\begin{align*}
& s_{11}=\sum_{j=0}^{m} p_{j}, \quad s_{12}=s_{21}=\sum_{j=0}^{m} p_{j} x_{j}, \quad s_{22}=\sum_{j=0}^{m} p_{j} x_{j}^{2},  \tag{16}\\
& b_{0}=\sum_{j=0}^{m} p_{j} f_{j}, \quad b_{1}=\sum_{j=0}^{m} p_{j} x_{j} f_{j} . \tag{17}
\end{align*}
$$

Parameters in approximation function can be obtained from

$$
\begin{equation*}
a_{0}=\frac{1}{D}\left(s_{22} b_{0}-s_{12} b_{1}\right), \quad a_{1}=\frac{1}{D}\left(s_{11} b_{1}-s_{21} b_{0}\right) \tag{18}
\end{equation*}
$$

where $D=s_{11} s_{22}-s_{12}^{2}$. SS
Suppose that we need to obtain relationship between some values, that we call a dependent variable (we can denote it with $y$ ) and one or more values, which we call independent variables (we can denote them with $x_{1}, x_{2} \ldots, x_{n}$ ), so that that model is a linear dependence on independent variables,

$$
\begin{equation*}
y=a_{1} x_{1}+a_{2} x_{2}+\cdots+a_{n} x_{n}+b \tag{19}
\end{equation*}
$$

where $a_{1}, a_{2}, a_{n}, b$ are real numbers. If the dependence is of one variable function has form $y=a x+b$ simple linear relation, where $a, b$ are real numbers. Those type of approximation functions are easy for use in practical applications, because of simple, linear dependence of parameters.

The least squares method (discrete mean square approximation) belongs to the so-called the best approximations, ie. in approximation methods in which the criterion is error minimization according to one of the norms. Specifically, this is the norm $L^{2}$, ie. the total sum of the squares of the errors in the approximation nodes is minimized.

Most applications of such approximation functions are when the goal is simulation of real situation or prediction based on that model, obtained from a data set of values of dependent and independent quantities. When we have approximation model, then value of the dependent variable $y$ can be calculated for any new value of independent variables $x_{i}$. Goal of analysis can also be quantification of strength of relationship between the dependent variable $y$ and each of the independent variables $x_{1}, x_{2} \ldots, x_{n}$. In this paper, we use first approach. We use an approximation procedure, known as the discrete mean square approximation.

## 3. About pension funds in Serbia

The Law on Voluntary Pension Funds and Pension Plans in Serbia is adopted in September 2005 and introduced April 2006, while its first amendment was made in May 2011 - providing the legal framework for pension system reform. This law introduces, so called, third pillar of pension insurance (see [10]).

There are two pension insurance systems in Serbia - mandatory (state) and voluntary (private)(see [9]). All employed persons in Serbia are members of mandatory pension Fund. Membership in voluntary pension fund is, of course, based on free will. At the end of the fourth quarter of 2019, there were 201587 members voluntary funds, in accumulation phase. Note that membership in the fund is divided into two phases - the accumulation phase (the period in which members pay) and the withdrawal phase (period when the member withdraws the accumulated funds). The strategic goal in this area is to introduce a healthy multi-pillar pension system.

Private pensions are completely independent of state pensions and are based on the principle of personal accounts. The funds of the private pension fund are invested in financial instruments that provide portfolio optimization, ie. give the best ratio of investment risk and rate of return. Voluntary pension fund funds are invested in accordance with the following investment principles prescribed by law:

The principle of security, which is achieved by investing in securities of issuers with a high rating;
The principle of portfolio diversification, which is achieved by investing in various financial instruments (government bonds, corporate bonds, treasury bills, shares, bank deposits, mortgage bonds, etc.). By applying different quantitative methods, horizontal diversification is performed, ie the selection of specific securities within different types of instruments on offer. The most important issuers of financial instruments are the state, commercial banks, companies, and local self-government.

The principle of maintaining liquidity, which is achieved by investing in securities that can be quickly sold and bought at a stable price. The fund's goal is to have a sufficient percentage of liquid financial instruments in its portfolio to be able to meet its obligations at any time.

The members of the Fund themselves choose the Fund to which they will pay the money, the manner and amount of payment, as well as the manner of payment of the pension. There are currently four voluntary pension fund management companies operating in Serbia, which manage seven voluntary pension funds.

Yield rates of voluntary pension funds are also favorable if the exchange rate movements during the last year are taken into account. Their are given in next table.

| Company | Members | Assets | Yield (2019) |
| :--- | :---: | :---: | :---: |
| Generali Basic | 46535 | $13.075,80$ | $9,14 \%$ |
| Generali Index | 4966 | $1.095,60$ | $8,34 \%$ |
| Raiffaisen Future | 35064 | $5.459,90$ | $4,87 \%$ |
| Raiffaisen Euro Future | 4464 | 225,00 | $2,91 \%$ |
| DDOR Garant Equilibrio | 53517 | $6.050,30$ | $5,63 \%$ |
| DDOR Garant Savings | 19287 | $1.328,40$ | $7,85 \%$ |
| DUNAV | 87195 | $18.010,50$ | $6,80 \%$ |

Table 1. Number of members, assets and yield rates for voluntary pension funds operating in Serbia (Source: NBS Statistical Annex for December 2019.)

| Indicators | 2010 | 2011 | 2012 | 2013 | 2014 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Companies | 6 | 6 | 5 | 4 | 4 |
| Funds | 8 | 9 | 9 | 6 | 6 |
| Members | 166780 | 174868 | 179823 | 183508 | 187997 |
| Contracts | 220451 | 234405 | 240369 | 244462 | 252072 |
| Assets | $9.862,70$ | $12.452,30$ | $16.011,30$ | $19.007,70$ | $23.565,30$ |
| Indicators | 2015 | 2016 | 2017 | 2018 | 2019 |
| Companies | 4 | 4 | 4 | 4 | 4 |
| Funds | 7 | 7 | 7 | 7 | 7 |
| Members | 190492 | 183553 | 185445 | 192295 | 201587 |
| Contracts | 25868 | 250460 | 253900 | 261726 | 275833 |
| Assets | $28.874,80$ | $32.790,10$ | $36.200,00$ | $40.185,00$ | $45.245,50$ |

Table 2. Key indicators of the development of voluntary pension funds in Serbia
(Source: National Bank of Serbia)
Based on the information from Table 2 can be observed positive trends in the growth of the Fund's net assets as well as in the number of beneficiaries. The influence of various factors in society on the development of pension funds, as well as me the possibility of predicting development in this domain is the subject of a number works from different countries and parts of the world, which we are founded our research in this paper (see [1], [2], [3], [11], [12], [4], [5]).

Pay as you go financing system can function well if the national economy is on the rise and when the number of employees is significantly higher than the number of pensioners. If there is no economic sustainability of the public pension fund, financed according to the pay as you go principle, the state inevitably intervenes as a financier using general budget funds, and if they are insufficient, it uses special taxes on tobacco, alcohol, gasoline, luxury goods, etc. (cee [8])Private pension funds function as a fully funded financing system, often referred to as a capital accumulation system or a system of capitalized funds. Basically, the amount of pension compensation depends on the amount of accumulated premiums (contributions) and the return on invested premiums (contributions) (see [9], [10]).

## 4. Main Results

In this paper, we apply discrete mean square approximation and we use data from the Table 3 , that shows the values of the average salary in Serbia, the value of parameter FONDEX, as well as the number of fund members in a period of 5 years (2015-2019).

| Year | Average sallary | FONDEX | Fund members |
| :---: | :---: | :---: | :---: |
| 2019 | $54.908,25$ | 3064,86 | 201587 |
| 2018 | $49.642,59$ | 2862,92 | 192295 |
| 2017 | $47.887,67$ | 2713,39 | 185445 |
| 2016 | $46.836,75$ | 2592.50 | 183553 |
| 2015 | $44.436,50$ | 2407.45 | 190490 |

Table 3. Data for the Republic of Serbia for the period 2015-2019
We first determine simple approximation function that gives the dependence of the number of fund members on the average salary, that is most important parameter for determining number of potential fund members. After that, we determine dependence of the number of fund members on both the average salary and the FONDEX, that should be more accure and precise approximation, because it connects two independent values (average salary and FONDEX). Obtained dependencies are given in the next subsections, together with corresponding prediction tables, containing prediction of fun members in accordance with obtained formula.

### 4.1. Approximation formula 1

Approximation formula, based on the data in Table 3, describes relation between the number of fund members (dependent variable $y$ ), and the average salary (independent variable $x$ ). We will calculate parameters in appropriate approximation function $\varphi_{1}(x)=a+b x$, using mean square method. We will start from the initial condition, that the error of approximation in the nodes is equal to zero, ie. that

$$
\begin{align*}
& \varphi_{1}(54.908,25)=a+b \cdot 54.908,25=201587 \\
& \varphi_{1}(49.642,59)=a+b \cdot 49.642,59=192295 \\
& \varphi_{1}(47.887,67)=a+b \cdot 47.887,67=185445  \tag{20}\\
& \varphi_{1}(46.836,75)=a+b \cdot 46.836,75=183553 \\
& \varphi_{1}(44.436,50)=a+b \cdot 44.436,50=190490
\end{align*}
$$

If we transform this system into a matrix form, we have

$$
\left[\begin{array}{ll}
1 & 54.908,25  \tag{21}\\
1 & 49.642,59 \\
1 & 47.887,67 \\
1 & 46.836,75 \\
1 & 44.436,50
\end{array}\right] \cdot\left[\begin{array}{l}
a \\
b
\end{array}\right]=\left[\begin{array}{l}
201587 \\
192295 \\
185445 \\
183553 \\
190490
\end{array}\right]
$$

and we can solve it, using

$$
\begin{align*}
& A \cdot \vec{x}=\vec{b} / A^{T} \cdot \Rightarrow A^{T} \cdot A \cdot \vec{x}=A^{T} \cdot \vec{b} \quad \Rightarrow \quad\left(A^{T} \cdot A\right) \cdot \vec{x}=A^{T} \cdot \vec{b} /\left(A^{T} \cdot A\right)^{-1}  \tag{22}\\
& \vec{x}=\left(A^{T} \cdot A\right)^{-1} \cdot A^{T} \cdot \vec{b}=\left[\begin{array}{c}
-790,405006 \\
3.915,118438
\end{array}\right] \tag{23}
\end{align*}
$$

So approximation function for this case is

$$
\begin{equation*}
\varphi_{1}(x)=-790,405006+3.915,118438 \cdot x \tag{24}
\end{equation*}
$$

Using obtained function (24), we can make a prediction of the number of fund members, depending on further possible changes of average salary, shown in the following table.

| Average sallary | Fund members |
| :---: | :---: |
| $50.000,00$ | 194966 |
| $52.500,00$ | 204753 |
| $55.000,00$ | 214541 |
| $57.500,00$ | 224329 |
| $60.000,00$ | 234117 |
| $62.500,00$ | 243904 |
| $65.000,00$ | 253692 |
| $67.500,00$ | 263480 |
| $70.000,00$ | 273268 |

Table 4. Prediction of the number of fund members depending on changes in the average salary
As we can see, we have some approximative values of potential number of fund members, but it is obvious that average salary is not the only important parameter that affects to increase(decrease) of fund members.

### 4.2. Approximation formula 2

More complex approximation formula is also based on the data from Table 3, but we will try to connect number of fund members (as dependent variable $z$ ), and average salary and value of FONDEX (as independent variable $x$ and $y$ ). We will apply similar procedure (mean squares method) for calculating of parameters of approximation function $f_{2}(x, y)=a+b x+c y$.

We will start from the initial condition, that error of approximation in the nodes is equal to zero, ie. that

$$
\begin{align*}
& \varphi_{2}(54.908,25)=a+b \cdot 54.908,25+c \cdot 3064,86=201587 \\
& \varphi_{2}(49.642,59)=a+b \cdot 49.642,59+c \cdot 2862,92=192295 \\
& \varphi_{2}(47.887,67)=a+b \cdot 47.887,67+c \cdot 2713,39=185445  \tag{25}\\
& \varphi_{2}(46.836,75)=a+b \cdot 46.836,75+c \cdot 2592,50=183553 \\
& \varphi_{2}(44.436,50)=a+b \cdot 44.436,50+c \cdot 2407,45=190490
\end{align*}
$$

If we transform this system into a matrix form, we have

$$
\begin{align*}
& {\left[\begin{array}{lll}
1 & 54.908,25 & 3064,86 \\
1 & 49.642,59 & 2862,92 \\
1 & 47.887,67 & 2713,39 \\
1 & 46.836,75 & 2592,50 \\
1 & 44.436,50 & 2407,45
\end{array}\right] \cdot\left[\begin{array}{l}
a \\
b \\
c
\end{array}\right]=\left[\begin{array}{l}
201587 \\
192295 \\
185445 \\
183553 \\
190490
\end{array}\right]}  \tag{26}\\
& A \cdot \vec{x}=\vec{b} / A^{T} \cdot \Rightarrow A^{T} \cdot A \cdot \vec{x}=A^{T} \cdot \vec{b} \quad \Rightarrow \quad\left(A^{T} \cdot A\right) \cdot \vec{x}=A^{T} \cdot \vec{b} /\left(A^{T} \cdot A\right)^{-1}  \tag{27}\\
& \vec{x}=\left(A^{T} \cdot A\right)^{-1} \cdot A^{T} \cdot \vec{b}=\left[\begin{array}{l}
-600,7212221 \\
9.158,414506 \\
-93,69044454
\end{array}\right] \tag{28}
\end{align*}
$$

So approximation function for this case is

$$
\begin{equation*}
\varphi_{1}(x)=-600,7212221+9.158,414506 \cdot x-93,69044454 \cdot y \tag{29}
\end{equation*}
$$

Using the obtained function (29), we can make a prediction of the number of fund members, depending on possible changes of the average salary and a changes of parameter FONDEX, shown in the following table.

| Av.sallary | FONDEX | Memb. | Av.sallary | FONDEX | Memb. | Av.sallary | FONDEX | Memb. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50.000 | 3.000 | 176249 | 52.500 | 3.000 | 199145 | 55.000 | 3.000 | 222041 |
| 50.000 | 3.050 | 171564 | 52.500 | 3.050 | 194460 | 55.000 | 3.050 | 217356 |
| 50.000 | 3.100 | 166880 | 52.500 | 3.100 | 189776 | 55.000 | 3.100 | 212672 |
| 50.000 | 3.150 | 162195 | 52.500 | 3.150 | 185091 | 55.000 | 3.150 | 207897 |
| 50.000 | 3.200 | 157511 | 52.500 | 3.200 | 180407 | 55.000 | 3.200 | 203303 |
| 50.000 | 3.250 | 152826 | 52.500 | 3.250 | 175722 | 55.000 | 3.250 | 198618 |
| 50.000 | 3.300 | 148142 | 52.500 | 3.300 | 171038 | 55.000 | 3.300 | 193934 |
| 50.000 | 3.350 | 143457 | 52.500 | 3.350 | 166353 | 55.000 | 3.350 | 189249 |
| 50.000 | 3.400 | 138772 | 52.500 | 3.400 | 161669 | 55.000 | 3.400 | 184565 |
| 50.000 | 3.450 | 134088 | 52.500 | 3.450 | 156984 | 55.000 | 3.450 | 179880 |
| 50.000 | 3.500 | 129403 | 52.500 | 3.500 | 152299 | 55.000 | 3.500 | 175196 |

Table 5a. Prediction of the number of fund members depending on average salary and FONDEX

| Av.sallary | FONDEX | Memb. | Av.sallary | FONDEX | Memb. | Av.sallary | FONDEX | Memb. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 57.500 | 3.000 | 244937 | 60.000 | 3.000 | 267833 | 62.500 | 3.000 | 290729 |
| 57.500 | 3.050 | 240252 | 60.000 | 3.050 | 263148 | 62.500 | 3.050 | 286044 |
| 57.500 | 3.100 | 235568 | 60.000 | 3.100 | 258464 | 62.500 | 3.100 | 281360 |
| 57.500 | 3.150 | 230883 | 60.000 | 3.150 | 253779 | 62.500 | 3.150 | 276675 |
| 57.500 | 3.200 | 226199 | 60.000 | 3.200 | 249095 | 62.500 | 3.200 | 271991 |
| 57.500 | 3.250 | 221514 | 60.000 | 3.250 | 244410 | 62.500 | 3.250 | 267306 |
| 57.500 | 3.300 | 216830 | 60.000 | 3.300 | 239726 | 62.500 | 3.300 | 262622 |
| 57.500 | 3.350 | 212145 | 60.000 | 3.350 | 235041 | 62.500 | 3.350 | 257937 |
| 57.500 | 3.400 | 207461 | 60.000 | 3.400 | 230357 | 62.500 | 3.400 | 253253 |
| 57.500 | 3.450 | 202776 | 60.000 | 3.450 | 225672 | 62.500 | 3.450 | 248568 |
| 57.500 | 3.500 | 198092 | 60.000 | 3.500 | 220988 | 62.500 | 3.500 | 243884 |

Table 5b. Prediction of the number of fund members depending on average salary and FONDEX

| Av.sallary | FONDEX | Memb. | Av.sallary | FONDEX | Memb. | Av.sallary | FONDEX | Memb. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 65.000 | 3.000 | 313625 | 67.500 | 3.000 | 336521 | 70.000 | 3.000 | 359417 |
| 65.000 | 3.050 | 308940 | 67.500 | 3.050 | 331836 | 70.000 | 3.050 | 354732 |
| 65.000 | 3.100 | 304256 | 67.500 | 3.100 | 327152 | 70.000 | 3.100 | 350048 |
| 65.000 | 3.150 | 299571 | 67.500 | 3.150 | 322467 | 70.000 | 3.150 | 345363 |
| 65.000 | 3.200 | 294887 | 67.500 | 3.200 | 317783 | 70.000 | 3.200 | 340679 |
| 65.000 | 3.250 | 290202 | 67.500 | 3.250 | 313098 | 70.000 | 3.250 | 335994 |
| 65.000 | 3.300 | 285518 | 67.500 | 3.300 | 308414 | 70.000 | 3.300 | 331310 |
| 65.000 | 3.350 | 280833 | 67.500 | 3.350 | 303729 | 70.000 | 3.350 | 326625 |
| 65.000 | 3.400 | 176149 | 67.500 | 3.400 | 299045 | 70.000 | 3.400 | 321941 |
| 65.000 | 3.450 | 271464 | 67.500 | 3.450 | 294360 | 70.000 | 3.450 | 317256 |
| 65.000 | 3.500 | 266780 | 67.500 | 3.500 | 289676 | 70.000 | 3.500 | 312572 |

Table 5c. Prediction of the number of fund members depending on average salary and FONDEX

## 5. Conclusions

In this paper we modelled the behavior and interdependence of the private pension fund members and two relevant parameters, average salary and FONDEX, which are in correlation with fund members. First approximation function represents a mathematical model of behavior, that is not relevant enough, because it takes into account only one of the factors, and that is the average salary in the country. Second approximation function represents the mathematical model of behavior, that is more relevant, because number of fund members is modelled over the average salary in country and size of FONDEX, both. This mathematical function gives a very precise picture of dependency and it gives opportunity for good prediction of growth and development of the pension system, number of fund members, and its relevance is based on interaction and influence of those two independent factors.

Using those two formulas, we calculated and estimated number of fund members, that is shown in Table $4,5 \mathrm{a}, 5 \mathrm{~b}$ and 5 c . Based on data shown in those tables (especially Tables $5 \mathrm{a}, 5 \mathrm{~b}$, and 5 c ), we can be sure about good estimation. If we analyze data in tables, we can notice direct proportion between average salary and number of fund members (increase of average salary gives increase of fund members). We can also notice that there is reverse proportion between FONDEX and number of fund members (increase of FONDEX gives decrease of fund members).

Further research and analysis should additional interesting parameters. For example structure of the population, which could affect the improvement of the performance of voluntary pension funds in Serbia.

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[^0]:    2010 Mathematics Subject Classification. Primary 41A45; Secondary 91G99, 65K05
    Keywords. (approximation, discrete mean square approximation, pension system, voluntary pension)
    Received: 27 July 2020; Accepted: 15 September 2020
    Communicated by Dragan S. Djordjević
    Research supported by MoESTD of Serbia, under grant TR-32012
    Email addresses: bane@elfak.ni.ac.rs (Branislav Randjelović), ivan.radojkovic@dunavpenzije.com (Ivan Radojković), boban.gajic@dunavosiguranje.com (Boban Gajić)

